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OPTIMAL GUIDANCE OF TORPEDOES

by

Erling Gunnar Wessel

Approved
Kjeller 16 June 1972

B Landmark
Superintendent

FORSVARETS FORSKNINGSINSTITUTT
Norwegian Defence Research Establishment
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OPTIMAL GUIDANCE OF TORPEDOES

SUMMARY

This report claims that the use of line-of-sight and collision-point guidance is not the best way to guide torpedoes in a fire control system which estimates uncertainties of the target parameters. It is shown that the hit-probability varies greatly with the angle-of-attack, and that it attains maximum and minimum at certain torpedo angles. The necessary mathematical formulas to calculate the angle-of-attack yielding maximum hit-probability are developed for any given orientation and size of the axes of the uncertainty-ellipse describing the uncertainties of the target estimate.

A practical guidance scheme called Optimal Guidance is described, which always gives a torpedo trajectory yielding the highest possible hit-probability obtainable through the use of position uncertainties. It automatically provides line-of-sight guidance when the uncertainties are large, respectively small, for distance and bearing between own ship and target. When the uncertainties of distance and bearing are fairly equal, the trajectory becomes more broadside on the target than that generally obtained from collision-point guidance.

The author considers that optimal guidance has definite advantages by comparison with conventional line-of-sight and collision-point guidance, and is confident that it should substitute these conventional guidance schemes in future fire control systems in which requirements for its application are met.

1 INTRODUCTION

In most conventional fire control systems designed for torpedo guidance the operator can choose between two automatic guidance modes: Line-of-sight guidance and collision-point guidance. The use of collision-point guidance is generally recommended when the uncertainties of both the distance and the bearing from own ship to the target are small. When the uncertainty of the bearing is small and the uncertainty of the distance is large, it is recommended to use line-of-sight guidance.

In most conventional fire control systems the uncertainties of the estimated target parameters (distance, bearing, course and speed) are not available. The operator's choice of guidance mode is therefore purely based on his intimate knowledge of the tracking-history of the target in question and on his former experience with the performance of the fire control system in similar tactical situations. It is obvious that decisions taken on such vague assumptions are not ideal, since the distance- and bearing-uncertainties can take on any values between the extremes mentioned above.

There exist, however, fire control systems (MSI-70U fire control system for Kobben class submarines) in which certain parameter-uncertainties are estimated. It is the scope of this report to investigate if it is possible to utilize the information of the parameter-uncertainties to guide torpedoes in a more optimal fashion. Or, in other words, to investigate if the knowledge of uncertainties makes it possible to find a guidance scheme leading to a higher hit-probability than that obtainable through the use of the conventional two-state choice between line-of-sight guidance and collision-point guidance.

The primary questions which must be answered in connection with such an optimalization of the torpedo hit-probability are:

a) What parameters govern the angle-of-attack of the torpedo leading to a maximum hit-probability, when the uncertainties of the relative position between the target and the torpedo are taken into account?

b) Is it possible to find a mathematical relationship between the wanted angle-of-attack and the relevant uncertainties?

c) If a mathematical relation can be found, is this relation suited for practical guidance of torpedoes?

It is the purpose of this report to answer and discuss all relevant problems in connection with these questions.

The report is divided into 8 chapters and 5 appendices, and it is written such that the basic information is contained in the chapters and the detailed mathematics in the appendices. A reader only interested in the basic outline of the optimal guidance scheme does not have to read the appendices and he may possibly also skip section 3.3 discussing some practical aspects of the employed numerical iteration.

A brief description of the content of each of the remaining chapters is given below in order to familiarize the reader with the general outline of the report:

a) Chapter 2 discusses first the general nature of the guidance problem and states the necessary assumptions for its mathematical formulation. The needed mathematical equations are developed for both the calculation of the hit-probabilities for any given angle-of-attack and for the calculation of the angle-of-attack yielding the maximum hit-probability. The chapter gives also a detailed description of the numerically calculated hit-probabilities and discusses the results.

b) Chapter 3 describes in detail how the obtained formulas for the angle-of-attack yielding the maximum hit-probability can be practically utilized to obtain a guidance scheme for torpedoes. Besides developing the necessary mathematical formulation of the guidance scheme called Optimal Guidance, practical problems in connection with the necessary numerical iteration are discussed. The optimal guidance scheme is divided into 3 separate stages, each of which is described in detail. The chapter ends with two sections discussing the prediction of the orientation and the size of the uncertainty-ellipse describing the position uncertainties of the target estimate.

c) Chapter 4 discusses some relevant salvo aspects of the developed optimal guidance scheme.

d) Chapter 5 discusses some aspects of the optimal guidance scheme which relates to problems in connection with multipass guidance or guidance after it becomes certain that hit was not obtained at last trial.

e) Chapter 6 states the assumptions made in the presented optimal guidance simulations, and discusses the results obtained from these simulations.

f) Chapter 7 makes several suggestion as to what future work should be carried out before the optimal guidance scheme can be fully recommended for practical application.

g) Chapter 8 gives a summary of the advantages that can be obtained through the use of optimal guidance.

2 CALCULATION OF OPTIMUM HIT-PROBABILITY

2.1 Assumptions

In a fire control system the position of the target is estimated relative to the current position of own ship. The positions of the torpedoes are estimated relative to the firing position by dead-reckoning. It is the relative positions of the target and the torpedoes which are of basic interest when guidance of torpedoes is considered. These positions are only known unaccurately and the primary sources of uncertainties are:

a) Uncertainties in the target position relative to own ship due to uncertainties in the target observations.

b) Uncertainties in the current own position relative to the positions when the torpedoes were fired due to errors in the dead-reckoning of own ship.

c) Uncertainties of the current position of the torpedoes relative to the positions of firing due to errors in the dead-reckoning of the torpedoes.

The estimates of the torpedoes relative to own ship are hence burdened both by uncertainties from the torpedo dead-reckoning and the own ship dead-reckoning from the time of firing.

An estimation process such as Kalman filtering provides estimates of the speed, course and position of the target relative to the current position of own ship. These estimates will later in this report be referred to as the state vector of the target. A new state vector is normally calculated at the time of each new observation, using this current observation and an updated version of the last computed state vector, taking into account the intermediate movement of both the target estimate and own ship. The uncertainties of the current state vector parameters are therefore dependent on the history of the state vector, the uncertainties of the observations and on the dead-reckoning errors between current time and last observation time. However, when the dead-reckoning errors are negligible it is sufficient to calculate the new state vector taking into account only errors in the observations.

Estimation processes such as Kalman filtering do not only have the ability to calculate each parameter of the state vector, but also to give estimates of the uncertainties of these parameters. As stated above, the primary interest when guiding torpedoes is focused on the relative position of the target and the torpedoes and their associated uncertainties. If the uncertainties of the dead-reckoning of the torpedo and own ship were known to the fire control system and these uncertainties were normally distributed, it is in principle possible to calculate a state vector and a total uncertainty which would reflect both errors in the observations and in the dead-reckoning. It would then be possible to calculate the guidance problem as if the torpedo position was known without error and as if the target position was known as an estimated state vector with uncertainties reflecting all relevant errors. If and how this could be practically achieved is considered to be outside the scope of this report.

An understanding of the results of this report is not conditional on whether the total uncertainties are known or not. It is, however, of importance to have a qualitative understanding of the fact that errors in the dead-reckoning of the torpedoes and own ship might impose changes in the uncertainties reflecting only errors in the observations. (Both an enlargement and a rotation of the uncertainty-ellipse described later, will result.) In every well-adjusted fire control system the influence of the dead-reckoning errors should be negligible, and the remaining part of this report should preferably be understood as if that is the case. In the considerations to follow, a Kalman filter is utilized to give estimates of target position, speed and course as visualized in Figure 2.1, together with an estimate of the uncertainty of these parameters. The probability-density function of the estimated target position is generally a complex function of both x and y (the origin of the x,y-coordinate system is located at the estimated target position as shown in Figure 2.1), but is reducible to a simple Gaussian form under most practical tactical situations

$$p(x,y) = \frac{1}{2\pi ab} \cdot \mathrm{EXP}\{-\tfrac{1}{2}((\tfrac{x}{a})^2 + (\tfrac{y}{b})^2)\} \qquad (2.1)$$

Where a and b are the standard deviation of the position in respectively the x- and y-directions. The simple form given in equation (2.1) is valid when a and b are small in comparison with the distance R between the target estimate and own ship.

The density function p(x,y) is constant when x and y describe concentric ellipses with centers located at the estimate position. The term UNCERTAINTY-ELLIPSE refers to the ellipse which has the standard deviations a and b as major and minor half-axes. The probability that the target-center (or any other specific point on the target to which the target-tracking refers) actually is located inside this uncertainty-ellipse is approximately 50 per cent.

The probability of finding the actual target center within a differential area dx·dy at a location (x,y) is given by

$$P(x,y) = \frac{1}{2\pi ab} \text{EXP}\{-\tfrac{1}{2}((\frac{x}{a})^2 + (\frac{y}{b})^2)\}dx\cdot dy \qquad (2.2)$$

The hit-probability is thus found by a summation (integration) of P(x,y) over the total area where hit is possible between the torpedo and the target.

The calculations shown in the next section are performed under the assumptions that orientation of the uncertainty-ellipse and the size of the half-axes a and b, do not change from the time of calculation until the time of hit with the estimate. In most practical applications the change in these parameters will be negligible. However, it is primarily the use of repetitive calculation of the relevant guidance parameters which justifies a calculation using these assumptions. Any changes in the orientation and/or the size of the axes of the uncertainty-ellipse will hence be reflected in slightly changed guidance parameters. This point will also be discussed in chapter 3 when the problem of torpedo guidance is treated.

The target is assumed to have the shape of an ellipse with half-axes a_1 and b_1. The length of the target is thereby $2a_1$ and the width $2b_1$. It is considered that this gives a very good approximation to the actual outline of the target.

The torpedo will be considered to be a mathematical point in the calculations in the next section. This should describe the torpedo very well, particularly when the torpedoes are furnished with impact fuses.

2.2 Calculation of optimum hit-corridor

Hit between a torpedo and the target becomes possible, as visualized in Figure 2.2, when the target center is actually located on the periphery of an ellipse having its center at the location of the torpedo. This ellipse is oriented with its major axis along the estimated course and the axes are equal to that of the target ($2a_1$ and $2b_1$). It is thus assumed that the real course of the target is equal to that of the estimate. (The implication of uncertainties in the estimated course will be discussed in section 3.9 dealing with prediction of the uncertainty-ellipse.)

The calculations given later are based on the following assumptions:

a) No error in estimated target course

b) The torpedo runs on a straight course with constant speed V_T

c) The estimate of the target runs on a straight course with constant speed V_S

d) The torpedo is guided before the integration starts on a course which leads to collision with the estimate of the target.

e) The torpedo runs at a proper depth, allowing hit with the target in question

Figure 2.3 shows a straight-running torpedo which will pass through the estimate at location S_o. The assumptions stated above will as indicated in Figure 2.3, assure that the angle Φ between the estimated target course and the bearing from the estimate to the torpedo, will not change with time. A consideration of the movements of the estimate- and torpedo-positions in a stationary coordinate system, can be reduced to a consideration of the relative movement between the torpedo and the estimate by a transfer to a moving coordinate system with its origin located at the estimate. Figure 2.4 shows the situation of Figure 2.3 when only relative movements between the torpedo and the estimated target position are considered. All possible locations of the target center with a possibility of hit by the torpedo will, as time elapses, cover an area designated as the HIT-CORRIDOR (TREFF-GATE). To avoid misinterpretations it must be clearly understood that the mid-line of this hit-corridor does not describe the actual movement of the torpedo, but only the movement of the torpedo relative to the estimate. As it will be shown in section 2.5 the actual angle-of-attack (angle C in Figures 2.3 and 2.4) is both dependent on the corridor angle Φ and the ratio of the target- and torpedo-speeds.

In Appendix 1 it is shown that in the X-Y coordinate system of Figure 2.5, the borderlines of the hit-corridor are given by the following equations

$$y = mx \pm n \tag{2.3}$$

where

$$m = tg\,(\Phi+\theta) \tag{2.4}$$

and

$$n = \sqrt{(a_1 \sin\Phi)^2 + (b_1 \cos\Phi)^2} \Big/ \cos(\Phi+\theta) \tag{2.5}$$

The total probability of hit, as the torpedo moves in a straight course as prescribed earlier, is given by combining equations (2.2) and (2.3), yielding

$$P = \frac{1}{2\pi ab} \int_{-\infty}^{\infty} \int_{mx-n}^{mx+n} \text{EXP}\{-\tfrac{1}{2}((x/b)^2 + (y/b)^2)\}dydx \tag{2.6}$$

The goal is to find the value of the hit-corridor angle Φ which yields the maximum probability of hit given the axes of the target-ellipse a_1 and b_1, the axes of the uncertainty-ellipse a and b, and the inclination angle θ between major axis a of the uncertainty-ellipse and the estimated target course. This optimum hit-corridor angle can be found by setting the derivative of the hit-probability equal to zero

$$\frac{\partial P}{\partial \Phi} = 0 \tag{2.7}$$

As shown in Appendix 2 the solution to this equation is

$$m_{12} = A \pm \sqrt{A^2 + C^2} \tag{2.8}$$

where

$$A = \{(D^2-C^2)\cot g\theta + (1-D^2C^2)tg\theta\}/2(1-D^2) \quad (2.9)$$

and

$$D = b_1/a_1 \quad (2.10)$$

$$C = b/a \quad (2.11)$$

$$m = tg(\Phi+\theta)$$

The development of equation (2.8) from (2.6) was done under the pretension that the hit corridor was infinitely long. Or more precisely that the corridor extended in both directions to a region where the probability of hit was zero for all practical purposes. However, in a practical utilization of the hit-corridor yielding the maximum hit-probability for torpedo guidance purposes, the optilization of the angle-of-attack must also be achieved from a distance to the target which is not infinite in the sense stated above. A closer reasoning (which will not be given in this report) will show that no alteration of the optimum angle-of-attack should be expected when the integration is started at a point closer to the target than that which corresponds to infinity. This cannot, unfortunately, be proven analytically because the equation corresponding to equation (A2.7) will yield a summation of analytic terms of the form

$$\int_{f+N}^{g+N} u\cdot EXP(-u^2/2\sigma^2)du$$

and non-analytic terms of the form

$$\int_{f+N}^{g+N} EXP(-u^2/2\sigma^2)du$$

where f and g are the non-infinite integration limits. N is given by equation (A2.6).

The proof that equation (2.8) can be used for calculation of the optimum hit-corridor angle Φ also in the cases when the integration limits are non-infinite, will therefore be given numerically. The results of such a numerical calculation will be discussed in sections 2.3 and 2.4.

There are however certain considerations which must be taken into account when such an integration is performed. The mathematically correct procedure would be to terminate the ends of the hit-corridor by the outline of a semi-ellipse corresponding to the tilt and the size of the target-ellipse. This procedure would add complexity to the integration-process, and it is found to have no practical implication on the results of the integration when some other termination of the hit-corridor is used. The most simple procedure would be to utilize equation (2.6) with non-infinite integration-limits. This yields, however, quite unwanted results of the corridor-termination when Φ is approximately equal to $-\theta$. The actual termination utilized in the numerical calculations with non-infinite integral limits, is therefore a termination which always makes the hit-corridor a rectangle with 90 degree corners, as shown in Figure 2.5. The probability of hit is therefore calculated from

$$P = \frac{1}{2\pi ab} \cdot \int_{f}^{g} \int_{-k}^{k} \mathrm{EXP}\{-\tfrac{1}{2}((x/a)^2 + (y/b)^2)\}du\, dv \qquad (2.12)$$

and according to Appendix 1

$$k = \sqrt{(a_1 \sin\Phi)^2 + (b_1 \cos\Phi)^2} \qquad (2.13)$$

and

$$x = v \cdot \cos(\Phi+\theta) - u \cdot \sin(\Phi+\theta)$$

$$y = v \cdot \sin(\Phi+\theta) + u \cdot \cos(\Phi+\theta)$$

The integration is also performed with the upper integration limit g equal to infinity, since no valid argument has been found to dictate a premature termination of the integration before the probability of hit is reduced to (practically) zero.

2.3 Numerically calculated hit-propability

Most fire control systems will use observations from one or more sensors which can measure the bearing to a target with high accuracy. Sensors having standard deviations on the bearing observations of less than 1 degrees are frequently used. It is expected that future fire control systems will utilize sensors yielding accuracies better than a few milliradians (1 mrad $\simeq$ 0.057 degrees). Systems employing sensors with such bearing accuracies will generally obtain target position estimates with uncertainties across the bearing of a few multiples of ten meters, when the target distance is less than 10 kilometers.

The minor axis b of the uncertainty-ellipse will, in accordance with the statements given above, generally be varied between 10 and 50 meters.

The major axis a of the uncertainty-ellipse is dependent on the particular use of the different sensors, the tracking time and the particular sailing of own ship. The variations used in the numerical calculation of the major axis a are therefore much greater and extend from 10 meters to several thousand meters.

Most of the numerical results are, for easy comparison, computed for a target width of $2b_1$ = 10 meters and a length of $2a_1$ = 100 meters. This corresponds to a smaller vessel of the Frigate-class.

The following parameter symbols are used:

a) Φ = HIT-CORRIDOR ANGLE measured from estimated tgt course to centerline of corridor. (Positive c c w)

b) θ = TILT ANGLE OF UNCERTAINTY-ELLIPSE measured from estimated tgt course to major axis of ellipse (positive c w): $-90 \leq \theta \leq 90$.

c) a = MAJOR UNCERTAINTY-ELLIPSE HALF-AXIS

d) b = MINOR UNCERTAINTY-ELLIPSE HALF-AXIS

e) a_1 = MAJOR TARGET-ELLIPSE HALF-AXIS

f) b_1 = MINOR TARGET-ELLIPSE HALF-AXIS

g) C = b/a

h) D = b_1/a_1

i) P = PROBABILITY OF HIT.

Figure 2.6 shows how the HIT-PROBABILITY varies for different tilt angles θ, when the corridor angle Φ is varied from -90 to +90 degrees. It is complete 180 degrees symmetry such that

$$P(180+\Phi) = P(\Phi)$$

Figures 2.7 and 2.8 show hit-probabilities for similar configurations to that of Figure 2.6 with different values of the major axis of the uncertainty-ellipse.

In order to give the reader a thorough insight into how the probability of hit varies when the uncertainty ellipse changes its size and orientation, Figure 2.9 through Figure 2.19 have been included. In each separate figure the tilt angle θ and one of the ellipse-axes is varied. Figure 2.18 makes an exception in that the ellipse-axis b, which is in all other figures the minor axis, in this figure is varied from a value smaller, to a value greater than the a-axis. When the b-axis takes on the role as the major axis the maximum hit-probability will not be at $\Phi = -\theta$, but at the complementary angle to the tilt angle

$$\Phi = 90 - \theta = 90 - 22.5 = 67.5 \text{ degrees}$$

Figure 2.19 shows a calculation of the corridor angle Φ for which the probability of hit is maximum and minimum (based on equation (2.8)), when the tilt angle θ is varied from 0 to 90 degrees. The calculation is shown for different ratios of the minor and major ellipse-axes $C = b/a$. The symmetry in this figure is such that

$$\Phi(-\theta) = -\Phi(\theta)$$

Figures 2.20, 2.21 and 2.22 are included to familiarize the reader with the change in hit probability when the size of the target is changed.

In all figures from Figure 2.6 to Figure 2.22 the integration limits (f and g) have been greater than 4 times the major axis of the uncertainty ellipse. This corresponds to infinite integration limits, since the probability of hit outside these limits is negligible.

Figures 2.23, 2.24 and 2.25 show how the hit-probability is changed when the lower integration limit f is changed from infinite to non-infinite values. The results are shown for three different values of the major ellipse-axis a.

2.4 Discussion on calculated results

The discussion to follow can only be understood in light of the assumptions made in sections 2.1 and 2.2.

A close study of Figure 2.6 through Figure 2.25 will among others reveal the following conclusions:

a) There exist in general distinct corridor angles Φ for which the probability of hitting the target with a torpedo is both minimum and maximum.

b) For infinite integration limits ($f,g \geq 4 \cdot a$) these corridor angles can be found from the relations

$$\Phi_1 = -\theta + \text{arctg}(A-\sqrt{A^2+C^2}) \qquad (2.14)$$

and

$$\Phi_2 = -\theta + \text{arctg}(A+\sqrt{A^2+C^2}) \qquad (2.15)$$

where

$$A = \{(D^2-C^2)\ \text{cotg}\theta + (1-D^2C^2)\text{tg}\theta\}/2\cdot(1-D^2)$$

$$C = b/a$$

$$D = b_1/a_1$$

When $0 \leq \theta \leq \pi/2$, Φ_1 will give the corridor angle leading to maximum hit-probability, and Φ_2 to minimum hit-probability. When $0 \geq \theta \geq -\pi/2$, Φ_1 will yield minimum and Φ_2 maximum hit-probabilities.

c) For $|\theta| > 5$ degrees and $C < 0.2$ the following approximations are valid

$$\Phi_{max} = -\theta$$

$$\Phi_{min} = 0$$

In other words the maximum hit-probability is obtained when the corridor runs parallel with the major axis of the uncertainty-ellipse. The minimum hit probability is obtained when the hit-corridor is aligned with the major axis of the target-ellipse.

d) When $4b < a_1$ and $|\theta| > 20$ degrees there exists a maximum hit-probability of approximately 100%, and a corresponding minimum hit-probability which rarely exceeds 20%.

e) The ratio between maximum and minimum hit-probability is increasing with decreasing $C = b/a$.

f) The width of the maximum (measured at any convenien percentage of the maximum value) is decreased with decreasing $C = b/a$. (It becomes in other words more critically dependent on the values of θ, C and D).

g) Orientations of the uncertainty-ellipse having small θ-values ($|\theta| < 20$ degrees) will in general yield lower maximum hit-probabilities than those orientations having larger θ-values ($|\theta| > 20$ degrees).

h) As $C = b/a$ is increased and approaching unity the angle of maximum hit-probability is shifted towards larger absolute values, i e the maximum is shifted more towards a corridor angle hitting broadside on the target.

i) In accordance with point c, the angular spacing between the maximum and the minimum is approximately equal to θ. For decreasing absolute value of θ, this means that the maximum and the minimum become closer together.

j) Equations (2.14) and (2.15) can be used to compute the corridor-angles leading to maximum and minimum hit-probabilities also when the lower integration limit is non-infinite. (If locking distances of more than 50 meters are employed these formulas will still very faithfully compute the actual values).

The practical implications of these conclusions will be discussed in the following sections.

2.5 Calculation of actual torpedo-angle

As stated earlier the hit-corridor angle Φ is only descriptive of the relative motion between the torpedo and the estimate. It is the task of this section to find the relationship between the motion described by the corridor and the actual torpedo motion.

From Figure 2.26 it can be found that

$$OD = V_S T \sin\Phi$$

and

$$BD = DC = \sqrt{(V_T T)^2 - (V_S T \sin\Phi)^2}$$

Or if

$$N = V_S/V_T \tag{2.16}$$

$$BD = CD = V_T T \cdot \sqrt{1-N^2 \sin^2\phi}$$

Thus

$$tg\alpha = \frac{OD}{BD} = \frac{OD}{DC} = \frac{V_S T \sin\Phi}{V_T T \sqrt{1-N^2 \sin^2\phi}}$$

$$\alpha = \arctg \{\frac{N \sin\Phi}{\sqrt{1-N^2 \sin^2\phi}}\} \tag{2.17}$$

where

$$-\pi/2 \leq \alpha \leq \pi/2$$

Also from Figure 2.26 it is obtained that

$$\alpha = \Phi - \Delta C_1 = \pi - \Phi - \Delta C_2$$

and

$$\Delta C_1 = C - C_1$$

$$\Delta C_2 = C_2 - C$$

where C is the estimated target course and C_1 and C_2 the two possible solutions of the actual torpedo courses.

Hence

$$\Delta C_1 = \Phi - \alpha \tag{2.18}$$

$$\Delta C_2 = \pi - \Phi - \alpha \tag{2.19}$$

Or

$$C_1 = C - \Phi + \alpha \tag{2.20}$$

$$C_2 = C + \pi - \Phi - \alpha \tag{2.21}$$

Given a wanted corridor angle Φ (such as that leading to maximum hit-probability) there exist in general two torpedo angles given by equations (2.20) and (2.21) which will yield the desired corridor.

2.6 Discussion of obtainability of the wanted hit-corridor

There are some limitations and points of interest in relation to the practical obtainability of a wanted hit-corridor. Figures 2.27 through 2.31 are designed to demonstrate these points:

a) At every instant the centerline of any hit-corridor will divide the sea into two regions. The two possible solutions obtainable from equations (2.20) and (2.21) will always be located in that region in which the centerline was located at an earlier instant.

b) When $V_T > V_S$, there always exist two torpedo-courses corresponding to any given corridor angle Φ.

c) When $V_T < V_S|\sin \Phi|$, no solution can be found for that particular wanted corridor angle Φ.

d) When $V_T = V_S$, one solution of the torpedo-course will be coincident with the target course. Such a solution is only of mathematical interest since the distance between the torpedo and the estimate necessarily is constant under these circumstances and no real hit-corridor is produced. (It can be shown that in the context of chapter 3, this means that the "optimal point" will be located infinitely far away from the estimate, i e $A=\infty$.)

e) The calculated wanted torpedo-course becomes less dependent on the knowledge of the actual target-course as the corridor angle becomes smaller, i e larger errors in the target velocity V_S can be tolerated at smaller corridor angles.

The next chapter will discuss how the results obtained in this chapter can be utilized in practical guidance of torpedoes.

3 OPTIMAL TORPEDO GUIDANCE

3.1 General

This chapter will develop a scheme to be used for guidance of torpedoes such that the wanted hit-corridor discussed in the former chapters will be obtained. As it turns out to be neither line-of-sight nor collision-point guidance, the guidance scheme to be developed will be called "Optimal guidance", since it assures an optimal torpedo trajectory toward the target. The development of the appropriate guidance equations are based on the following assumptions:

a) That the estimate- and torpedo-positions and -courses are known at the time of calculation. (Exemplified by X,Y,C and X_o, Y_o, C_o respectively in Figure 3.1.)

b) That the tilt angle θ of the uncertainty-ellipse is known at the time of calculation. (For prediction of θ see section 3.8.)

c) That the wanted optimum hit-corridor-angle Φ can be calculated from equations (2.14) and (2.15).

d) That there exist solutions for the two possible torpedo courses C_1 and C_2 using equations (2.20) and (2.21).

e) That it is desirable to guide the torpedo from its present position to an "Optimal point" (X_3, Y_3 in Figure 3.1) in order to control the starting point of the hit-corridor. The torpedo course in this optimal point (angle C_3 in Figure 3.1) must be such that if the torpedo is to receive no further steering information from this point on, it will continue on a course yielding the wanted hit-corridor. The optimal point must be located such that it corresponds to a location where there is negligible probability of hit.

f) That the torpedo trajectory between the current location and the optimal point in general will consist of two turns employing minimum turn radius separated by a straight trajectory. The choice of two turns and one straight is dictated by the necessity of guiding the torpedo from its present position where the torpedo course is given, to some other point (here the optimal point) at which the torpedo course must have a desired value. The use of two turns separated by a straight path is the minimum number of turns and straights and hence the minimum number of unknowns in the corresponding mathematical formulation, by which this can be achieved. When the turns employ minimum turn radius it is also assured that the trajectory becomes the shortest possible under the given requirements.

g) That the relevant guidance parameters can be calculated at any chosen repetition-rate and at any intermediate instances between two repetitive calculation times.

h) Though not strictly necessary, the guidance scheme is developed to incorporate prediction of the tilt angle θ at some convenient future time. (Section 3.8 discusses θ-prediction in detail.)

Using these assumptions the following section develops the mathematical formulation of the optimal guidance scheme.

3.2 The mathematical formulation of the optimal guidance scheme

Given the wanted hit-corridor angle Φ such that

$$-\pi/2 \leq \Phi \leq \pi/2$$

The corresponding torpedo angles C_6 and C_7 can be found as shown in chapter 2 by calculating

$$\alpha = \text{arctg} \left\{ \frac{N\sin\Phi}{\sqrt{1-N^2\sin^2\Phi}} \right\}$$

From which

$$C_6 = C - \Phi + \alpha \qquad (3.1)$$

$$C_7 = C + \pi - \Phi - \alpha \qquad (3.2)$$

Solutions will be found whenever

$$(N\sin\Phi)^2 \leq 1$$

and the torpedo angles must be adjusted such that

$$0 \leq C_6, C_7 < 2\pi$$

As it was stated in the last section it is desirable to locate the optimal point at the starting point of the hit-corridor, i e at a point where the probability of hit is still negligible. For a wanted torpedo-course C_3 given by equation (3.1) or (3.2), Appendix 3 shows that the optimal distance "E" and the distance "A" between the optimal point and the hit point can be calculated from

$$E = 3 \cdot a \qquad (3.3)$$

and

$$A = \max\{A1, \left|\frac{E \cos \alpha}{1-N\cdot\cos(C_3-C)}\right|\} \tag{3.4}$$

where "a" is the major half-axis of the uncertainty-ellipse, "A1" a convenient fixed distance and "C_3" and "C" respectively the wanted torpedo-course and the estimate-course.

The mathematical formulation of the optimal guidance scheme employs for convenience a parameter for each turn called the "turn parameter". As it is shown in detail in the beginning of Appendix 4, the absolute values of these turn parameters are always unity, whereas its sign is used to describe whether a given torpedo turn is a left turn or a right turn. The convention for the turnparameter K is such that

K = +1 for a right torpedo turn
K = -1 for a left torpedo turn

The following notation (see Figure 3.1) is used for the development of the necessary mathematical formulas

X, Y	=	estimate coordinates
C	=	estimate course
X_o, Y_o	=	torpedo coordinates
C_o	=	torpedo course
β_1	=	first turn angle
K_1	=	turn parameter for the first turn
β_2	=	second turn angle
K_2	=	turn parameter for the second turn
C_1	=	course of straight path between turns
C_3	=	wanted torpedo course at optimal point
$N = V_S/V_T$	=	ratio of target and torpedo speeds
R	=	minimum turn radius
A	=	distance between optimal point and collision point

Using this notation it is shown in Appendix 4 that the principle unknowns are

K_1, K_2 and C_1

and that, given the values of K_1 and K_2, the value of C_1 can be solved from the following equation

$$F(C_1) = D_3\sin(C-C_1)-NRK_1(\cos(C-C_1)-\cos(C-C_o)) + NRK_2(\cos(C-C_1)-\cos(C-C_3))$$
$$-K_1R(\cos(C_o-C_1)-1)+K_2R(\cos(C_3-C_1)-1)-Q_1\cos C_1+P_1\sin C_1+ND_5 = 0 \qquad (3.5)$$

where

$$D_3 = N(A+(\beta_1+\beta_2)R) = N(A+(K_1(C_1-C_o)+Z_1+K_2(C_3-C_1)+Z_2)R) \qquad (3.6)$$

$$P_1 = Y_o-Y + A\cos C_3 \qquad (3.7)$$

$$Q_1 = X_o-X + A\sin C_3 \qquad (3.8)$$

$$D_5 = Q_1\cos C - P_1\sin C \qquad (3.9)$$

Since D_3 is generally dependent on the value of C_1 the first term in equation (3.5) makes it impossible to obtain any values of C_1 for which

$$F(C_1) = 0$$

by analytical means.

The other parameters necessary for the guidance are:

The time required for the first turn

$$T_1 = \beta_1R/V_T = (K_1(C_1-C_o) + Z_1)R/V_T \qquad (3.10)$$

The time required for the straight trajectory

$$T_2 = \frac{P_1 - K_1 R\sin C_o + K_2 R\sin C_3 + R(K_1 - K_2)\sin C_1 - D_3\cos C}{(N\cos C - \cos C_1)V_T} \quad (3.11)$$

or

$$T_2 = \frac{Q_1 + K_1 R\sin C_o - K_2 R\cos C_3 - R(K_1 - K_2)\cos C_1 - D_3\sin C}{(N\sin C - \sin C_1)V_T} \quad (3.12)$$

The time required for the second turn

$$T_3 = \beta_2 R/V_T = (K_2(C_3 - C_1) + Z_2)R/V_T \quad (3.13)$$

The time required from the optimal point to the collision point

$$T_4 = A/V_T \quad (3.14)$$

The total time required from the current position to the collision point

$$T = T_1 + T_2 + T_3 + T_4 \quad (3.15)$$

Before giving the general outline of the practical optimal guidance, the next section will discuss some practical problems connected with the iteration process which must be employed in the calculations.

3.3 Some practical aspects of the numerical iteration

Certain precautions must be incorporated before equation (3.5) becomes suitable for numerical iteration. Any sinusoidal function is well suited for numerical iteration by the use of the Newton-Raphson method (see reference such as C E Frøberg: Lärobok i numerisk analys, page 16), in which the (n+1)'th term is obtained from the n'th term using the following formulas

$$C_{1,n+1} = C_{1,n} - \Delta C_{1,n} \quad (3.16)$$

where

$$\Delta C_{1,n} = F(C_{1,n})/F'(C_{1,n}) \quad (3.17)$$

The use of these formulas requires that $F(C_1)$ and $F'(C_1)$ (the derivative with respect to C_1) must be continuous between $C_{1,n}$ and $C_{1,n+1}$.

$F(C_1)$ as given by equation (3.5) is a modified sinusoid, which is also well suited for iteration by the above mentioned method, provided $F(C_1)$ and $F'(C_1)$ are continuous for all values of C_1.

If both the first and second turn (β_1 and β_2) are allowed to take on any value between 0 and 2π radians, it is proved in Appendix 5 that $F(C_1)$ and $F'(C_1)$ are continuous for all values of C_1, provided that K_1 changes sign at $C_1 = C_o$ and K_2 changes sign at $C_1 = C_3$. These requirements are quite natural since a change of a turn parameter is the mathematical means by which a left turn is transformed into a right turn of equal magnitude. This is exactly what is needed to approximately maintain the same solution when C_1 is made to pass either C_o or C_3 in the process of iteration.

As stated in Appendix 5 the change of K_1 and K_2 at respectively $C_1 = C_o$ and $C_1 = C_3$ implies the following natural definitions

$$K_1 \text{ for } C_o > C_1 \geq 0 \quad \text{and} \quad -K_1 \text{ for } 2\pi > C_1 \geq C_o \tag{3.18}$$

and

$$K_2 \text{ for } C_3 > C_1 \geq 0 \quad \text{and} \quad -K_2 \text{ for } 2\pi > C_1 \geq C_3 \tag{3.19}$$

since C_1 is restricted in the following manner

$$2\pi > C_1 \geq 0 .$$

Such a definition of K_1 and K_2, implies that K_1 and K_2 also change sign at $C_1 = 0$. This is an unwanted effect which must be avoided in order to maintain $F(C_1)$ and $F'(C_1)$ continuous at $C_1 = 0$.

To select the solution best suitable for torpedo-guidance purposes, all possible solutions must be found and compared. There are hence 4 different combinations of K_1 and K_2 which need investigation: 2 right turns, 2 left turns, left and right turn, and right and left turn. For each pair of values of K_1 and K_2 it must be assured that all possible values of C_1 acquiring $F(C_1) = 0$ can be found. It becomes therefore necessary to start the iteration at several values of C_1, i e at

$$C_1 = \varepsilon, 2\varepsilon, 3\varepsilon, \ldots\ldots, 2\pi$$

However, when such a scheme is employed, it also becomes certain that any iteration leading to a value of $\Delta C_{1,n}$ in equation (3.16) for which

$$|\Delta C_{1,n}| \geq \varepsilon \qquad (3.20)$$

will be found from some other starting-point of the iteration. It is hence logical to make the iteration procedure test for values of $\Delta C_{1,n}$ using equation (3.20) and if found to interrupt the iteration and go to the next start value of the iteration. The author has found by careful examination of the employed iteration process, that a value of $\varepsilon = \pi/2$ radians gives a desired compromise between a large enough value of ε to reduce the number of iterations and a small enough value of ε to assure that all possible solutions are found. Hence it is recommended that the iteration is started at

$$C_1 = 0, \pi/2, \pi, 3\pi/2 \text{ and } 2\pi \text{ radians}$$

when all solutions are to be investigated.

By employing these starting points, the solution of C_1 is found to an accuracy of 0.1 degree in most circumstances using from 2 to 4 iterations.

The definition of K_1 and K_2 stated in equations (3.18) and (3.19) respectively will be employed except when C_1 passes 0 and hence the starting points of iteration at $C_1 = 0$ and $C_1 = 2\pi$ will not reveal the same solution. It can not be assured either that some solution found by starting at $C_1 = 0$ for one combination of K_1 and K_2 is found by the starting of $C_1 = 2\pi$ for some other combination of K_1 and K_2.

Since there are two optimal torpedo courses as given by equations (2.20) and (2.21), 4 combinations of K_1 and K_2 and 5 starting points for the iteration, a total of 40 iterations must be undertaken in order to investigate all possible solutions. In any real time system employing torpedo guidance the time consumption is of vital interest and every scheme in which the number of iterations can be cut down will be advantageous provided it does not gravely degrade the total performance of the guidance systems.

In the search for some means to cut down the number of required iterations the author has tried a method in which the first turn angle is not directly incorporated in equation (3.5), which is achieved by letting $\beta_1 = K_1 = Z_1 = 0$ in equation (3.5). The straight trajectory angle C_1 is then calculated as if the torpedo could take on the angle C_1 instantaneously. The actual first turn β_1 is then calculated from the present torpedo course C_o and the wanted torpedo course C_1. The torpedo guidance is carried out by repetitive calculation of C_1 and corresponding turning until $C_o = C_1$. Such a guidance scheme cannot fully be recommended since the total time to collision with the target estimate required to find the best solution will be inaccurate. The advantage of the scheme is of course that the number of iterations is reduced to 20 since K_1 no longer enters equation (3.5). A necessary consequence of the scheme is that β_1 must be restricted to a value not exceeding π radians.

Another method which also will yield 20 iterations totally to find all possible solutions available, is one which uses the formula for C_1 as given by equation (3.5), but which restricts the first turn angle β_1 such that

$$\pi \geq \beta_1 \geq 0 \qquad (3.21)$$

Such a restriction on β_1 implies the following values of K_1

For $\pi \geq C_o \geq 0$

$$K_1 = -1 \quad \text{for} \quad C_o > C_1 \geq 0 \quad \text{and} \quad 2\pi > C_1 > C_o + \pi \qquad (3.22)$$

$$K_2 = +1 \quad \text{for} \quad C_o + \pi \geq C_1 \geq C_o \qquad (3.23)$$

and for $2\pi > C_o > \pi$

$$K_1 = -1 \quad \text{for} \quad C_o > C_1 > C_o - \pi \qquad (3.24)$$

$$K_2 = +1 \quad \text{for} \quad 2\pi > C_1 \geq C_o \quad \text{and} \quad C_o - \pi > C_1 \geq 0 \qquad (3.25)$$

Hence for any given iteration value of C_1, the corresponding value of K_1 can be determined uniquely by the use of equations (3.22) through (3.25). Now that K_1 is known, only K_2 and C_1 are unknown in equation

(3.5) and 20 iterations will determine all obtainable solutions.

The restriction on the value of β_1 has the following two distinct disadvantages:

a) In some configurations the very best solution will not be found because it actually yields $\beta_1 > \pi$ radians.

b) $F(C_1)$, but not $F'(C_1)$, becomes discontinuous at $C_1 = C_o \pm \pi$ radians as proved in Appendix 5.

It is only in the short time intervals around firing time and around the start of next multipass that all possible solutions needs investigation (see sections 3.4 - 3.6), and that these disadvantages can possibly have any consequence. At all other instances the torpedo has turned such that the value of β_1 is not in the vicinity of π radians and the listed disadvantages will no longer have any consequence.

Though both experience and reasoning tells that the restriction of β_1 does not have any practical consequence at firing time, the author will not recommend the implementation of such a restriction. There are two reasons for this:

c) There are certain important configurations when the target is approaching own ship head-on, that the best multipass-solution becomes significantly better when β_1 is allowed to be greater than π radians.

d) The calculations of all solutions are only required at firing time and start of each multipass under normal circumstances. Since this means that such calculations are fairly rare, at every 150 - 300 seconds in normal cases, the saving of the time required to find all solutions is found to be of less importance than the time-doubling to find the very best solution.

As a summary of this chapter it should be observed that the following is recommended for every implementation of optimal guidance:

e) K_1 changes sign at $C_1 = C_o$.

f) K_2 changes sign at $C_1 = C_3$.

g) Neither K_1 nor K_2 changes sign at $C_1 = 0$, except when respectively $C_o = 0$ or $C_3 = 0$.

h) Before starting any iteration, values of K_1 and K_2 must be assigned and C_1 given a start value of iteration.

i) When all possible solutions are investigated the following start angles are recommended: $C_1 = 0$, $\pi/2$, π, $3\pi/2$ and 2π radians.

3.4 An outline of optimal guidance

The computations of the relevant guidance parameters will be divided into two modes of calculation:

Mode 1:

In this mode of calculation the turn parameters K_1 and K_2, the straight path course C_1 and the end course C_3 are unknown. It is therefore necessary to obtain and compare all possible solutions. The best solution which will be used for guidance purposes is that solution requiring the least time to hit the target estimate.

This mode of calculation is normally only required at firing time and at the start of each new multipass. However, when Mode 2 calculations fails to find any solution or finds an unacceptable solution, the calculation is transferred to mode 1. Flowchart of mode 1 calculation is shown in Figure 3.4 and Figure 3.15.

Mode 2:

This mode of calculation is used for the repetitive computation of the running values of the guidance parameters, when a decision has been taken on the principle nature of the solution used for guidance purposes. Each new solution is in this mode based on the last found values of K_1, K_2 and C_1. Of the two possible values of C_3, the one used will be that one which differs the least from the formerly used value. Flowchart of mode 2 calculation is shown in Figure 3.4 and Figure 3.21.

If no solution is obtained or the new solution differs in the value of hit-time by a predetermined time (20 seconds are used in the later mentioned simulations), mode 1 calculation is performed bef re the guidance is reassumed.

No solution should be accepted from either mode 1 or mode 2 calculations if the found value of the straight path T_2 is less than a predetermined time G_1. In the simulated case described later G_1 is set to 10 seconds in mode 1 and 2 seconds in mode 2. This precaution is necessary in order to prevent that minor jumps in the state vector data will result in the formerly found best solution being lost.

Furthermore the optimal guidance will be divided into three distinct stages, each with a slightly different utilization of the developed mathematical equations. The stages are as follows:

Stage 1:

Guidance between firing time or the time of multipass calculation and an instant 15 seconds before the start of the second turn.

Stage 2:

Guidance between a time 15 seconds before the start of the second turn and the time when the second turn is finished. The torpedo is hence located at the optimal point at the end of stage 2 guidance.

Stage 3:

Guidance between the time of arrival at the optimal point and the time of next multipass calculation.

The following sections will describe these three stages of guidance in more detail.

3.5 Stage 1 Guidance

As stated in the previous section stage 1 guidance is employed from firing time or the time of a multipass-calculation to an instant 15 seconds before the start of the second turn. The following outline is recommended:

a) The stage 1 guidance is always initiated by a mode 1 calculation finding the best of all possible solutions.

b) The new guidance parameters are thereafter found by repetitive calculations using the current values of the state vector (including the relevant data of the uncertainty-ellipse) and the last found values of the guidance parameters by mode 2 calculations.

c) No new solution is accepted if $T_2 \leq G_1$ or if the new time to collision T exceeds the last found value of T updated to current time (i e last found time minus the elapsed time since last calculation) by a certain predetermined value (20 seconds used in simulations).

d) At each new calculation time, the time between the next calculation and the start of the second turn is checked. If the time becomes less than 15 seconds,(15 seconds are the experimentally used value in the simulations) stage 2 guidance is adopted.

e) If a stage 1 solution is found the torpedo guidance is performed in accordance with the values of β_1 and K_1. If the first turn is completed before the next repetitive time of calculation the torpedo will be guided in a straight path until the instant of next calculation.

f) When current time is coincident with the next calculation time, a new solution is found as outlined from point b.

A flowchart of stage 1 guidance is shown in Figure 3.5.

3.6 Stage 2 Guidance

Stage 2 guidance is employed from the end of stage 1 guidance until coincidence with the optimal point. The following outline is recommended:

a) Stage 2 guidance is always started with guidance of the torpedo by the use of the guidance parameters found by the last calculation in stage 1, i e the torpedo is guided through a possible first turn β_1 and an accompanying straight path until a new calculation becomes necessary.

b) Before calculation it is tested if the next calculation time will exceed the time at which the second turn should be started. If such is the case guidance will be conducted as outlined from point e, if not the new guidance parameters are calculated by mode 2 calculation.

c) If no solution or an unacceptable solution (same tests as for stage 1) is found, this is taken to mean that the remaining straight path of the previously found solution has become too small (i e $T_2 \leq G_1$) for any solution to be obtained. In that case the guidance is re-assumed as outlined from point e.

d) If a solution is found, guidance is performed as dictated by the guidance parameters until the next time for calculation. Guidance is then continued as outlined from point b.

e) Guidance is performed along a straight path until the last accepted start of the second turn.

f) At this point it is desired to start the second turn, but in the so far developed method the guidance is only performed through first turns and straights. The turn formerly considered as the second turn is therefore transferred into a first turn. This is best done by renewed calculation of the guidance parameters with a different value of the distance "A" between the last calculated optimal point and the collision point. The new value of "A" is taken to be the last found value minus a predetermined distance D_9. (The value of D_9 will be discussed in the section 3.8.) The starting point of the iteration angle C_1 is taken to be equal to the last found value of C_3. The value of K_2 corresponding to $C_1 = C_3$ can be uniquely determined from C_1 and C_3, since β_2 is necessarily a small angle, at least smaller than π radians. The value of K_1 is taken to be the former value of K_2, when β_2 was found to be greater than $\pi/2$ radians. If β_2 was found to be less than $\pi/2$ radians K_1 can be uniquely determined by C_o and the new value of the start angle C_1.

g) If a solution is found, guidance is conducted by turning as prescribed by the found values of β_1 and K_1 until $C_o = C_1$ at which time a transfer to stage 3 guidance is undertaken or until the next time of calculation. Renewed calculation is performed as in stage 1 and turning is resumed.

h) If no solution is found while turning through the formerly called second turn, calculation is tried several times with smaller value of "A", as described for stage 3 guidance.

i) When C_o is turned to a value closest to C_1 within the turning ability of the torpedo, stage 3 guidance is undertaken.

A flowchart of stage 2 guidance is shown in Figure 3.6.

3.7 Stage 3 Guidance

Stage 3 guidance is undertaken from the optimal point until the start of next multipass. It is at this stage of the trajectory that the torpedo has a definite probability of hitting the target, and it might be advantageous to conduct the repetitive calculation more often than that employed in stage 1 and 2. The following outline for stage 3 guidance is recommended:

a) At intervals, when the second turn is completed as outlined later, renewed calculation is conducted with a value of A such that $A_{n+1} = A_n - D_9$, (where the subscripts denotes the n'th and n+1'st calculation). The value of D_9 should depend on the value of A in some crude fashion. (D_9 = 100+10 INT(ABS(A)/200) is used for simulation). The reason is that any small change in C_3 from calculation to calculation will yield a transversal displacement of the trajectory which is dependent on A. When D_9 is made larger for a given change in C_3, the corresponding calculated values of β_1 and β_2 become smaller. Hence when D_9 is made dependent on A in an appropriate fashion it serves to make both the trajectory smoother and a solution more readily obtainable.

b) The calculation is performed as mode 2 calculation as outlined before. If no solution is obtained this is not necessarily because the general outline of the solution is wrong, but because C_3 has changed so much that a solution using the particular value of A is not found. Hence, several values of A should be tried before a transfer to stage 1 guidance and mode 1 calculation is undertaken.

c) When a solution is found, the torpedo is guided in accordance with the calculated guidance parameters through a first turn, a straight and a second turn. The guidance through the second turn is in stage 3 most readily performed by letting $K_1 = K_2$ and $\beta_1 = \beta_2$, and turning as it was a first turn.

d) At any appropriate time in accordance with the repetitive calculation, a new solution will be calculated in accordance with mode 2 calculations using the former values of K_1, K_2 and C_1, and the value of C_3 which differs the least from the formerly used value. In these repetitive calculations the value of A is not changed by D_9. If a solution is obtained the guidance is conducted according to the found parameters. If a solution is not found the guidance is continued using the accepted values of last calculation until the completion of the second turn. When the second turn is finished a new calculation is performed as outlined in point a, using this time a new value of A.

e) When stage 3 guidance is entered the guidance is performed as outlined from point c, and at this moment the time to start the multipass is saved as twice the current value of the hit-time T.

f) At the time of each repetitive calculation a check if current time has passed the time of multipass calculation should be performed.

When this is found to be the case, a transfer to stage 1 guidance and mode 1 calculation should be undertaken.

After collision with the estimate the value of A will become negative since the solution now is based on a collision with the estimate which has already taken place.

In all three stages of the guidance it is relevant to finish the first turn, such that C_o becomes approximately equal to C_1. Since the torpedo can not turn through arbitrarily small angles, but is limited to say turns of 1 degree, the torpedo angle C_o becomes practically never equal to C_1. To start a new mode 2 calculation one must be sure that there is correspondence between the values of K_1 and C_1. However, since the remaining turn angle β_1 is small when the first turn is finished, K_1 can be uniquely determined from the values of C_o and C_1.

In stage 3 the torpedo is turned through both the first and second turns, and it is therefore necessary to calculate both K_1 and K_2 from the values of C_1 and C_o, respectively from C_1 and C_3 before a mode 2 calculation is performed.

Though the above stipulated stage 3 guidance works very well, it does have a distinct disadvantage under certain special circumstances. When a high bearing rate is combined with a large value of A, the transversal displacement corresponding to the transposed distance D_9 becomes significant. The corresponding values of β_1 and β_2 become larger in such a case, and as stated earlier D_9 must be made dependent on A in order to compensate for this effect. Though there are no mathematical objections to such a guidance, it is in practice unsatisfactory since increased turning leads to increased errors in the dead-reckoning of the torpedo.

The logical consequence of the above mentioned facts is that since C_3 is steadily changing, it becomes unnecessary to turn the torpedo onto the course dictated by C_3. Hence the proper thing to do is to set β_2 equal to zero in stage 3. Though the simulations will demonstrate that this will smoothen the stage 3 guidance wonderfully, it cannot be fully re-recommended before simulations with sensor errors and dead-reckoning errors are performed. Not only will the incorporation of $\beta_2 = 0$ in stage 3 have a very accurate way of predicting future dévelopment of the transversal displacement, but it also allows use of a value of D_9 which is much smaller and independent of A. As the next paragraph on prediction will demonstrate, a smaller value of D_9 is of great importance for an accurate trajectory.

In stage 2 guidance it is not incorporated any future development of the transversal displacement corresponding to the transposed distance D_9, and the stage 2 guidance is ended with the torpedo course C_o equal to C_3. When the transversal displacement is significant, the first stage 3 calculation with renewed transposed D_9, must respond with an equivalent significant turn, even in the case when β_2 is set to zero. Employing $D_9 = 40$ meters for stage 2 and $D_9 = 150$ meters for stage 3 guidance for the smoothed simulation, this minor redundant turning at the start of stage 3 guidance seems to be of no practical importance.

A flowchart of stage 3 guidance when β_2 is allowed to take on non-zero values is shown in Figure 3.7.

3.8 Prediction of the tilt angle

When the bearing rate is different from zero, the value of the tilt angle θ of the uncertainty-ellipse will change with time. Every change in the tilt angle will inflict a corresponding change in the value of the end-torpedo course C_3.

It is possible not to employ any prediction of the tilt angle and always to use the current value of θ for the calculation of C_3. The proposed straight trajectory in stage 1 guidance would hence be slightly curved when the bearing rate is non-zero.

The prediction of the tilt angle θ becomes more important with increased bearing rates. There are two basically different approaches to obtain a proper prediction of the θ-value at some future time:

a) To calculate a tilt angle rate $\dot{\theta}$ by observation of θ at different instances. The tilt angle at some future time T_9 becomes accordingly

$$\theta_{T9} = \theta + \dot{\theta} \cdot T_9 \qquad (3.26)$$

when θ is the current value of the tilt angle as estimated by the K-filter.

b) It is only in rare cases that the current value of the tilt angle is not coincident with the bearing from own ship to the target. The tilt angle at some future time T_9 can hence be calculated from

$$\theta = \psi_{T9} - C \tag{3.27}$$

if the bearing ψ_{T9} at time T_9 can be calculated. By assuming that both the estimate and own ship will travel on a straight path the future positions of both vessels can be calculated from their current location, their courses and their velocities. Given the positions the bearing can be calculated directly.

In the simulations later shown the second method using bearing calculation is employed.

In both stage 1 and 2 guidance the prediction of the tiltangle θ is calculated for the instant when the torpedo reaches the point displaced by D_9 from the optimal point. The prediction time for stage 1 and 2 guidance is hence

$$T_9 = T_1 + T_2 + T_3 + D_9/V_T + U_6 - U \tag{3.28}$$

where U is the current time and U_6 is the time for which $T_1+T_2+T_3$ is valid.

For stage 3 guidance the prediction time is

$$T_9 = D_9/V_T \tag{3.29}$$

for every calculation performed when a new displacement D_9 is incorporated. At every intermediate repetitive calculation, the prediction time becomes

$$T_9 = T_9 + U_6 - U \tag{3.30}$$

where T_9 on the right side of the equality sign has been computed at time U_6.

It should be evident that the smaller the value of D_9 which can be employed, the more accurate the actual torpedo trajectory will yield the ideal trajectory.

The actual torpedo end courses are calculated from the predicted value of θ as earlier discribed. In mode 2 calculations the value of C_3 will be used which differs the least from the formerly used value.

In mode 1 calculation employed principally at firing time and at the start of each multipass, the prediction time is not readily available, since the future trajectory is still completely unkonwn. In the simulation discribed later a crude prediction time is calculated before mode 1 calculations are performed. This prediction time is set equal to the collision time calculated when no turns are considered. Such a calculation is analytical with no need for iteration. For moderate values of "A" this gives a good starting point for obtaining the best solution, particularly at firing time when the collision time is generally larger than that of multipass calculations. Since the prediction time might be in error at the first mode 1 calculations, stage 1 guidance is never started before an acceptable solution is found by a mode 2 calculation. The relevant tests for acceptance are found in the flowchart in Figure 3.4.

It is of importance to notice that for each new calculated value of C_3, it must be investigated if this new value demands a corresponding change in the second turn paramter K_2. The rule must be to change K_2 if the angle C_1 is passed when going from the old to the new value of C_3.

3.9 Prediction of the axes of the uncertainty-ellipse

As is evident from the description of the optimal guidance in sections 3.2 through 3.6, the guidance parameters are dependent on the values of the axes of the uncertainty-ellipse in all three guidance stages. In stage 1 and stage 2 guidance the location of the optimal point is dependent on the values of these axes through both the distance "A" between the optimal point and the collision point, and through the torpedo end-course C_3. In stage 3 guidance the displaced end point of the second turn is at all times dependent on the value of C_3.

The nature of the optimal guidance and the employment of repetitive calculations will however, make it rather unsusceptible to smaller adjustments of ellipse-axes. Such adjustments are readily taken care of by calculation of a corresponding small change in the guidance parameters. Though, not yet proven by simulation, it is expected that even larger adjustments of the ellipse-axes towards smaller values will readily be accepted by the optimal guidance in all three stages such that the new guidance parameters can be found by mode 2 calculations. Note that in both stage 2 and stage 3 guidance, when the guidance is most susceptible to changes in the ellipse-axes, the calculation is first tried with several increased values of the displacement before the general nature of the formerly accepted solution is no longer considered valid and a transfer to mode 1 calculation is undertaken. Larger instantaneous changes in the major ellipse axis (distance-uncertainty) could result in a new location of the optimal point behind the current location of the torpedo, when guidance in stage 2 or the latter part of stage 1 is employed. However, as argued below such sudden increases in the distance uncertainty are not expected to take place in practical, tactical situations.

In common practical, tactical situations in which for example frequent use of passive bearing sensors are used, the minor ellipse-axis (bearing uncertainty) will rapidly stabilize at a low and fairly constant level and the major axis (distance uncertainty) will steadily decrease, but never surpass a minimum value. Every singular use of a distance measuring sensor with better accuracy than that which corresponds to the current distance uncertainty, will yield a sudden jump of the major axis towards a smaller value. The distance uncertainty will, however, slowly increase as the time since last distance observation elapses, as estimated by the K-filter. Frequent use of distance measuring sensors will tend to stabilize the ellipse-axes at some fairly constant values dependent on the accuracy of the sensors in use. It can hence from these arguments be concluded that a fairly normal development of the tracking history of a target is such that the bearing uncertainty is fairly constant, whereas the distance uncertainty is decreasing either continuously or abruptly towards a minimal value. There is one exception from this after a singular use of a distance measuring sensor when the distance uncertainty might slowly increase.

The Kalman filter is at any time capable of predicting the state vector uncertainties at some future time. Such a prediction would both reflect the current position-uncertainties and the added uncertainties due to inaccuracies of the course and speed. The greatest deficiency by a K-filter uncertainty prediction (P-matrix prediction) is, however, that it cannot take into account the use of any sensors during the prediction time. The ellipse-axes predicted by the K-filter are therefore generally larger than the current values, in direct contradiction to the arguments above stating a normal decrease in these values. It must therefore be concluded that K-filter uncertainty-prediction is not desirable.

Other means of uncertainty-prediction might be employed. Similarly to that suggested in the last section, the computation of the time-derivatives of the ellipse-axes could be utilized. This kind of prediction could only predict continuous changes of the uncertainties, for which the optimal guidance scheme is rather unsusceptible as argued above. The more drastic changes of the ellipse-axes, to which the scheme is somewhat more susceptible, can never be predicted.

It is therefore concluded that prediction of the uncertainty-ellipse-axes seems hardly worth while, and the use of the current values of the uncertainties is therefore recommended.

4 SALVO ASPECTS OF OPTIMAL GUIDANCE

The author of this report has not analysed the different aspects of salvo guidance in great detail. However, certain suggestions seem evident from the experience gained in optimal guidance:

a) Angular spreading should definitely not be used to guide salvoes consisting of more than one torpedo when optimal guidance is employed.

b) It is recomended to use spreading distances which are slightly less than the target length.

c) As far as possible it is recomended to conduct mode 1 calculation for only one (central) torpedo in the salvo to deduce the general nature of the best solution. It is further recommended to use individual guidance and mode 2 calculations for each individual torpedo.

d) Displacement of fictitious targets along the estimate course of one spreading distance and half the spreading distance for three, respectively two, torpedoes in a salvo, should be employed. It is vital that the distribution is along the estimate course, since this is the only way in which slightly overlapping hit-corridors can be assured. It should be evident that particularly for 2 torpedoes in a salvo, non-overlapping hit-corridors will leave the most probable area for hit uncovered. This fact should also demonstrate how important it is never to use a spreading distance which is greater than the actual target length.

5 MULTIPASS ASPECTS OF OPTIMAL GUIDANCE

At the start of every multipass one additional information is available: The data used for the last guidance (preferably stage 3 guidance) was not accurate enough to allow hit. If the data is not significantly more accurate when the best solution for next multipass is calculated, it seems reasonable to ultilize the information of no hit to displace the estimate used for guidance in relation to that given by the K-filter. Such a displacement should preferably be conducted along the estimate course, but there is no information to advise if the displacement should be in front or behind the given estimate. If the next multipass also fails to guide the salvo to a hit and the data have still not bettered significantly, the following multipass should be guided onto an estimate located on the oposite side of the estimate of that used in the former multipass.

The third multipass should preferably again be directed onto the target estimate given by the K-filter. It seems natural to use the same displacement-distance for this purpose as that used to displace torpedoes in a salvo.

The crucial point in estimate displacements for multipass guidance is that the estimate by the K-filter has not changed drastically in comparison with the data used to guide the salvo when no hit was obtained. There are two basically different ways to detect if such a change has taken place or not:

a) Manual surveillance and decision

A significant improvement of the uncertainties over a short time-interval is usually the result of one or more observations with a better sensor, of which the operator of the torpedoes should have knowledge. The operator is also normally given a plot of the tactical situation in which changes in estimate uncertainties are incorporated. It seems therefore reasonable to allow the operator to make the decision if the development of the estimate uncertainties has been such that estimate displacements for multipass should be employed or not. The most effective way to incorporate this, is to allow such a decision to be made prior to the calculation of the multipass guidance by selection on a separate controller on the fire control console.

b) Automatic decision

At the time of every transition to stage 3 guidance when no multipass displacement is employed, the major and minor axes of the uncertainty-ellipse should be saved for later comparisons. Before conducting any multipass calculation a specially designed test should take a decision if estimate-displacement should be incorporated or not for the oncoming multipass. If also a next multipass should be undertaken, the same test should be performed both using the formerly saved - and the current - values of the ellipse axes. If the test show no significant changes, estimate displacement on the oposite side to that used in the last multipass should be employed. At least every third multipass should be directed towards the estimate as given by the K-filter.

6 SIMULATIONS

6.1 Assumptions used in the simulation

In order to investigate and develop the optimal guidance scheme it has been necessary to undertake guidance simulations in a controlled and accountable environment. The following restriction has therefore been imposed on the simulation:

a) Own ship moves on a straight course with no dead-reckoning errors. Variations in speed and course are allowed from run to run.

b) The torpedo moves with a constant speed of 30 knots and is allowed to turn 1 degree per 0.2 seconds yielding a minimum turn radius of 177 meters. The torpedo course at the time of firing is equal to that of own ship.

c) The movement of the estimate of the target is simulated on a straight course without the use of a K-filter. The axes of the uncertainty-ellipse can be varied from run to run, but remains constant throughout a complete run. The orientation of the ellipse is always such that the major axis coincide with the bearing between own ship and the target estimate. Variations in speed and course are allowed from run to run. As it can be seen from the mathematical equations only the ratio of the target-ellipse axes enters the optimal guidance computations. This ratio is kept constant at 1/10 for all runs.

d) A salvo consists of only one torpedo, and no displacement of the estimate has been incorporated at the multipass calculations.

e) Each run has a maximum length of 800 seconds, corresponding to an approximate wirelength of 12000 meters. The next multipass is not started if collision takes place after 800 seconds from firing time. No special termination procedure has been incorporated, such as to turn the torpedo away from the general direction of own ship just before the wirelength is exceeded.

f) Stage 3 guidance employs no calculation of the second turn β_2, in order to make the trajectory smooth.

A printout of the simulation program has been included at the end of this report.

6.2 Explanation of trajectory plots

In the headings of the trajectory plots shown from Figure 6.1 through Figure 6.14, the following abbreviations have been used:

VO = own ship speed
VS = estimated target speed
VT = torpedo speed
A = major axis of uncertainty-ellipse
B = minor axis of uncertainty-ellipse

The plots show the positions of respectively the estimate, the torpedo and own ship every 10 seconds. At every 100 seconds these positions are marked with a symbol twice the size of that used for every 10 seconds. The position of the target estimate at firing time is marked with a star twice the size used for every 100 seconds. Own ship posision at firing time is recognizable by its coincidence with the torpedo position. The torpedo course is visualized by a vector representation starting out from the center of the torpedo symbol. The position of the torpedo both at collision with the estimate and at the start of a multipass is represented by a squared symbol. The calculation is undertaken in a north oriented coordinate system such that the positive Y-axis points to the north.

Some comments on the plots included in the report will be given in the next section.

6.3 Discussion on the trajectory plots

The trajectory plots shown in Figure 6.1 through 6.14, do not necessarily demonstrate practical and recommendable tactical situations. The plots are only designed to demonstrate the behaviour of the optimal guidance scheme in different situations.

Figures 6.1 through 6.10 must be considered as normal runs. In Figure 6.1 and Figure 6.2 which are very similar, it is demonstrated how different own ship velocities and thereby different bearing rates, alter the optimal guidance trajectory. When the uncertainty-ellipse is circular as demonstrated in Figure 6.3, the hit-corridor is oriented 90 degrees on the estimated target course, and a broadside collision is effectuated. Also Figures 6.4 and 6.5 demonstrate broadside collision when the value of the major uncertainty-ellipse axis approached that of the minor axis. Several runs such as that shown in Figures 6.9 and 6.10 anticipate the shorter optimal guidance trajectory in comparison with a line-of-sight guidance. In Figure 6.11 a complete line-of-sight trajectory has been included for easy comparison. A tactical situation with fairly extreme bearing rate is chosen to magnify the difference between the two different guidance schemes.

It should be noted that it is very common that the best solution found at the start of a multipass will cross the estimate course before reaching the new optimal point. Though this might seem unorthodox at first glance, the reason behind such a solution is clear: it yields the shortest trjactory under the given circumstances. Any approach to try to make the trajectory shorter must hence change the circumstances to which the calculations are subjected. Several suggestion as to how this can be done will be given in the next section. A suggestion is demonstrated in Figures 6.12, 6.13 and 6.14 in which the multipass calculation is postponed somewhat until the torpedo has moved further away from the estimate. From the time that the multipass should have been calculated as suggested in Chapter 3, the torpedo is moved in a straight trajectory dependent on the current value of the distance "A" to the former collision point. The best dependence is in these three runs found to be a straight pass equal to 50 percent of the distance "A". The time required for this straight pass is, however, not allowed to exceed 40 seconds. Not enough runs have been investigated at the time of writing, to yield a conclusion as to the efficiency of such an approach.

Before turning to the next chapter dealing with suggestions for further work which should be undertaken, the reader should notice that in most cases the crossing of the estimate course is done in the immediate vicinity of the target estimate.

7 PROPOSAL OF CONTINUED WORK

This report is not complete in every detail and before the optimal guidance can be fully recommended for practical adoption in a fire control system which meets its requirements for application, the author will propose some further areas of investigations. It is however with full confidence that such investigation will only lead to fairly minor changes in the general consept of optimal guidance. The proposed investigations are:

a) Simulations with turning own ship and target.

b) Simulation incorporating sensor inaccuracies and a K-filter yielding variable values of the uncertainty-ellipse axes during the time of guidance.

c) Simulation with both own ship - and torpedo-dead-reckoning errors of different magnitudes corresponding to realistic configurations of logs and gyroes.

d) Monte Carlo simulations calculating hit-probability or any other convenient measure to compare optimal guidance to line-of-sight guidance and collision-point guidance.

e) Adjustments of incorporated experimental parameters employed in the optimal guidance scheme should be performed with relevant inaccuracies in both dead-reckoning and sensor observations.

f) It is particularly advantageous to find a minimum experimental value of the distance between the estimate and the optimal point, set to $3 \cdot a$ in the presented simulation. This number will greatly influence the number of possible multipasses.

g) The work started on postponement of the multipass calculation described in the last section should be continued.

h) Since the torpedo crossing of the estimate course is commonly in the immediate vicinity of the estimate, an investigation should be undertaken to modify the stage 1 guidance to yield a slightly different trajectory leading to a collision at the crossing of the estimate course. Uncritical collision-point guidance when the torpedo is located closer to the estimate than a minimum distance (this distance should be less than the minimum allowed value of "A") suggests itself.

i) In systems in which mode 1 calculation-time is of no great concern, several values of postponed multipass calculation can be compared. Even iteration might be employed to find the best possible solution.

j) In tactical situations with extreme bearingrates the predicted value of C_3 will not coincide with the desired torpedo course at the optimal point (see Figures 6.9 and 6.10), and a redundant turning results. A better prediction of C_3 for stage 1 and stage 2 guidance taking into account the future development of C_3, can be investigated to overcome this minor problem.

It is only considered necessary to undertake all these points of investigation, when an extremely careful and complete investigation is to be undertaken.

8 CONCLUSIONS

The purpose of the work described in this report was to investigate the possibility of designing a guidance scheme which would yield a higher hit probability than that obtainable through the conventional line-of-sight and collision-point guidance. The reason why such an improvement was expected is that certain fire control systems supplement the estimates of the usual tracking parameters with estimates of the uncertainty of these parameters.

A mathematical solution which theoretically will yield the optimal hit-probability by using the position uncertainties of the target estimate has been given. A workable guidance scheme using this mathematical solution has been found to guide torpedoes in practical, tactical situations with dynamically changing tracking parameters.

It is hoped that this report has given sufficient material to convince the reader that the utilization of the designed optimal guidance yields definite advantages over that obtainable through a choice between line-of-sight guidance and collision-point guidance. For clarity, some of the arguments showing such advantages will be summarized below:

a) Calculations of hit probabilities when position uncertainties are incorporated show distinct maxima and minima, even when the bearing and distance uncertainties are almost equal. This disfavours the use of uncritical collision point guidance, since such guidance does not incorporate a calculation of the angle-of-attack yielding neither maxima nor minima.

b) Line-of-sight guidance is favourable in most circumstances even up to a ratio of the axes of the uncertainty ellipse of $b/a = 1/5$.

c) The optimal guidance yields guidance after the line-of-sight principle in all circumstances favouring such guidance. It does not, however, start line-of-sight guidance before the data predicts that the hit-probability is non-zero and hence such guidance becomes meaningful. The optimal guidance yields therefore a shorter trajectory than that obtainable through the use of line-of-sight guidance.

d) The optimal guidance will only rarely yield pure collision-point guidance. In cases where collision-point guidance is traditionally recommended, the optimal guidance tends to yield higher hit-probability since it guides the torpedo optimally to a collision more broadside on the target.

e) In every situation, also in the situation in which a choice between the two conventional guidance schemes seems difficult, the optimal guidance yields a trajectory which is very close to optimal in relation to those data entering the guidance computation.

f) It becomes possible when optimal guidance is implemented to use only one automatic guidance mode (automatic as contrary to manual) to assure the best possible guidance. The choice between two guidance modes, each serving very different circumstances is hence eliminated.

The optimal guidance will of course yield the highest hit-probability when the estimated position uncertainties faithfully represent the true uncertainties of a given tactical situation. It is, however, expected that optimal guidance, will yield better results than that obtainable conventionally, even in cases when fairly coarse errors in dead-reckoning are not incorporated in the uncertainty estimation process.

It is hoped that this report will raise enough interest to enable the work with optimal guidance to continue as suggested in chapter 7.

APPENDIX 1

DEVELOPMENT OF EQUATIONS DESCRIBING THE HIT-CORRIDOR

With reference to Figure 2.5, the equation for the target ellipse can be written as

$$(s/a_1)^2 + (r/b_1)^2 = 1$$

or

$$(a_1 r)^2 + (b_1 s)^2 = (a_1 b_1)^2 \qquad (A1.1)$$

which by differentiation yields

$$2a_1^2 r\, dr + 2b_1^2 s\, ds = 0$$

or

$$\frac{dr}{ds} = -\frac{b_1^2 s}{a_1^2 r}$$

A straight line with inclination tgΦ with respect to the s-axis will be tangential to the ellipse in a point described by

$$tg\Phi = -\frac{b_1^2 s}{a_1^2 r}$$

or

$$s = -(\frac{a_1}{b_1})^2 r\, tg\Phi \qquad (A1.2)$$

A substitution of equation (A1.2) into (A1.1) yields

$$(a_1 r)^2 + (b_1 (\frac{a_1}{b_1})^2 r\, tg\Phi)^2 = (a_1 b_1)^2$$

$$(a_1 b_1 r)^2 + (a_1^2 r\, tg\Phi)^2 = a_1^2 b_1^4$$

or

$$r_{12} = \pm \frac{b_1^{\ 2}}{\sqrt{(a_1 tg\Phi)^2 + b_1^{\ 2}}}$$

and correspondingly

$$s_{12} = \pm \frac{a_1^{\ 2} tg\Phi}{\sqrt{(a_1 tg\Phi)^2 + b_1^{\ 2}}}$$

From Figure 2.5 it is seen that

$$k_r = r_1 - s\ tg\Phi$$

or

$$k_r = \frac{b_1^{\ 2} + (a_1 tg\Phi)^2}{\sqrt{(a_1 tg\Phi)^2 + b_1^{\ 2}}} = \sqrt{(a_1 tg\Phi)^2 + b_1^{\ 2}} \qquad \text{(A1.3)}$$

And hence the equations for the two lines defining the hit corridor are

$$r = s\ tg\Phi \pm k_r$$

Rotation to the x-y coordinate system is governed by

$$r = y \cos\theta - x \sin\theta$$

$$s = y \sin\theta + x \cos\theta$$

Hence

$$y \cos\theta - x \sin\theta = tg\Phi(y \sin\theta + x \cos\theta) \pm k_r$$

$$y(\cos\theta + \sin\theta\ tg\Phi) = x(\sin\theta + \cos\theta\ tg\Phi) \pm k_r$$

$$y(\cos\theta \cos\Phi + \sin\theta \sin\Phi) = x(\sin\theta \cos\Phi + \cos\theta \sin\Phi) \pm k_r \cos\Phi$$

$$y \cos(\Phi+\theta) = x \sin(\Phi+\theta) \pm k_r \cos\Phi$$

$$y = mx \pm n \qquad \text{(A1.5)}$$

where

$$m = tg(\Phi+\theta) \qquad (A1.6)$$

and

$$n = \frac{\cos\Phi}{\cos(\Phi+\theta)} \sqrt{(a_1 tg\Phi)^2 + b_1{}^2}$$

$$n = \frac{\sqrt{(a_1\sin\Phi)^2 + (b_1\cos\Phi)^2}}{\cos(\Phi+\theta)} \qquad (A1.7)$$

A rotation to the u-v coordinate system is goverened by

$$r = v\sin\Phi + u\cos\Phi$$

$$s = v\cos\Phi - u\sin\Phi$$

Hence

$$v\sin\Phi + u\cos\Phi = tg\Phi(v\cos\Phi - u\sin\Phi) \pm k_r$$

$$u(\cos\Phi + tg\Phi\sin\Phi) = v(tg\Phi\cos\Phi - \sin\Phi) \pm k_r$$

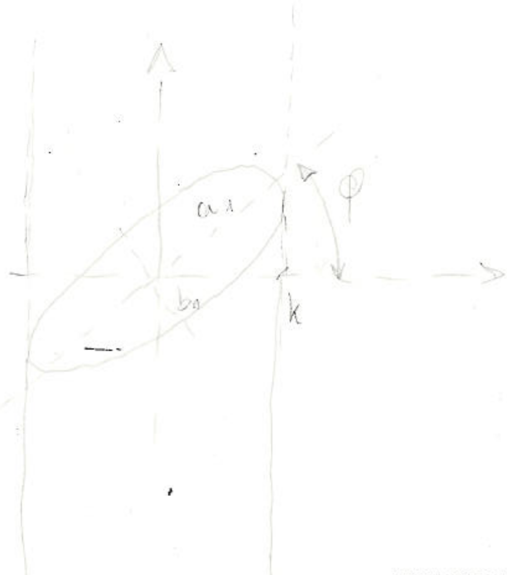
$$u(\cos^2\Phi + \sin^2\Phi) = \pm k_r\cos\Phi$$

$$u = \pm\cos\Phi\sqrt{a_1{}^2 tg^2\Phi + b_1{}^2} \qquad (A1.8)$$

$$u = \pm k \qquad (A1.8)$$

where

$$k = \sqrt{(a_1\sin\Phi)^2 + (b_1\cos\Phi)^2} \qquad (A1.9)$$



APPENDIX 2

CALCULATION OF THE CORRIDOR ANGLE YIELDING MAXIMUM HIT-PROBABILITY

The corridor angle yielding the maximum probability of hit can be found by combinding equations (2.6) and (2.7)

$$\frac{\partial P}{\partial \Phi} = \frac{\partial}{\partial \Phi} \{\frac{1}{2\pi ab} \int_{-\infty}^{\infty} \int_{mx-n}^{mx+n} \mathrm{EXP}\left(-\tfrac{1}{2}((x/a)^2 + (y/b)^2)\right) dy\ dx\} = 0 \qquad (A2.1)$$

An integral of the form

$$\int_a^b \mathrm{EXP}(-u^2)\ du$$

has unfortunately no arithmetic solution and hence a solution of (A2.1) cannot be obtained by a differentiation of the solution of the double-integral. However from the intergral-calculus the following two formulas can be used to obtain a solution

$$\frac{\partial}{\partial x} \int_a^b f(x,t)dt = \int_a^b \frac{\partial}{\partial x} \{f(x,t)\}dt \qquad (A2.2)$$

and

$$\frac{\partial}{\partial x} \int_{v(x)}^{u(x)} f(t)dt = f(u)\frac{\partial u}{\partial x} - f(v)\frac{\partial v}{\partial x} \qquad (A2.3)$$

where a and b must be constants.

By the use of (A2.2), equation (A2.1) reduces to

$$\frac{\partial P}{\partial \Phi} = \frac{1}{2\pi ab} \int_{-\infty}^{\infty} \mathrm{EXP}\left(-\tfrac{1}{2}(x/a)^2\right) A(x)dx$$

where

$$A(x) = \frac{\partial}{\partial \Phi} \int_{mx-n}^{mx+n} \mathrm{EXP}\left(-\tfrac{1}{2}(y/b)^2\right) dy$$

Application of (A2.3) yields

$$A(x) = \text{EXP}\left(-(mx+n)^2/2b^2\right) \left(x \frac{\partial m}{\partial \Phi} + \frac{\partial n}{\partial \Phi}\right)$$

$$- \text{EXP}\left(-(mx-n)^2/2b^2\right) \left(x \frac{\partial m}{\partial \Phi} - \frac{\partial n}{\partial \Phi}\right)$$

For convenience the following parameters are included

$$A = = \frac{\partial m}{\partial \phi} \quad \text{and} \quad B = \frac{\partial n}{\partial \Phi} \tag{A2.4}$$

giving

$$A(x) = \text{EXP}\left(-(mx+n)^2/2b^2\right) \left(Ax+B\right) + \text{EXP}\left(-(mx-n)^2/2b^2\right) \left(Ax-B\right)$$

Hence

$$\frac{\partial P}{\partial \Phi} = \frac{1}{2\pi ab} \int_{-\infty}^{\infty} \{\text{EXP}\left(-\tfrac{1}{2}(x^2/a2 + (mx+n)^2/b^2)\right) \left(Ax+B\right)$$

$$- \text{EXP}\left(-\tfrac{1}{2}(x^2/a^2 + (mx-n)^2/b^2)\right) \left(Ax-B\right)\}dx$$

Again for convenience use

$$\frac{1}{\sigma^2} = \frac{1}{a^2} + \frac{m}{b^2} = \frac{a^2m^2 + b^2}{a^2b^2} \tag{A2.5}$$

and

$$N = \frac{mn\sigma^2}{b^2} = \frac{mna^2}{a^2b^2 + b^2} \tag{A2.6}$$

and hence obtain

$$\frac{\partial P}{\partial \Phi} = C_1 \int_{-\infty}^{\infty} \{\text{EXP}\left(-(X+N)^2/2\sigma^2\right) \left(Ax+B\right) - \text{EXP}\left(-(X-N)^2/2\sigma^2\right) \left(Ax-B\right)\}dx$$

where

$$C_1 = \frac{1}{2\pi ab} \text{EXP}\left(-n^2/2b^2 + N^2/2\sigma^2\right)$$

By the use of

$$u = X+N \quad \text{and} \quad v = X-N$$

one has

$$\frac{\partial P}{\partial \Phi} = C_2 \int_{-\infty}^{\infty} (A(u-N)+B)\ EXP\left(-u^2/2\sigma^2\right) du - C_2 \int_{-\infty}^{\infty} (A(v+N)-B)\ EXP\left(-v^2/2\sigma^2\right) dv$$

or

$$\frac{\partial P}{\partial \Phi} = C_2(B-AN) \int_{-\infty}^{\infty} EXP\left(-u^2/2\sigma^2\right) du + C_2(B-AN) \int_{-\infty}^{\infty} EXP\left(-v^2/2\sigma^2\right) dv$$

$$+ C_2A \int_{-\infty}^{\infty} u\ EXP\left(-u^2/2\ \ ^2\right) du - C_2A \int_{-\infty}^{\infty} v\ EXP\left(-v^2/2\sigma^2\right) dv \qquad (A2.7)$$

Let u = v and find

$$\frac{\partial P}{\partial \Phi} = 2C_2(B-AN) \int_{-\infty}^{\infty} EXP\left(-u^2/2\sigma^2\right) du$$

or

$$\frac{\partial P}{\partial \Phi} = 2C_2(B-AN)\sqrt{2\pi}\ \sigma$$

To find the maximum and minimum

$$\frac{\partial P}{\partial \Phi} = 0$$

or

$$\sigma(B-AN) = 0$$

Solving $\sigma=0$ does not give any general solution for max and min, and hence it must be found from

$$B - AN = 0$$

or rather

$$\frac{\partial n}{\partial \Phi} = \frac{\partial m}{\partial \Phi} N \qquad (A2.8)$$

Solving this equation in terms of $tg\Phi$ yields a cumbersome third degree equation, whereas a solution in terms of $m' = tg(\Phi+\theta)$ yields a second degree equation. The latter is chosen for convenience, but first n must be found as a function of m.

Equation (A1.7) defined n as

$$n = \sqrt{(a_1 \sin\Phi)^2 + (b_1 \cos\Phi)^2}/\cos(\Phi+\theta)$$

which by substitution of

$$D = b_1/a_1 \qquad \text{(A2.9)}$$

yields

$$n = a_1\sqrt{\sin^2\Phi + D^2\cos^2\Phi}/\cos(\Phi+\theta)$$

or

$$n = a_1\sqrt{\sin^2(\Phi+\theta-\theta) + D^2\cos^2(\Phi+\theta-\theta)}/\cos(\Phi+\theta)$$

$$n = a_1\sqrt{(\sin(\Phi+\theta)\cos\theta - \cos(\Phi+\theta)\sin\theta)^2 + D^2(\cos(\Phi+\theta)\cos\theta - \sin(\Phi+\theta)\sin\theta)^2}/\cos(\Phi+\theta)$$

$$n = a_1\sqrt{(\cos^2\theta + D^2\sin^2\theta)\sin^2(\Phi+\theta) - 2(1-D^2)\sin(\Phi+\theta)\cos(\Phi+\theta)\sin\theta\cos\theta + (\sin^2\theta + D^2\cos^2\theta)\cos^2(\Phi+\theta)}/\cos(\Phi+\theta)$$

$$n = a_1 \cos\theta\sqrt{(1+D^2tg^2\theta)\,tg^2(\Phi+\theta) - 2(1-D^2)\,tg\theta\,tg(\Phi+\theta) + tg^2\theta + D^2}$$

$$n = a_1\cos\theta\sqrt{(1+D^2tg^2\theta)\,m^2 - 2(1-D^2)\,tg\theta\,m + tg^2\theta + D^2} \qquad \text{(A2.10)}$$

and

$$\frac{\partial n}{\partial \Phi} = \frac{a_1\cos\theta\{2(1+D^2tg^2\theta)m - 2(1-D^2)tg\theta\}}{2\sqrt{(1+D^2tg^2\theta)m^2 - 2(1-D^2)tg\theta\,m + tg^2\theta + D^2}} \cdot \frac{\partial m}{\partial \Phi}$$

and hence equation(A2.8) yields

$$\frac{a_1\cos\theta\{(1+D^2tg^2\theta)m - (1-D^2)tg\theta\}}{\sqrt{(1+D^2tg^2\theta)m^2 - 2(1-D^2)tg\theta\ m + tg^2\theta + D^2}} \cdot \frac{\partial m}{\partial\Phi} =$$

$$\frac{ma^2}{a^2m^2 + b^2}\, a_1\cos\theta\sqrt{(1+D^2tg^2\theta)m^2 - 2(1-D^2)tg\theta\ m + tg^2\theta + D^2} \cdot \frac{\partial m}{\partial\Phi}$$

Which by substitution of

$$C = b/a \qquad (A2.11)$$

yields

$$\{(1+D^2tg^2\theta)m - (1-D^2)tg\theta\}\ \{m^2+C^2\} =$$

$$m\{(1+D^2tg^2\theta)m^2 - 2(1-D^2)tg\theta\ m + tg^2\theta + D^2\}$$

$$(1-D^2)tg\theta\ m^2 + (C^2+D^2C^2tg^2\theta - D^2 - tg^2\theta)m - C^2(1-D^2)tg\theta = 0$$

or

$$(1-D^2)m^2 - \{(D^2-C^2)\text{cotg}\theta + (1-C^2D^2)tg\theta\}m - C^2(1-D^2) = 0 \qquad (A2.12)$$

When

$$D^2 \neq 1$$

$$m^2 - m\{(D^2-C^2)\text{cotg}\theta + (1-C^2D^2)tg\theta\}/(1-D^2) - C^2 = 0$$

If

$$A = \{(D^2-C^2)\text{cotg}\theta + (1-C^2D^2)tg\theta\}/2(1-D^2) \qquad (A2.13)$$

Then the solution becomes

$$m_{12} = A \pm \sqrt{A^2 + C^2} \qquad (A2.14)$$

APPENDIX 3

CALCULATION OF THE OPTIMAL DISTANCE

From Figure 3.2 it is obtained that the uncertainty ellipse is given by

$$b^2x^2 + a^2y^2 = a^2b^2$$

and the line OP is given by

$$y = x\, tg(\Phi+\theta) = m\, x$$

Hence

$$b^2x^2 + a^2x^2m^2 = a^2b^2$$

$$x^2 = \frac{a^2b^2}{b^2 + a^2m^2}$$

$$FP^2 = x^2 + y^2 = (1+m^2)x^2 = (1+m^2)\frac{a^2b^2}{b^2 + a^2m^2}$$

or

$$FP = b\sqrt{\frac{1 + m^2}{C^2 + m^2}} \qquad (A3.1)$$

where $C = b/a$ and $m = tg(\Phi+\theta)$.

It is required to locate the optimal point at a distance from the position of the estimate at which there is only a very negligible probability of hit. Such a requirements is met if the optimal distance E is from 2.5 to 4 times the distance FP in Figure 3.2.

A practical solution is

$$E = 3\cdot FP = 3b\sqrt{\frac{1 + m^2}{C^2 + m^2}} \qquad (A3.2)$$

The distance between the optimal point and the hit point can be found from Figure 3.2:

When

$N \leq 1$, and $|\Phi| \geq |\alpha|$

$$A = V_T T = E \cos\alpha + V_S T \cos (C_3 - C)$$

$$T = \frac{E \cos\alpha}{V_T - V_S \cos (C_3 - C)}$$

$$T = \frac{V_T E \cos\alpha}{V_T - V_S \cos (C_3 - C)} = \frac{E \cos\alpha}{1 - N \cos (C_3 - C)}$$

When

$N > 1$, and $|\Phi| < |\alpha|$

$$A = V_T T = V_S T \cos (C_3 - C) - E \cos\alpha$$

and

$$A = \frac{E \cos\alpha}{N \cos(C_3 - C) - 1}$$

Hence in general

$$A = \left|\frac{E \cos\alpha}{1 - N \cos (C_3 - C)}\right|$$

"A" is infinitely large when $N \cos (C_3 - C) = 1$.

As the discussion of section 2.3 has shown

$\Phi \simeq -\theta$ when $C = b/a < 0.2$

Hence for $C < 0.2$

$m = 0$ and $E = 3 \cdot a$

For c > 0.2 value of A can become quite small, which is for example in disagreement with conventional use of fixed locking distances.

Hence the following value of A is recommended

$$A = \max\left(A1, \left|\frac{3a \cos\alpha}{1 - N \cos(C_3 - C)}\right|\right) \qquad (A3.3)$$

where A1 is a predetermined fixed distance.

APPENDIX 4

DEVELOPMENT OF EQUATIONS FOR OPTIMAL GUIDANCE

A thorough analysis of left and right runs, such as visualized in Figure 3.3, reveals that the turn angle is given by

$$\beta = K(C_2 - C_1) + Z \qquad (A4.1)$$

where $Z = 2\pi$ when $K(C_2-C_1) < 0$ and $Z = 0$ when $K(C_2-C_1) \geq 0$. Using this convention for the value of "Z" assures that

$$2\pi > \beta \geq 0 \quad \text{whenever} \quad 2\pi > C_1, C_2 \geq 0$$

The values of the turn parameter "K" is furthermore such that $K = 1$ for right turns and $K = -1$ for left turns.

The time required for the torpedo to turn an angle β becomes

$$T = \beta R/V \qquad (A4.2)$$

where R is the turn radius and V the torpedo velocity.

The corresponding movements in the X- and Y-directions are correspondingly

$$X_2 - X_1 = K\,R\,(\cos C_1 - \cos C_2) \qquad (A4.3)$$

$$Y_2 - Y_1 = K\,R\,(\sin C_2 - \sin C_1) \qquad (A4.4)$$

It must be clearly understood that equations (A4.1) through (A4.4) and the mathematical relations to follow are only valid when all course angles (torpedo- and estimate-courses) are adjusted to angles between 0 and 2π.

Using these equations for the turns and the notation ada[illegible]ed in Figure 3.1, the coordinates of the hit-point can be found to be

$$Y_H = Y + V_S T \cos C$$

$$Y_H = Y_o + K_1 R(\sin C_1 - \sin C_o) + V_T T_2 \cos C_1 + K_2 R(\sin C_3 - \sin C_1) + A \cos C_3$$

and

$$X_H = X + V_S T \sin C$$

$$X_H = X_o + K_1 R(\cos C_o - \cos C_1) + V_T T_2 \sin C_1 + K_2 R(\cos C_1 - \cos C_3) + A \sin C_3$$

where

$$T = T_1 + T_2 + T_3 + T_4$$

From which the following can be obtained

$$V_S T \cos C = P_1 - K_1 R \sin C_o + K_2 R \sin C_3 + R(K_1 - K_2)\sin C_1 + V_T T_2 \cos C_1 \quad \text{(A4.5)}$$

$$V_S T \sin C = Q_1 + K_1 R \cos C_o - K_2 R \cos C_3 - R(K_1 - K_2)\cos C_1 + V_T T_2 \sin C_1 \quad \text{(A4.6)}$$

Where the following terms are independent of K_1, K_2 and C_1

$$P_1 = Y_o - Y + A \cos C_3 \quad \text{(A4.7)}$$

$$Q_1 = X_o - X + A \sin C_3 \quad \text{(A4.8)}$$

The two turns are described by

$$\beta_1 = K_1(C_1 - C_o) + Z_1 \quad \text{(A4.9)}$$

$$T_1 = \beta_1 R / V_T \quad \text{(A4.10)}$$

and

$$\beta_2 = K_2(C_3-C_1) + Z_2 \quad (A4.11)$$

$$T_3 = \beta_2 R/V_T \quad (A4.12)$$

where the rules governing K_1, K_2, Z_1 and Z_2 are as stated in the begining of this appendix.

For abbreviation the following notations are adapted

$$D_5 = Q_1\cos C - P_1\sin C \quad (A4.13)$$

and

$$D_3 = (T_1 + T_3 + T_4)V_S = (\beta_1 R/V_T + \beta_2 R/V_T + A/V_T)V_S$$

$$D_3 = N(A + (\beta_1 + \beta_2)R) \quad (A4.14)$$

where

$$N = V_S/V_T \quad (A4.15)$$

Equation (A4.5) yields

$$V_S T_2\cos C + D_3\cos C = P_1 - K_1R\sin C_o + K_2R\sin C_3 + R(K_1-K_2)\sin C_1 + V_T T_2\cos C_1$$

or

$$T_2 = \frac{P_1 - K_1R\sin C_o + K_2R\sin C_3 + R(K_1-K_2)\sin C_1 - D_3\cos C}{(N\cos C - \cos C_1)V_T} \quad (A4.16)$$

and correspondingly

$$T_2 = \frac{Q_1 + K_1R\cos C_o - K_2R\cos C_3 - R(K_1-K_2)\cos C_1 - D_3\sin C}{(N\sin C - \sin C_1)V_T} \quad (A4.17)$$

A combination of equations (A4.16) and (A4.17) yields

$$(P_1-K_1R\ \sin C_o + K_2R\ \sin C_3 + R(K_1-K_2)\sin C_1 - D_3\cos C)(N\ \sin C-\sin C_1) =$$

$$(Q_1+K_1R\ \cos C_o - K_2R\ \cos C_3 - R(K_1-K_2)\cos C_1-D_3\sin C)(N\ \cos C-\cos C_1)$$

$$D_3(\sin C\cos C_1-\cos C\sin C_1) - NRK_1(\cos C\cos C_1+\sin C\sin C_1-\cos C\cos C_o-\sin C\sin C_o)$$

$$+ NRK_2(\cos C\ \cos C_1+\sin C\ \sin C_1-\cos C\ \cos C_3-\sin C\ \sin C_3)$$

$$- K_1R(\cos C_o\cos C_1+\sin C_o\sin C_1-\cos^2C_1-\sin^2C_1)+$$

$$+ K_2R(\cos C_3\cos C_1+\sin C_3\sin C_1-\cos^2C_1-\sin^2C_1)$$

$$- Q_1\cos C_1 + P_1\sin C_1 + N(Q_1\cos C - P_1\sin C) = 0$$

or

$$D_3\sin(C-C_1)-NRK_1(\cos(C-C_1)-\cos(C-C_o)) + NRK_2(\cos(C-C_1)-\cos(C-C_3))$$

$$- K_1R(\cos(C_o-C_1)-1) + K_2R(\cos(C_3-C_1)-1) - Q_1\cos C_1 + P_1\sin C_1 + N\ D_5 = 0$$

Since D_3 in general is dependent on C_1, this equation cannot be solved analytically, and the solution to the guidance problem becomes a iteration problem of finding the value of C_1 for which

$$F(C_1) = 0$$

where

$$\begin{aligned} F(C_1) = {} & D_3\sin(C-C_1) - NRK_1(\cos(C-C_1)-\cos(C-C_o)) \\ & + NRK_2(\cos(C-C_1)-\cos(C-C_3)) - K_1R(\cos(C_o-C_1)-1) \\ & + K_2R(\cos(C_3-C_1)-1) - Q_1\cos C_1 + P_1\sin C_1 + N\ D_5 \end{aligned} \qquad (A4.18)$$

APPENDIX 5

A CONTINUETY ANALYSIS OF $F(C_1)$

The iteration function was in Appendix 4 found to be

$$F(C_1) = D_3\sin(C-C_1) - NRK_1(\cos(C-C_1)-\cos(C-C_o)) + NRK_2(\cos(C-C_1)-\cos(C-C_3)) - K_1R(\cos(C_o-C_1)-1) + K_2R(\cos(C_3-C_1)-1) - Q_1\cos C_1 + P_1\sin C_1 + N\,D_5 \qquad (A5.1)$$

In order for this function to be suitable for the type of iteration (Newton-Raphson-iteration) employed in chapter 3, both the function $F(C_1)$ and its derivative should be continous for all values of C_1. An uncritical use of $F(C_1)$ will not fullfill these requirements, and it is the purpose of this appendix to show what precautions must be taken in order to make the function suitable for iteration.

In the discussion to follow the following notation will be adapted

$$C_1^- = C_9 - \varepsilon$$

$$C_1^+ = C_9 + \varepsilon$$

where C_9 is either C_o or C_3 and where ε is an arbitrary small positive angle.

A substitution of C_1^- and C_1^+ into equation (A5.1) when $C_9 \neq C$, yields

$$F(C_1^-) = D_3^- \sin(C-C_9) + D_6^-$$

and

$$F(C_1^+) = D_3^+ \sin(C-C_9) + D_6^+$$

For $C_9 = C_o$, the constants D_3 and D_6 becomes

$$D_3^-(C_o) = N(A+R(-K_1^-\varepsilon + Z_1^- + \beta_2)) \tag{A5.2}$$

$$D_3^+(C_o) = N(A+R(K_1^+\varepsilon + Z_1^+ + \beta_2)) \tag{A5.3}$$

$$D_6(C_o) = NRK_2(\cos(C-C_o)-\cos(C-C_3)) + K_2R(\cos(C_3-C_o)-1)$$
$$- Q_1\cos C_o + P_1\sin C_o + N\,D_5 \tag{A5.4}$$

and for $C_9 = C_3$

$$D_3^-(C_3) = N(A+R(\beta_1 + K_2^-\varepsilon + Z_2^-)) \tag{A5.5}$$

$$D_3^+(C_3) = N(A+R(\beta_1 - K_2^+\varepsilon + Z_2^+)) \tag{A5.6}$$

$$D_6(C_3) = - NRK_1(\cos(C-C_3)-\cos(C-C_o)) - K_1R(\cos(C_o-C_3)-1)$$
$$- Q_1\cos C_3 + P_1\sin C_3 + N\,D_5 \tag{A5.7}$$

Dependent on if $K_{1,2}^-$ and $K_{1,2}^+$ are equal or unequal, the continuety of D_3 needs further investigation wheras D_6 is obviously continuous both at $C_1 = C_o$ and $C_1 = C_3$.

Since $\varepsilon > 0$ and

$$2\pi > -K_1^-\varepsilon + Z_1^- \geq 0$$

Z_1^- can mathematically be expressed as

$$Z_1^- = \pi(1+K_1^-)$$

and similarly

$$Z_1^+ = \pi(1 - K_1^+)$$

And hence at $C_1 = C_o$

$$D_3^{+}(C_o) - D_3^{-}(C_o) = (K_1^{+} + K_1^{-})N\varepsilon + (K_1^{+} + K_1^{-})N\pi \qquad (A5.8)$$

Similarly at $C_1 = C_3$

$$Z_2^{-} = \pi(1-K_2^{-})$$

$$Z_2^{+} = \pi(1+K_2^{+})$$

and

$$D_3^{+}(C_3) - D_3^{-}(C_3) = -(K_2^{+} + K_2^{-})N\varepsilon + (K_2^{+} + K_2^{-})N\pi \qquad (A5.9)$$

Only when

$$K_1^{-} = -K_1^{+} \quad \text{and} \quad K_2^{-} = -K_2^{+}$$

equations (A5.8) and (A5.9) become zero.

Hence when K_1 and K_2 changes sign at $C_1 = C_o$ and $C_1 = C_3$ respectively

$$F(C_o+\varepsilon) = F(C_o-\varepsilon) \quad \text{and} \quad F(C_3+\varepsilon) = F(C_3-\varepsilon)$$

The derivative of $F(C_1)$ with respect to C_1 is

$$F'(C_1) = NR(K_1-K_2)\sin(C-C_1) - D_3\cos(C-C_1) - NRK_1\sin(C-C_1)$$
$$+ NRK_2\sin(C-C_1) - K_1R\sin(C_o-C_1) + K_2R\sin(C_3-C_1)$$
$$+ Q_1\sin C_1 + P_1\cos C_1$$

or

$$F'(C_1) = -D_3\cos(C-C_1) - K_1R\sin(C_o-C_1) + K_2R\sin(C_3-C_1)$$
$$+ Q_1\sin C_1 + P_1\cos C_1$$

Similarly for that obtained for $F(C_1)$

$$F'(C_1^-) = -D_3^- \cdot \cos(C-C_9) + D_7^+$$

$$F'(C_1^+) = -D_3^+ \cdot \cos(C-C_9) + D_7^+$$

where

$$D_7(C_o) = K_2R \sin(C_3-C_o) + Q_1\sin C_o + P_1\cos C_o$$

$$D_7(C_3) = K_1R \sin(C_o-C_3) + Q_1\sin C_3 + P_1\cos C_3$$

Since D_7 is obviously continuous at $C_1 = C_o$ and $C_1 = C_3$ and proof already has been given on the continuety of D_3 at these angles, it must be concluded that

$$F'(C_o+\varepsilon) = F'(C_o-\varepsilon) \quad \text{and} \quad F'(C_3+\varepsilon) = F'(C_3-\varepsilon)$$

whenever K_1 changes sign at $C_1 = C_o$ and K_2 changes sign at $C_1 = C_3$.

The continuity requirement of changing the sign of the proper turn parameter when $C_1 = C_o$ and $C_1 = C_3$ is by no mean a surprise, since this is the mathematical method to transform a left turn into a right turn of equivalent size, or vica versa a right turn into a left turn.

In the mathematical formulation of the iteration process it becomes natural to define the turn parameter of for example the second turn in the following manner

$$K_2 \quad \text{for} \quad C_3 > C_1 \geq 0$$

and

$$-K_2 \quad \text{for} \quad 2\pi > C_1 \geq C_3$$

This implies mathematically a change of K_2 from $C_1 = 2\pi-\varepsilon$ to $C_1 = \varepsilon$ (or vica versa), which must be avoided to keep $F(C_1)$ and $F'(C_1)$ continuous at $C_1 = 0$. The proof that K_1 and K_2 must be kept unchanged at $C_1 = 0$ is quite straight forward using the above adapted notation and will not be given in this text.

If the first turn β_1 is restricted to a value not exceeding π radiaus as suggested in chapter 3, the value of the turn parameter K_1 must change at $C_o+\pi$. In order to investigate the continuety of $F(C_1)$ and $F'(C_1)$ at $C_1 = C_o+\pi$, the following notation is adapted

$$C_1^- = C_o + \pi - \varepsilon$$

$$C_1^+ = C_o + \pi + \varepsilon$$

(If $C_o \geq \pi$, the π radians should be subtracted in the above formulas, but this will make no difference in the proof.)

$$F(C_1^-) = D_3^- \sin(C-C_o-\pi) + NRK_1^-(\cos(C-C_o-\pi)-\cos(C-C_o))$$
$$+ NRK_2(\cos(C-C_1)\cos(C-C_3)) - K_1^-R(\cos(C_o-C_o-\pi)-1)$$
$$+ K_2R(\cos(C_3-C_1)-1) - Q_1\cos C_1 + P_1\sin C_1 + N\,D_5$$

or

$$F(C_1^-) = -D_3^- \sin(C-C_o) + D_8^-$$

where

$$D_3^- = N(A+R(K_1^-(\pi-\varepsilon) + Z_1^- + \beta_2))$$

$$D_8^- = -2NRK_1^-\cos(C-C_o) + 2K_1^-R + D_6$$

$$D_6 = NRK_2(\cos(C-C_1)-\cos(C-C_3)) + K_2R(\cos(C_3-C_1)-1)$$
$$-Q_1\cos C_1 + P_1\sin C_1 + N\,D_5$$

Similarly

$$F(C_1^+) = D_3^-\sin(C-C_o) + D_8^+$$

where

$$D_3^+ = N(A+R(K_1^+(\pi+\varepsilon) + Z_1^+ + \beta_2)$$

$$D_8^+ = -2NRK_1^+ \cos(C-C_o) + 2K_1^+R + D_6$$

The restriction of β_1 to a value not exceeding π radians imply that

$$K_1^- = 1$$

$$K_1^+ = -1$$

such that

$$D_3^- = N(A+R(\pi-\varepsilon+0+\beta_2)) = N(A+R(\pi-\varepsilon+\beta_2))$$

$$D_8^- = -2R(N \cos(C-C_o)-1) + D_6$$

and

$$D_3^+ = N(A+R(-\pi-\varepsilon+2\pi+\beta_2)) = N(A+R(\pi-\varepsilon+\beta_2))$$

$$D_8^- = 2R(N \cos(C-C_o)-1) + D_6$$

From which the following conclusion must be drawn

$$D_3^- = D_3^+ \quad \text{and} \quad D_8^- \neq D_8^+$$

and hence

$$F(C_o+\pi-\varepsilon) \neq F(C_o+\pi+\varepsilon) \tag{A5.10}$$

Similar values of $F'(C_1)$ becomes

$$F'(C_1^-) = D_3^- \cos(C-C_o) + D_7$$

where

$$D_7 = -K_2R \sin(C_3-C_o) - Q_1\sin C_o - P_1\cos C_o$$

and

$$F'(C_1^{+}) = D_3^{+} \cos(C-C_o) + D_7$$

Since it is already proven that $D_3^{-} = D_3^{+}$ it must be concluded that

$$F'(C_o+\pi-\varepsilon) = F'(C_o+\pi+\varepsilon) \qquad (A5.11)$$

The restriction on the value of β_1 not to exceeded π radians, will hence make $F(C_1)$ discontinuous and $F'(C_1)$ continuous at $C_1 = C_o+\pi$. The implication of this will be discussed in chapter 3. Note however that the continuous term D_6 is of the order of P_1 or Q_1, wheras the discontinuity is of the order of R.

TABLE OF SIMULATION PROGRAMS

PROGRAM NAME	PROGRAM FUNCTION	FLOWCHART FIGURE NUMBER
MAIN	Main Program for torpedo guidance	3.4 to 3.7
CCT	Checking of collision time and start of multipass	3.8
COP	Guidance program for stage 3	3.9 and 3.10
CT	Compare total time for new and last best solution	3.11
FA	Finds distance A	3.12
FAZ	Finds argument making F(C5) = 0	3.13
FBA	Finds torpedo end-course closest to the last one used	3.14
FBS	Finds the best of all possible solutions	3.15
FCA	Finds wanted corridor angle	3.16
FCT	Find time to collision with no turns	3.17
FES	Find every solution of F(C1) = 0	3.18
FET	Finds the functional values and turns	3.19
FK	Finds the values of the turn parameters	3.20
FNS	Finds a new solution using the latest value of C1	3.21
FST	Finds the shortest time to collision with no turns	3.22
FTS	Finds transposed solution	3.23
FTT	Finds total time required to hit the estimate	3.24
NEP	Finds new estimate position	3.25
NTS	Finds new torpedo poisiton in straight path	3.26
NTT	Finds new torpedo position in turns	3.27
STR	Finds new positions of both torpedo and estimate	3.28
TA	Finds wanted torpedo end-course from corridor angle	3.29
TEST	Test for necessary change of second turnparameter	3.30
TURN	Finds new positions of both torpedo and estimate	3.31

TABLE OF PROGRAM SYMBOLS

Temporary storage symbols are not listed

A	=	Distance between optimal point and hit point
A0	=	Major axis of uncertainty-ellipse
A3	=	Tiltangle θ at prediction time
A7	=	Distance from optimal point to estimate position
B0	=	Minor axis of uncertainty-ellipse
C	=	Ratio of minor and major axes of uncertainty-ellipse
C	=	Estimated target course
C0	=	Current torpedo course
C1	=	Torpedo course in straight trajectory
C3	=	Currently used torpedo end-course
C6	=	Possible torpedo end-course
C7	=	Possible torpedo end-course
ΔC1	=	Accuracy of iteration angle C1
D	=	Ratio of minor and major axes of target ellipse
D8	=	Displacement between each tried solution
D9	=	Transposed displacement
E1	=	First turn angle in radians
E2	=	Second turn angle in radians
G	=	Time between repetitive computations
G1	=	Constant used for acceptance of straight path
G2	=	Time between repetitive computations in stage 3
H1	=	Corridor angle Φ
H2	=	Angle α
I	=	Trajectory number used for plotting
J1	=	Number of accepted solutions
J3	=	Number of 1 degree turns of torpedo

J4 = Number of employed iterations
J5 = Number of tried transposed solutions
J6 = Number of trieded mode 1 solutions

K1 = Turn parameter of first turn
K2 = Turn parameter of second turn

L = Total time to collision
L1 = Time required for the first turn
L2 = Time required for the straight path
L3 = Time required for the second turn
L4 = Time required from the optimal point to the hit point

M1 = Indicator for passing hit-time
M2 = Stage indicator
M3 = Angle indicator
M4 = Carry out plan indicator
M5 = Indicator of plan progress

N = Ratio of estimate and torpedo velocities
N9 = Distance between next optimal point and hit point

O2 = Own ship velocity

P2 = 2π
P3 = 1 degree in radians
P4 = π
P5 = $\pi/2$
P6 = $3\pi/2$
P7 = 5 degrees in radians

R = Minimum turn radius of torpedo

S = Scalefactor for plotting
S1 = Start angle of iteration process
S2 = Functional value of F(C1)
S5 = Angle variable
S6 = Derivative of F(C1)

T = Total time to collision
T9 = Prediction time

U = Current time measured from firing time
U0 = Last time for updating of own ship position
U1 = 15 seconds before start of second turn
U2 = Time of start of second turn
U4 = Collision time
U5 = Time of next multipass calculation
U6 = Time of last computation
U7 = Last time for updating of torpedo position
U8 = Last time for updating of estimate position
U9 = Time for next updating of torpedo position

V = Estimated target velocity
VS = Estimated target velocity
VT = Torpedo velocity
V9 = Torpedo velocity

W = Time of next repetitive computation

X = Current X-position of target estimate
X0 = Current X-position of torpedo
X8 = Current X-position of own ship

Y = Current Y-position of target estimate
Y0 = Current Y-position of torpedo
Y8 = Current Y-position of own ship

Z6 = Error or no solution indicator
Z7 = Error or no solution indicator
Z8 = Error or no solution indicator
Z9 = Error or no solution indicator

```
3     REM ***  PRØGRAM LISTING ØF ØPTIMAL GUIDANCE  ***

5     EXTERNAL BPLØTS,BPLØT,BWHERE,BSYMBL,BNUMBR,BAXIS
8     REM ******************************************************
9     REM ** INPUT DATA **
10    LET B=0
11    REM ************************************************************
14    PRINT"Ø2=";
15    INPUT 02
16    PRINT "     ";Ø2
20    PRINT "AO=";
22    INPUT AO
24    PRINT "     ";AO
26    PRINT "BO=";
28    INPUT BO
30    PRINT "     ";BO
32    PRINT "XO=";
34    INPUT XO
36    PRINT "     ";XO
38    PRINT "YO=";
40    INPUT YO
42    PRINT "     ";YO
44    PRINT "CO=";
46    INPUT CO
48    PRINT "     ";CO
50    PRINT "X=";
52    INPUT X
54    PRINT "     ";X
56    PRINT "Y=";
58    INPUT Y
60    PRINT "     ";Y
62    PRINT "C=";
64    INPUT C
66    PRINT "     ";C
68    PRINT "V=";
70    INPUT V
72    PRINT "     ";V
74    PRINT "S=";
76    INPUT S
78    PRINT "     ";S
80    PRINT "I=";
82    INPUT I
84    PRINT "     ";I
112   LET X8=XO
114   LET Y8=YO
116   LET X7=X
118   LET Y7=Y
119   LET Ø2=Ø2*0.5144
120   LET P2=6.28318530
121   LET P3=P2/360
122   LET P4=0.5*P2
123   LET P5=0.25*P2
124   LET P6=P4+P5
125   LET P7=5*P3
130   LET S5=CO*P3
135   GØSUB 1700
140   LET CO=S5
145   LET Ø1=S5
150   LET S5=C*P3
155   GØSUB 1700
160   LET C=S5
170   LET V9=30*0.5144
```

```
180      LET N=V/30
210      LET R=V9*72/P2
212      CALL BPLØTS(0)
214      LET I7=16+I
216      IF I#0 THEN 256
218      CALL BAXIS(0.0,"X-AXIS",-6,6,0,0,S)
220      CALL BAXIS(0.0,"Y-AXIS",6,8,90,0,S)
222      CALL BSYMBL(0.5,9,0.14,"TØRPEDØ TRAJECT",0,15)
224      CALL BWHERE(I3,I4,I5)
226      CALL BSYMBL(I3,9,0.14,"ØRY PLØT",0,8)
227      GØSUB 8100
228      LET U=0 ; U8=0
230      CALL BSYMBL(X/S,Y/S,0.21,25,0,-1)
232      LET U=U+10
233      IF U >800 THEN 254
234      GØSUB 2000
236      IF X>6*S THEN 254
238      IF Y>8*S THEN 254
240      IF X<0 THEN 254
242      IF Y<0 THEN 254
244      LET I1=INT(U/100)-U/100
245      LET I2=0.14;I4=25
246      IF ABS(I1)<0.001 THEN 250
248      LET I2=0.07;I4=18
250      CALL BSYMBL(X/S,Y/S,I2,I4,0,-1)
252      GØTØ 232
254      GØTØ 255
255      GØSUB 8000
256      LET I6=8.5-I*0.2
257      CALL BSYMBL(0.5,I6+0.07,0.14,I7,0,-1)
258      CALL BWHERE(I3,I4,I5)
259      CALL BSYMBL(I3,I6,0.14," : WITH A=",0,10)
260      CALL BWHERE(I3,I4,I5)
262      CALL BNUMBR(I3,I6,0.14,A0,0,-1)
264      CALL BWHERE(I3,I4,I5)
266      CALL BSYMBL(I3,I6,0.14," AND B=",0,7)
268      CALL BWHERE(I3,I4,I5)
270      CALL BNUMBR(I3,I6,0.14,B0,0,-1)
272      CALL BWHERE(I3,I4,I5)
274      CALL BSYMBL(I3,I6,0.14," METERS",0,7)
284      REM ************************************************************
286      REM ** MAIN PRØGRAM **
288      LET B=0
290      REM ************************************************************
291      LET       U0=0
292      LET U=0
294      LET W=0
296      LET X=X7
298      LET Y=Y7
300      LET U8=0
302      LET G2=5
303      LET J6=0
305      LET G=5
310      LET M2=0
315      LET G1=10
320      GØSUB 4700
322      IF Z7=0 THEN 336
325      LET T9=0
330      GØTØ 336
335      LET T9=L-L4+D9/V9
336      LET J6=J6+1
337      IF J6#10 THEN 340
338      PRINT "NØ SØLUTIØN ØBTAINABLE AT U=";U
339      GØTØ 4350
340      GØSUB 3000
```

```
345   GØSUB 3500
346   LET G1=2
348   LET U6=U
350   LET T=L+U6-U
351   LET U6=U
352   LET A9=A
355   GØSUB 910
360   IF Z7=1 THEN 335
362   IF ABS(A-A9)>50 THEN 350
365   IF ABS(T-L)>20 THEN 335
367   LET J6=0
370   LET U2=U+L1+L2
380   LET U1=U2-15
390   LET W=W+G
400   IF W>U1 THEN 450
410   GØSUB 1000
420   IF E1#0 THEN 350
430   GØSUB 810
440   GØTØ 350
450   LET M2=1
455   GØSUB 1000
470   GØSUB 810
475   GØSUB 4200
480   LET W=W+G
481   IF W>U2 THEN 500
482   LET T=L+U6-U
483   LET U6=U
484   GØSUB 910
486   IF Z7=1 THEN 500
488   IF ABS(T-L)>10 THEN 500
490   LET U2=U+L1+L2
495   GØTØ 455
500   LET U9=U2
510   GØSUB 830
512   GØSUB 4200
513   LET S5=C3-P3
514   GØSUB 1700
515   LET C1=S5
516   LET T9=T9+U6-U
517   LET T=T+U6-U
518   LET K9=K2
520   GØSUB 3900
521   IF E2<P5 THEN 523
522   LET K6=K9
523   LET A=N9
524   LET G9=C1
525   LET M9=0
526   GØSUB 6500
527   IF Z8=1 THEN 305
530   LET M9=1
532   LET U6=U
534   GØTØ 547
535   LET T9=T9+U6-U
536   LET T=L+U6-U
538   LET K6=K1
539   LET K8=K2
540   LET G9=C1
542   LET U6=U
543   GØSUB 950
544   IF Z7=1 THEN 526
545   IF ABS(T-L)>20 THEN 526
547   GØSUB 1000
550   IF U#W THEN 555
552   LET W=W+G
555   IF E1#0 THEN 535
```

```
560   LET M2=2
565   LET M1=0
570   LET U5=U+L+L
575   LET G=G2
580   LET M4=0
585   LET U4=U+L
592   LET M5=C
595   GØSUB 6800
600   LET R3=K1
605   LET R4=K2
610   LET R5=E1
615   LET R6=E2
620   LET R7=L2
622   LET R8=C1
625   IF Z8=1 THEN 305
627   IF M5=3 THEN 705
630   LET T9=T9+U6-U
632   LET T=L+U6-U
633   LET U6=U
634   GØSUB 3900
635   GØSUB 950
640   IF Z7=1 THEN 660
645   IF ABS(T-L)<10 THEN 580
660   LET K1=R3
665   LET K2=R4
670   LET E1=R5
675   LET E2=R6
680   LET L2=R7
682   LET C1=R8
685   LET M4=1
690   GØSUB 6800
700   IF Z8=1 THEN 305
705   LET D9=10*INT(ABS(A)/200)+100
707   LET N9=N9-D9
710   LET T9=D9/V9
720   LET G9=C1
736   GØSUB 3900
737   LET T=T+U6-U
738   LET U6=U
742   GØSUB 950
745   IF Z7=1 THEN 755
750   IF ABS(T-L)<20 THEN 580
755   LET M9=1
757   GØSUB 6500
760   IF Z8=1 THEN 305
765   GØTØ 580
798   REM ***********************************************************
799   REM ** PRØGRAM STR **
800   LET B=0
801   REM ***********************************************************
810   LET U7=U
815   LET U=W
820   GØTØ 840
830   LET U7=U
835   LET U=U9
840   GØSUB 2000
850   GØSUB 2100
855   LET L2=L2-U+U7
870   RETURN
898   REM ***********************************************************
899   REM ** PRØGRAM FNS. FIND NEW SØLUTIØN **
900   LET B=0
901   REM ***********************************************************
910   LET T9=L-L4-G+D9/V9
915   GØSUB 5500
```

```
920     GØSUB 5800
925     IF Z6=1 THEN 990
930     GØTØ 960
950     LET A=N9
955     GØSUB 5500
960     LET S5=C1
962     GØSUB 1400
964     IF Z9=1 THEN 990
966     LET C1=S5
968     GØSUB 1600
970     IF Z7=1 THEN 990
975     LET Z7=0
980     GØSUB 4200
985     RETURN
990     LET Z7=1
995     RETURN
998     REM ***********************************************************
999     REM ** PRØGRAM TURN **
1000    LET B=0
1001    REM ***********************************************************
1010    LET J3=0
1020    LET S5=INT(E1/P3+0.5)
1025    IF S5=0 THEN 1075
1030    IF U+0.19>W THEN 1051
1035    LET J3=J3+1
1040    LET S5=S5-1
1045    LET U=U+0.2
1050    GØTØ 1025
1051    LET E1=S5*P3
1052    LET L1=E1*R/V9
1053    IF ABS(W-U)>0.01 THEN 1056
1054    LET U=W
1055    GØTØ 1060
1056    GØSUB 1900
1057    GØSUB 810
1059    GØTØ 1064
1060    GØSUB 1900
1062    GØSUB 2000
1064    IF M2=2 THEN 1074
1066    IF E1/P3>2 THEN 1074
1070    GØSUB 3950
1074    RETURN
1075    LET E1=0
1080    LET L1=0
1085    GØTØ 1060
1098    REM ***********************************************************
1099    REM ** PRØGRAM TEST. CHANGE K2 IF C3 IS PASSED FRØM S8 TØ S5
1100    LET B=0
1101    REM ***********************************************************
1110    IF S3>=0 THEN 1150
1120    IF S5<C3 THEN 1180
1130    IF S8>=C3 THEN 1180
1140    GØTØ 1170
1150    IF S8<C3 THEN 1180
1160    IF S5>=C3 THEN 1180
1170    LET K2=-K2
1180    IF S3>=0 THEN 1188
1182    IF S5<C0 THEN 1195
1184    IF S8>=C0 THEN 1195
1186    GØTØ 1192
1188    IF S8<C0 THEN 1195
1190    IF S5>=C0 THEN 1195
1192    LET K1=-K1
1195    RETURN
```

```
1198    REM ***********************************************************
1199    REM ** PRØGRAM FP. FIND PARAMETERS **
1200    LET B=0
1201    REM ***********************************************************
1202    REM FIND CØNSTANTS NECESSARY FØR FINDING F(C1)
1210    LET W0=CØS(C-C3)
1215    LET W2=CØS(C-C0)
1220    LET V3=SIN(C3)
1225    LET W3=CØS(C3)
1230    LET P1=Y0-Y+A*W3
1235    LET Q1=X0-X+A*V3
1240    LET V6=SIN(C)
1245    LET W6=CØS(C)
1250    LET D5=Q1*W6-P1*V6
1260    RETURN
1298    REM ***********************************************************
1299    REM ** PRØGRAM FFT. FIND FUNCTIØN AND TURNS **
1300    LET B=0
1301    REM ***********************************************************
1302    REM CALCULATE F(S5) FØR GIVEN S5
1305    LET E1=K1*(S5-C0)
1306    IF E1>=0 THEN 1310
1307    LET E1=E1+P2
1310    LET W7=CØS(C3-S5)
1315    LET W8=CØS(C0-S5)
1317    LET R1=K1*R
1320    LET R2=K2*R
1325    LET E2=K2*(C3-S5)
1330    IF E2>=0 THEN 1340
1335    LET E2=E2+P2
1340    LET D3=N*(A+(E1+E2)*R)
1345    LET V4=SIN(C-S5)
1350    LET W4=CØS(C-S5)
1355    LET V1= SIN(S5)
1360    LET W1=CØS(S5)
1365    LET S2=D3*V4+N*R2*(W4-W0)-N*R1*(W4-W2)
1370    LET S2=S2+P1*V1-Q1*W1-R1*(W8-1)+R2*(W7-1)+N*D5
1380    RETURN
1398    REM ***********************************************************
1399    REM ** PRØGRAM FAZ. FIND ARGUMENT MAKING F(S5) ZERØ **
1400    LET B=0
1401    REM ***********************************************************
1402    REM START ITERATIØN AT S5
1410    LET Z9=0
1420    LET J4=0
1425    GØSUB 1200
1440    GØSUB 1300
1445    LET V7=SIN(C3-S5)
1447    LET V8=SIN(C0-S5)
1450    LET S6=-D3*W4+P1*W1+Q1*V1-R1*V8+R2*V7
1455    IF S6#0 THEN 1470
1462    LET Z9=1
1465    GØTØ 1595
1470    LET S3=S2/S6
1471    IF S3=0 THEN 1595
1472    IF ABS(S3)>P5 THEN 1462
1475    LET J4=J4+1
1478     LET S8=S5
1480    LET S5=S5-S3
1482    GØSUB 1700
1484    REM CAREFUL IF S5 PASSES 0/360 DEGREES
1485    IF ABS(S8-S5)>P4 THEN 1500
1490    GØSUB 1100
1495    GØTØ 1545
1500    IF C3#0 THEN 1504
```

```
1502    LET K2=-K2
1504    IF CO#0 THEN 1510
1506    LET K1=-K1
1510    LET S6=S5
1512    IF S3<0 THEN 1526
1514    LET S5=0
1516    GØSUB 1100
1518    LET S8=P2
1520    LET S5=S6
1522    GØSUB 1100
1524    GØTØ 1545
1526    LET S5=P2
1528    GØSUB 1100
1530    LET S8=0
1532    LET S5=S6
1534    GØSUB 1100
1545    IF ABS(S3)<0.001 THEN 1585
1550    IF J4=40 THEN 1570
1560    GØTØ 1440
1570    LET Z9=1
1580    GØTØ 1595
1585    GØSUB 1300
1595    RETURN
1598    REM ********************************************************
1599    REM ** PRØGRAM FTT. FIND TØTAL TIME **
1600    LET B=0
1601    REM ********************************************************
1602    REM FIND RELEVANT TIMES FRØM GIVEN S5
1603    REM ASSUMES PARAMETERS FRØM 1200 AND 1300 TØ BE VALID
1610    LET Z7=0
1612    LET S6=(N*W6-W1)*V9
1614    IF ABS(S6)<0.00001 THEN 1625
1616    LET L2=(P1+R1*(V1-SIN(CO))+R2*(V3-V1)-D3*W6)/S6
1620    GØTØ 1640
1625    IF N=1 THEN 1690
1630    LET L2=(Q1+R1*(CØS(CO)-W1)+R2*(W1-W3)-D3*V6)/(V9*(N*V6-V1))
1640    LET L1=E1*R/V9
1642    IF L2<G1 THEN 1690
1645    LET L3=E2*R/V9
1650    LET L4=A/V9
1655    LET L=L1+L2+L3+L4
1660    GØTØ 1695
1690    LET Z7=1
1695    RETURN
1698    REM ********************************************************
1699    REM ** PRØGRAM AZ2P. ADJUST ANGLE TØ VALUE FRØM ZERØ TØ 2*PI **
1700    LET B=0
1701    REM ********************************************************
1702    REM ADJUST ANGLES5 TØ 360>S5>=0
1710    IF S5>=0 THEN 1740
1720    LET S5=S5+P2
1730    GØTØ 1710
1740    IF S5<P2 THEN 1760
1750    LET S5=S5-P2
1760    RETURN
1798    REM ********************************************************
1799    REM ** PRØGRAM AZP. ADJUST ANGLE TØ VALUE FRØM ZERØ TØ PI **
1800    LET B=0
1801    REM ********************************************************
1802    REM ADJUST ANGLE S5 TØ < PI AND PØSITIVE
1810    LET S5=ABS(S5)
1820    IF S5<=P4 THEN 1840
1830    LET S5=P2-S5
1840    RETURN
```

```
1848    REM ***************************************************
1849    REM ** PRØGRAM A2P/2. ADJUST ANGLE TØ VALUE FRØM -PI/2 TØ PI/2
1850    LET B=0
1851    REM ***************************************************
1852    REM ADJUST ANGLE S5  TØ -P5<=S5<=P5
1860    GØSUB 1700
1865    IF S5<=P5 THEN 1890
1870    IF S5>P6 THEN 1885
1875    LET S5=S5-P4
1880    GØTØ 1890
1885    LET S5=S5-P2
1890    RETURN
1898    REM ***************************************************
1899    REM ** PRØGRAM NTT. FIND NEW TØRPEDØ PØSITIØN AND CØURSE IN TURN
1900    LET B=0
1901    REM ***************************************************
1902    REM ASSUMES J3 TØ CØNTAIN THE NUMBER ØF 1 DEGREE PULSES
1910    IF J3=0 THEN 1970
1915    LET K=K1
1920    LET S5=C0+K*P3*J3
1930    GØSUB 1700
1940    LET X0=X0+K*R*(CØS(C0)-CØS(S5))
1950    LET Y0=Y0+K*R*(SIN(S5)-SIN(C0))
1960    LET C0=S5
1965    LET J3=0
1970    RETURN
1998    REM ***************************************************
1999    REM **PRØGRAM NEP. FIND NEW ESTIMATE AND ØWN PØSITIØN**
2000    LET B=0
2001    REM ***************************************************
2002    REM CURRENT TIME IS U. LAST CALCULATIØNIS SAVED IN U8
2010    LET T5=(U-U8)*V9*N
2015    IF T5=0 THEN 2050
2020    LET X=X+T5*SIN(C)
2030    LET Y=Y+T5*CØS(C)
2040    LET U8=U
2050    RETURN
2060    LET X8=X8+Ø2*SIN(Ø1)*(U-U0)
2062    LET Y8=Y8+Ø2*CØS(Ø1)*(U-U0)
2064    LET U0=U
2066    RETURN
2098    REM ***************************************************
2099    REM ** PRØGRAM NTS. FIND NEW TØRPEDØ PØSITIØN IN STREIGHT PATH
2100    LET B=0
2101    REM ***************************************************
2110    LET T5=(U-U7)*V9
2120    IF T5=0 THEN 2160
2130    LET X0=X0+T5*SIN(C0)
2140    LET Y0=Y0+T5*CØS(C0)
2160    RETURN
2398    REM ***************************************************
2399    REM ** PRØGRAM FAT. FIND BEST SØLUTIØN ØF ALL TURNS **
2400    LET B=0
2401    REM ***************************************************
2402    REM FIND ALL SØLUTIØNS FØR WHICH F(S)=0 WHEN C3 IS GIVEN
2403    REM BEST SØLUTIØN WILL BE GIVEN IN T,F1,F2,C1
2404    REM WILL INCREMENT J1 FØR EACH SØLUTIØN FØUND
2410    LET K1=1
2420    LET K2=1
2430    GØSUB 2600
2440    LET K1=1
2450    LET K2=-1
2460    GØSUB 2600
2470    LET K1=-1
2480    LET K2=1
```

```
2490  GØSUB 2600
2500  LET K1=-1
2510  LET K2=-1
2520  GØSUB 2600
2530  RETURN
2598  REM ****************************************************
2599  REM ** PRØGRAM FES. FIND EVRY SØLUTIØN **
2600  LET B=0
2601  REM ****************************************************
2602  REM FIND S5 FØR WHICH F(S5)=0 WHEN K1 AND K2 ARE GIVEN
2603  REM WILL INCREMENT J1 FØR EACH SØLUTIØN FØUND
2610  LET S1=0
2620  LET S5=0
2630  GØTØ 2670
2640  LET S5=S1
2650  LET S3=-P5
2660  GØSUB 1100
2670  LET K5=K1
2680  LET K3=K2
2690  GØSUB 1400
2700  IF Z9=1 THEN 2740
2710  GØSUB 2900
2720  IF Z6=1 THEN 2740
2730  LET J1=J1+1
2740  LET K1=K5
2750  LET K2=K3
2760  LET S8=S1
2770  LET S1=S1+P5
2780  IF S1<P2+0.1 THEN 2640
2790  RETURN
2898  REM ****************************************************
2899  REM ** PRØGRAM CT. CØMPARE TIMES **
2900  LET B=0
2901  REM ****************************************************
2902  REM TEST IF GIVEN SØLUTIØN IS BETTER THAN THE FØRMER SØLUTIØN
2905  LET Z6=0
2910  GØSUB 1600
2912  IF Z7=1 THEN 2980
2940  IF L>=T THEN 2990
2950  LET T=L
2954  LET A8=C3
2956  LET A9=A
2960  LET C1=S5
2965  LET F1=K1
2970  LET F2=K2
2975  GØTØ 2990
2980  LET Z6=1
2990  RETURN
2998  REM ****************************************************
2999  REM ** PRØGRAM TA. FIND TØRPEDØ HIT ANGLES **
3000  LET B=0
3001  REM ****************************************************
3002  REM FINDS TØRPEDØ HIT-ANGLES FRØM CØRRIDØR ANGLE H1
3005  GØSUB 5000
3010  LET H2=ABS(H1)
3020  IF H2=0 THEN 3130
3030  LET H3=N*SIN(H2)
3040  LET H4=1-H3*H3
3050  IF H4>0 THEN 3080
3060  PRINT "NØ SØLUTIØN FØR PHI =";H1/P3;" AND V=";V
3070  GØTØ 4350
3080  LET S5=ATN(H3/SQR(H4))
3090  GØSUB 1850
3100  LET H2=S5
3110  IF H1>=0 THEN 3130
```

```
3120    LET H2=-H2
3130    LET S5=C-H1+H2
3140    GØSUB 1700
3150    LET C6=S5
3160    LET S5=C+P4-H1-H2
3170    GØSUB 1700
3180    LET C7=S5
3197    RETURN
3498    REM ************************************************************
3499    REM ** PRØGRAM FBS. FIND BEST SØLUTIØN **
3500    LET B=0
3501    REM ************************************************************
3502    REM FIND TØRPEDØ PATH REQUIRING LEAST TIME
3503    REM IF J1=0 NØ SØLUTIØN HAS BEEN FØUND
3504    REM GIVES NØ SØLUTIØN WITH T>=800 SECØNDS
3505    REM ASSUMES PARAMETERS IN 3000 TØ BE VALID
3520    LET T=800
3530    LET J1=0
3540    LET C3=C6
3545    GØSUB 5800
3550    IF Z6=1 THEN 3580
3555    PRINT "A=";INT(A);" C6=";INT(C6/P3)
3560    GØSUB 2400
3580    LET C3=C7
3585    GØSUB 5800
3590    IF Z6=1 THEN 3610
3595    PRINT"A=";INT(A);" C7=";INT(C7/P3)
3605    GØSUB 2400
3610    IF J1#0 THEN 3670
3620    PRINT"NØ GEØMETRICAL SØLUTIØN"
3630    GØTØ 3850
3670    IF T=800 THEN 3774
3675    LET C3=A8
3680    LET A=A9
3700    LET K1=F1
3710    LET K2=F2
3720    LET S5=C1
3730    GØSUB 1200
3740    GØSUB 1300
3750    GØSUB 1600
3760    IF Z7=1 THEN 3780
3770    IF ABS(T-L)<1 THEN 3830
3772    GØTØ 3780
3774    PRINT "NØ SØLUTIØN WITH T<=800 SEC "
3776    GØTØ 3850
3780    PRINT "PRØGRAM ERRØR AT 3780"
3790    GØTØ 3850
3830    IF T+U<800 THEN 3870
3840    PRINT "NEXT HIT AT U=";INT(U+T)
3850    GØTØ 4350
3870    RETURN
3898    REM************************************************************
3899    REM ** PRØGRAM FK. FIND K1 AND K2 **
3900    LET B=0
3901    REM************************************************************
3902    REM MUST NØT BE USED FØR TURNS >= PI RADIANS
3905    LET S5=C3-C1
3910    IF ABS(S5)<P4 THEN 3920
3915    LET S5=-S5
3920    IF S5>=C THEN 3935
3925    LET K2=-1
3930    GØTØ 3940
3935    LET K2=1
3940    LET K8=K2
3950    LET S5=C1-C0
```

```
3955    IF ABS(S5)<P4 THEN 3965
3960    LET S5=-S5
3965    IF S5>=0 THEN 3980
3970    LET K1=-1
3975    GØTØ 3985
3980    LET K1=+1
3985    LET K6=K1
3990    RETURN
4198    REM ************************************************************
4199    REM ** PRØGRAM PTP. PLØTT TØRPEDØ PØSITIØN **
4200    LET B=0
4201    REM ************************************************************
4210    IF U=0 THEN 4250
4240    IF ABS(U3-U+10)<0.01 THEN 4250
4245    GØTØ 4340
4250    LET S5=P5-C0
4260    GØSUB 1700
4270    LET C5=S5
4280    LET U3=U
4300    IF U>800 THEN 4350
4303    LET I1=INT(U/100)-U/100
4304    LET I2=0.14
4305    IF ABS(I1)<0.001 THEN 4315
4310    LET I2=0.07
4315    CALL BSYMBL(X0/S,Y0/S,I2,I7,0,-1)
4325    LET I8=X0+150*CØS(C5)
4330    LET I9=Y0+150*SIN(C5)
4335    CALL BPLØT(I8/S,I9/S,2)
4340    RETURN
4350    CALL BPLØT(0,0,3)
4360    GØTØ 14
4698    REM********************************************************
4699    REM ** PRØGRAM FCT. FIND CØLLITIØN TIME **
4700    LET B=0
4701    REM********************************************************
4702    REM FIND CØLLITIØN TIME FØR A=K1=K2=0
4705    LET Z7=0
4710    LET Q1=X0-X
4715    LET P1=Y0-Y
4720    IF P1#0 THEN 4760
4725    LET S7=N*CØS(C)
4730    LET S6=1-S7*S7
4735    IF S6<0 THEN 4860
4740    LET S6=ATN(S7/SQR(S6))
4745    LET S3=0
4750    GØTØ 4800
4760    LET S3=ATN(Q1/P1)
4765    LET S7=N*SIN(C-S3)
4770    LET S6=1-S7*S7
4775    IF S6<0 THEN 4860
4780    LET S6= ATN(S7/SQR(S6))
4800    LET S5=S6+S3
4805    LET T9=800
4810    GØSUB 4900
4815    LET S5=S5+P4
4820    GØSUB 4900
4825    LET S5=-S6+S3
4830    GØSUB 4900
4835    LET S5=S5+P4
4840    GØSUB 4900
4845    IF T9=800 THEN 4860
4846    PRINT
4847    PRINT"T=";INT(T9);"  C1=";INT(S4/P3)
4850    RETURN
4860    LET Z7=1
4865    RETURN
```

```
4898   REM*****************************************************
4899   REM ** PRØGRAM FST. FIND SHØRTEST TIME **
4900   LET B=0
4901   REM*****************************************************
4905   GØSUB 1700
4907   LET S2=N*CØS(C)-CØS(S5)
4910   IF S2=0 THEN 4925
4915   LET L=P1/(S2*V9)
4920   GØTØ 4940
4925   LET S2=N*SIN(C)-SIN(S5)
4930   IF S2=0 THEN 4955
4935   LET L=Q1/(S2*V9)
4940   IF L<0 THEN 4955
4945   IF L>=T9 THEN 4955
4950   LET T9=L
4952   LET S4=S5
4955   RETURN
4998   REM ****************************************************
4999   REM ** PRØGRAM FCA. FIND CØRRIDØR ANGLE **
5000   LET B=0
5001   REM ****************************************************
5005   LET C9=C
5007   GØSUB 2060
5010   LET T5=N*V9*T9
5015   LET X9=X+T5*SIN(C)-(X8+Ø2*T9*SIN(Ø1))
5020   LET Y9=Y+T5*CØS(C)-(Y8+Ø2*T9*CØS(Ø1))
5030   LET Z1=SGN(X9)
5040   LET Z2=SGN(Y9)
5050   IF X9#0 THEN 5080
5060   LET H5=(1-Z2)*P5
5070   GØTØ 5140
5080   IF Y9#0 THEN 5110
5090   LET H5=P4-Z1*P5
5100   GØTØ 5140
5110   LET H6=ATN(ABS(Y9/X9))
5120   LET H5=P4-Z1*P5-Z1*Z2*H6
5140   LET S5=H5-C
5150   GØSUB 1850
5155   LET A3=S5
5160   LET A4=TAN(A3)
5170   LET D=10/100
5200   LET C=B0/A0
5210   LET C=C*C
5220   LET D=D*D
5230   IF A4#0 THEN 5260
5240   LET H1=P5-A3
5250   GØTØ 5340
5260   LET A2=((D-C)/A4+(1-C*D)*A4)/(2*(1-D))
5270   IF A3>=0 THEN 5300
5280   LET A5=A2+SQR(A2*A2+C)
5290   GØTØ 5310
5300   LET A5=A2-SQR(A2*A2+C)
5310   LET S5=ATN(A5)
5320   GØSUB 1850
5330   LET H1=S5-A3
5340   LET A7=3*A0
5345   LET C=C9
5350   RETURN
5498   REM ****************************************************
5499   REM ** PRØGRAM FBA. FIND BEST ANGLE **
5500   LET B=0
5501   REM ****************************************************
5510   GØSUB 3000
5525   LET C9=C3
5530   LET S5=C3-C6
```

```
5540    GØSUB 1800
5550    LET S6=S5
5560    LET S5=C3-C7
5570    GØSUB 1800
5580    IF S5<S6 THEN 5610
5590    LET C3=C6
5600    GØTØ 5620
5610    LET C3=C7
5615    REM CAREFULL IF C3 PASSES 0/360 DEGREES
5620    LET C8=C3
5630    IF ABS(C8-C9)<P4 THEN 5680
5640    IF C8>C9 THEN 5670
5644    IF C1<=P4 THEN 5650
5646    LET C8=C8+P2
5648    GØTØ 5680
5650    LET C9=P2-C9
5660    GØTØ 5680
5670    IF C1<=P4 THEN 5676
5672    LET C9=C9+P2
5674    GØTØ 5680
5676    LET C8=P2-C8
5680    IF C1=C3 THEN 5730
5690    IF C1>C8 THEN 5720
5700    IF C1<=C9 THEN 5740
5710    GØTØ  5730
5720    IF C1>=C9 THEN 5740
5730    LET K2=-K2
5740    RETURN
5798    REM ***************************************************
5799    REM ** PRØGRAM FA. FIND A **
5800    LET B=0
5801    REM ***************************************************
5805    LET Z6=0
5810    LET S5=C3-C
5820    GØSUB 1700
5825    LET B=S5
5830    LET B=N*CØS(B)
5840    IF B=1 THEN 5870
5850    LET A=ABS(A7*CØS(H2)/(1-B))
5854    IF A>200 THEN 5860
5856    LET A=200
5860    LET D9=10*INT(A/200)+100
5862    LET N9=A-D9
5865    RETURN
5870    LET Z6=1
5880    RETURN
6498    REM ****************************************************
6499    REM ** PRØGRAM FTS.FIND TRANSPØSED SØLUTIØN **
6500    LET B=0
6501    REM ****************************************************
6505    LET Z8=0
6520    LET J5=0
6525    LET D8=25*(INT(ABS(A)/1000)+1)
6527    IF M9=1 THEN 6690
6530    LET K1=K6
6535    LET K2=K8
6540    LET C1=G9
6545    GØSUB 960
6550    IF Z7=1 THEN 6690
6555    IF ABS(T-L)<20 THEN 6770
6690    LET J5=J5+1
6700    IF J5=20 THEN 6750
6720    LET A=A-D8
6730    LET N9=N9-D8
6732    LET T9=T9+D8/V9
```

```
6734   GØSUB 5500
6740   GØTØ 6530
6750   LET Z8=1
6760   RETURN
6770   LET T=L
6790   RETURN
6798   REM **********************************************************
6799   REM ** PRØGRAM CØP. CARRY ØUT PLAN **
6800   LET B=0
6801   REM **********************************************************
6803   IF M5=1 THEN 6860
6804   IF M5=2 THEN 6915
6805   IF INT(E1/P3+0.5)<1 THEN 6860
6810   GØSUB 1000
6815   IF U#W THEN 6860
6820   LET W=W+G
6825   GØSUB 4200
6830   GØSUB 7000
6835   IF Z8=0 THEN 6850
6840   RETURN
6850   IF M4=1 THEN 6857
6855   RETURN
6857   IF E1#0 THEN 6810
6860   LET U9=U+L2
6862   LET M5=1
6865   IF U9<W THEN 6910
6870   GØSUB 810
6875   GØSUB 4200
6880   LET W=W+G
6885   GØSUB 7000
6890   IF Z8=0 THEN 6900
6895   RETURN
6900   IF M4=1 THEN 6865
6905   RETURN
6910   GØSUB 830
6915   LET E1=E2
6920   LET K1=K2
6922   LET M5=2
6925   GØSUB 1000
6927   LET E2=E1
6930   IF U#W THEN 6972
6935   LET W=W+G
6940   GØSUB 4200
6945   GØSUB 7000
6950   IF Z8=0 THEN 6960
6955   RETURN
6960   IF M4=1 THEN 6970
6965   RETURN
6970   IF E1#0 THEN 6925
6972   LET M5=3
6975   RETURN
6998   REM **********************************************************
6999   REM ** PRØGRAM CCT. CHECK CØLLITIØN TIME **
7000   LET B=0
7001   REM **********************************************************
7010   LET Z8=0
7015   IF M1=1 THEN 7030
7020   IF W>U4 THEN 7040
7025   RETURN
7030   IF W>U5 THEN 7090
7035   RETURN
7040   LET U9=U4
7045   PRINT
7050   PRINT"CØLLITIØN AT U=";INT(U4)
7055   GØSUB 830
```

```
7060    GØSUB 4200
7062    CALL BSYMBL(X0/S,Y0/S,0.14,15,0,-1)
7065    LET M1=1
7070    LET M5=3
7071    REM COMPENSATE FØR UNCØVERED DISTANCE
7072    LET N9=N9+(T9+U6-U)*V9
7075    RETURN
7090    GØSUB 810
7094    GØSUB 4200
7106    CALL BSYMBL(X0/S,Y0/S,0.14,15,0,-1)
7110    PRINT
7112    PRINT"MULTIPASS AT U=";INT(U)
7115    LET Z8=1
7120    RETURN
7990    REM **************************************************
7992    REM ** PRØGRAM TØ DRAW ØWN SHIP
7994    REM **************************************************
8000    CALL BSYMBL(X8/S,Y8/S,0.14,25,0,-1)
8002    LET U=0
8003    LET U0=0
8004    LET     U=U+10
8005    IF U> 800 THEN 8056
8006    GØSUB 2060
8008    IF X8>6*S THEN 8056
8010    IF Y8>8*S THEN 8056
8012    IF X8<0 THEN 8056
8014    IF Y8<0 THEN 8056
8016    LET I1=INT(U0/100)-U0/100
8018    LET I2=0.14;I4=25
8020    IF ABS(I1)<0.001 THEN 8050
8022    LET I2=0.07;I4=18
8050    CALL BSYMBL(X8/S,Y8/S,I2,I4,0,-1)
8052    GØTØ 8004
8056    LET X8=X0
8058    LET Y8=Y0
8059    LET U0=U
8062    RETURN
8090    REM **************************************************
8092    REM  ** PRØGRAM TØ DRAW TEXT
8094    REM **************************************************
8100    CALL BSYMBL(0.5,8.8,0.14,"V0=",0,3)
8105    CALL BWHERE(I3,I4,I5)
8107    LET Ø=Ø2/0.5144
8110    CALL BNUMBR(I3,8.8,0.14,Ø,0,-1)
8115    CALL BWHERE(I3,I4,I5)
8120    CALL BSYMBL(I3,8.8,0.14,", VS=",0,5)
8125    CALL BWHERE(I3,I4,I5)
8130    CALL    BNUMBR(I3,8.8,0.14,V,0,-1)
8135    CALL BWHERE(I3,I4,I5)
8140    CALL BSYMBL(I3,8.8,0.14," AND VT=30 KTS",0,14)
8145    RETURN

        REM FOR SMOOTHING OF STAGE 3 GUIDANCE
635     LET K2=0
636     GØSUB 950
705     LET D9=150
740     LET K2=0
756     LET K8=0
5860    LET D9=40
6525    LET D8=25

*ØT,SNL,*IT
```

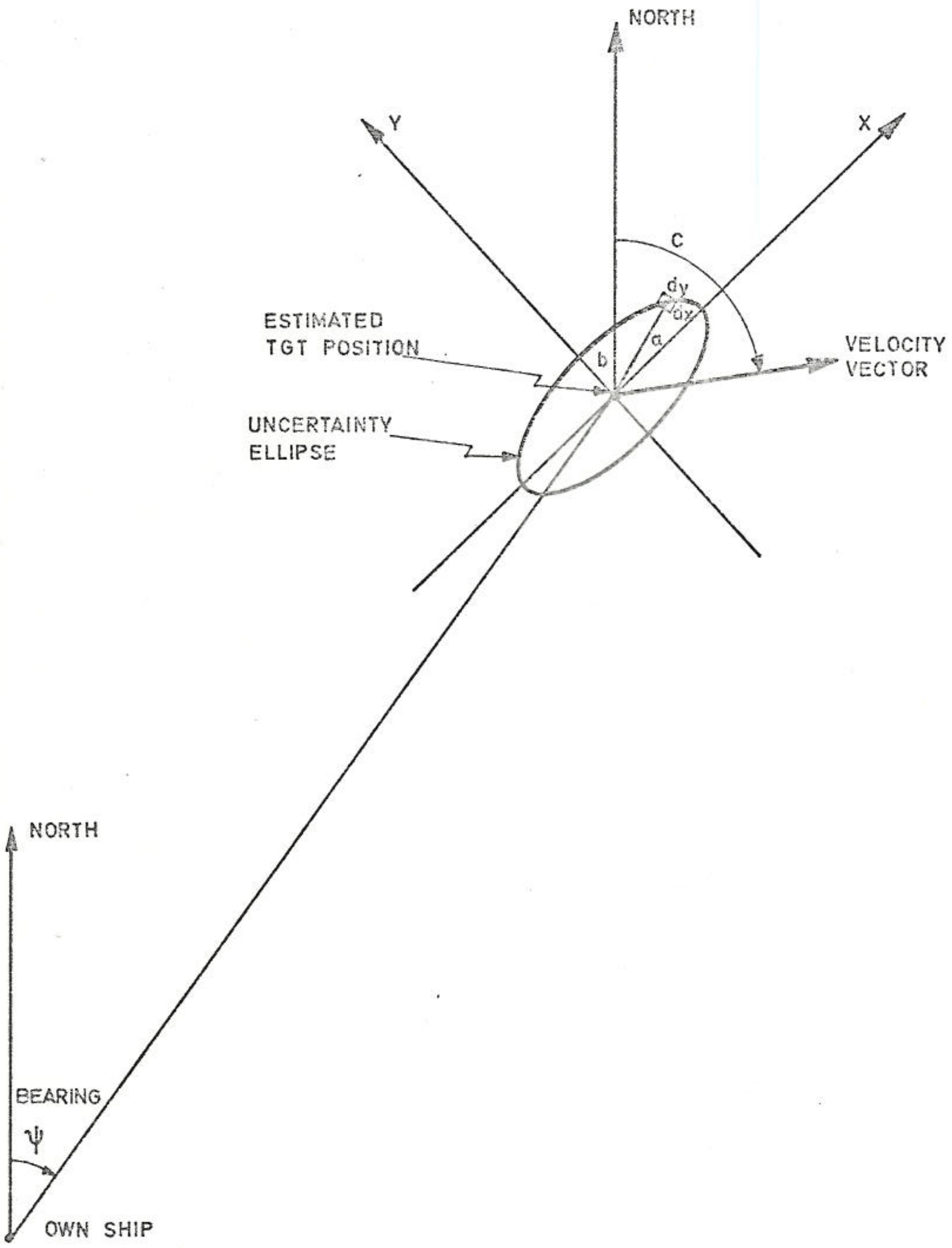


Figure 2.1 Own position and target state vector data with uncertainty-ellipse

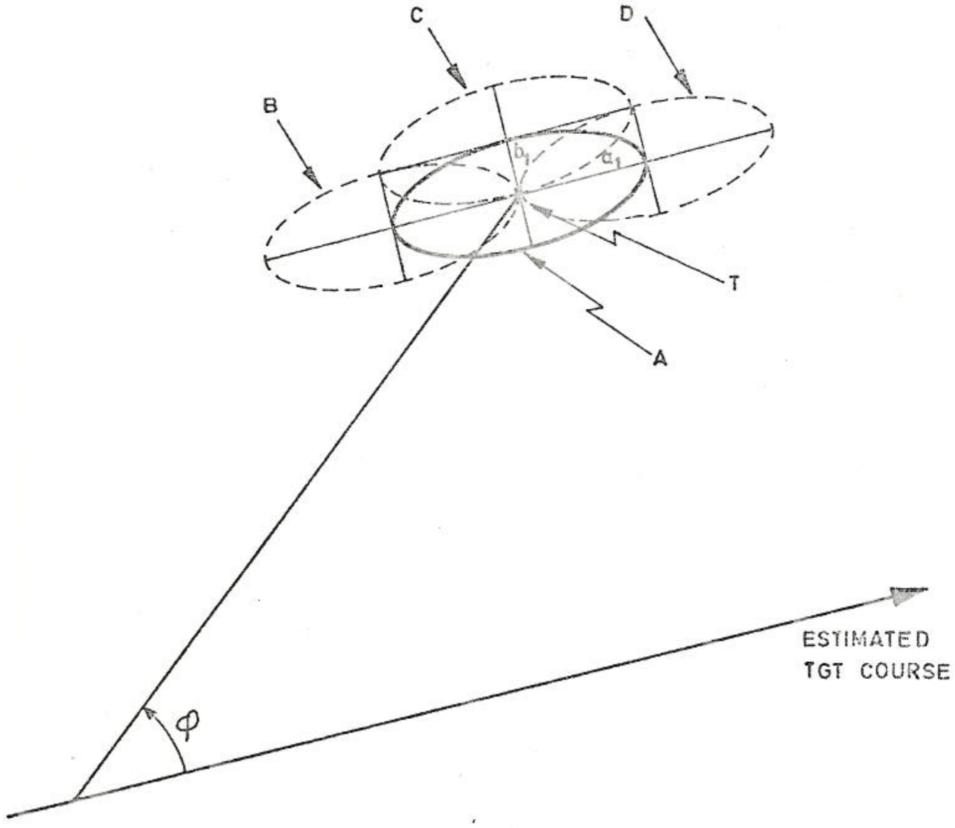


Figure 2.2 The torpedo position T and the ellipse A at which periphery the actual target center must be located in order for the torpedo to hit the target

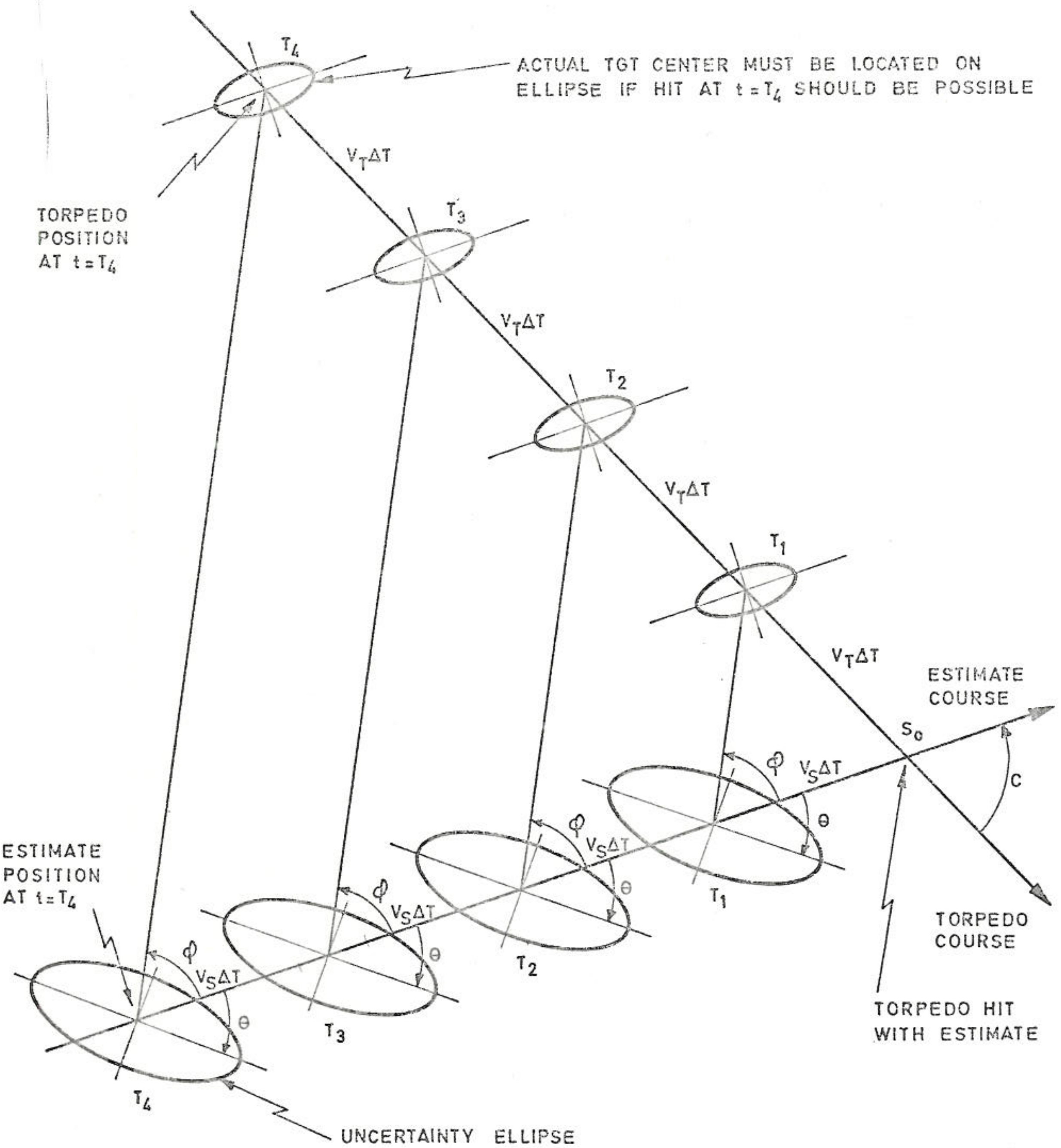


Figure 2.3 Different relative positions of the torpedo and the estimated target position

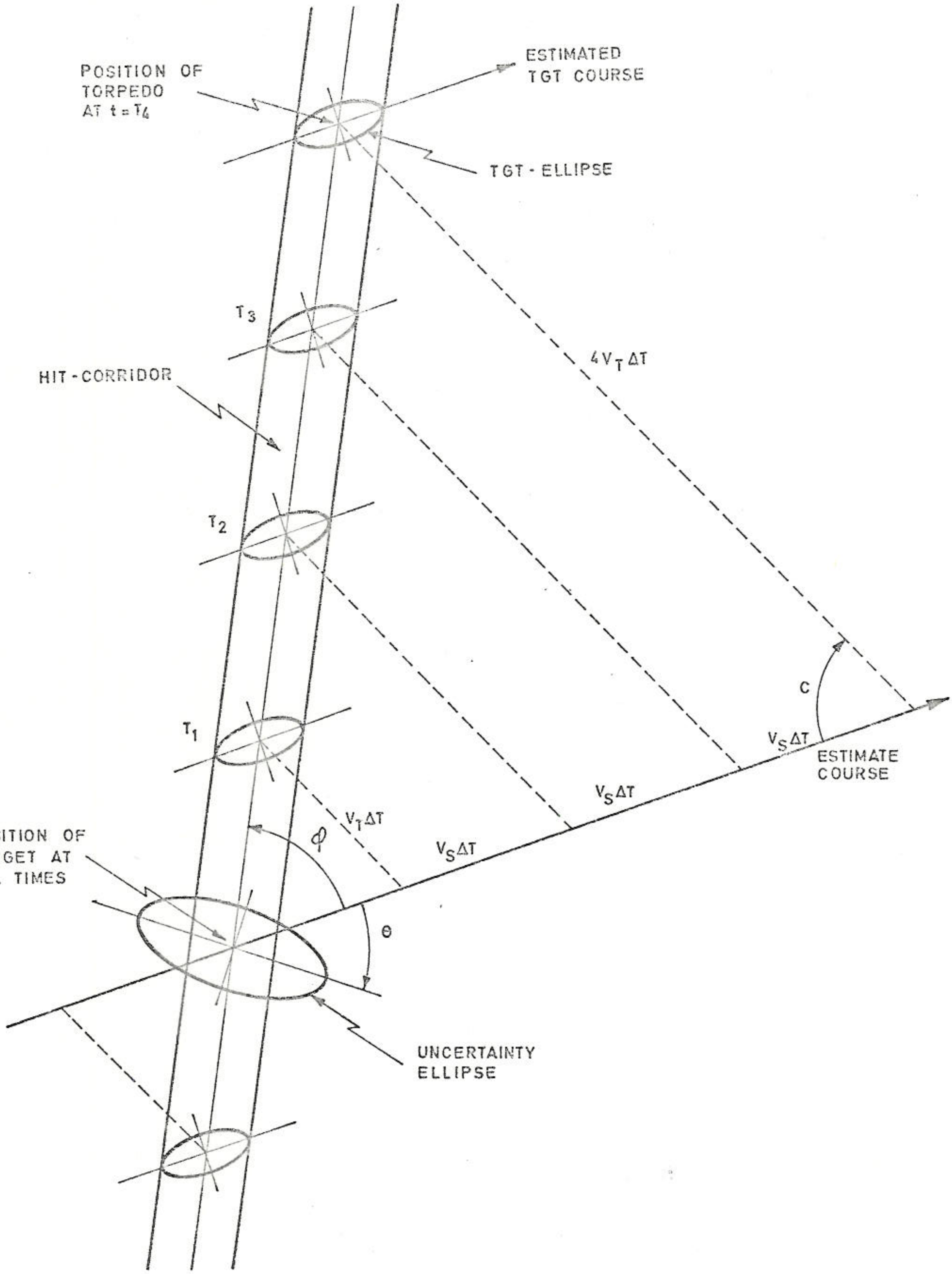


Figure 2.4 The establishment of the hit-corridor when only relative motion between the torpedo and the estimate is considered

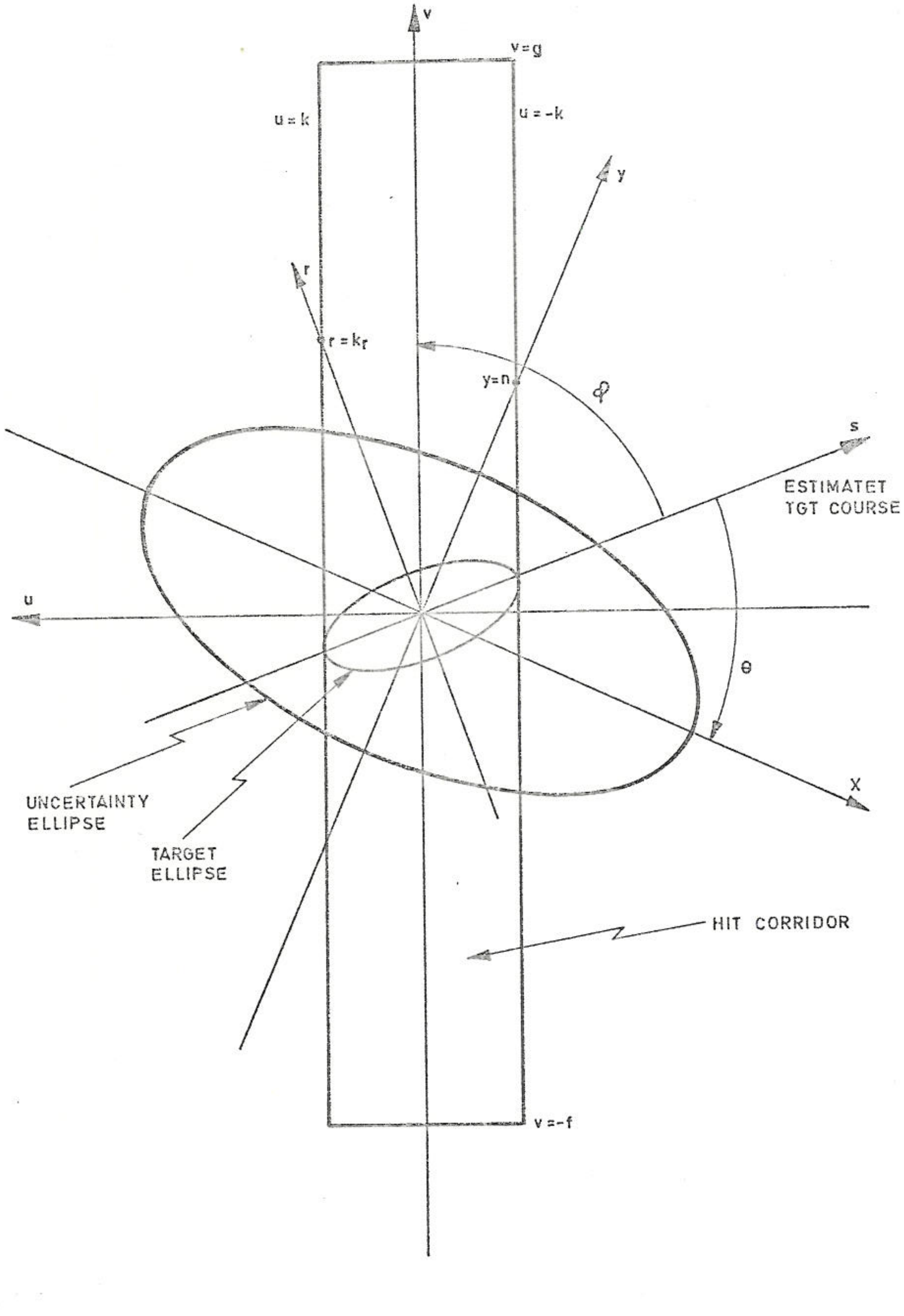


Figure 2.5 The hit-corridor, the target-ellipse and the uncertainty-ellipse

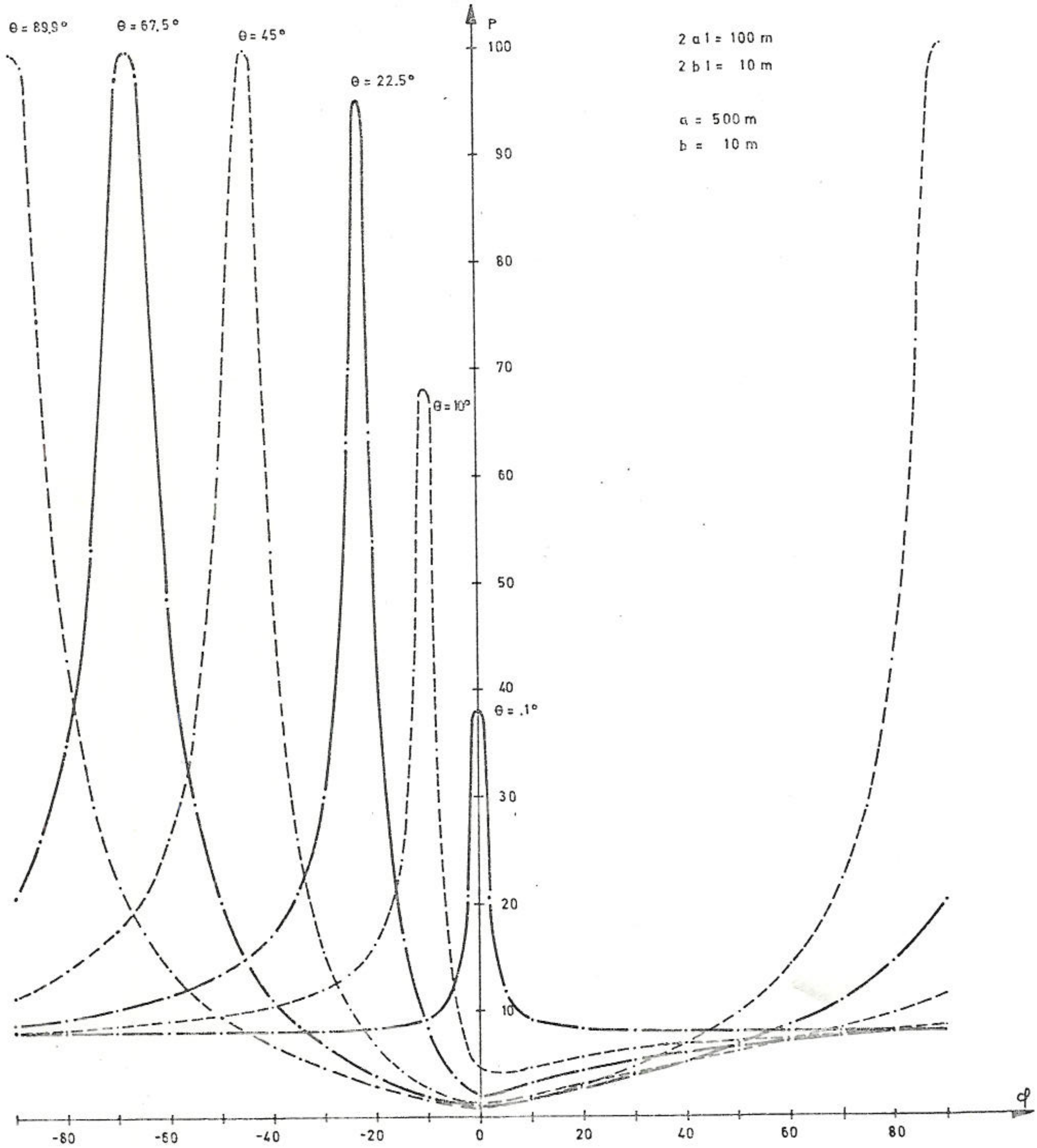


Figure 2.6 Hit-probability vs corridor angle Φ, with θ as parameter
a = 500 m, b = 10 m

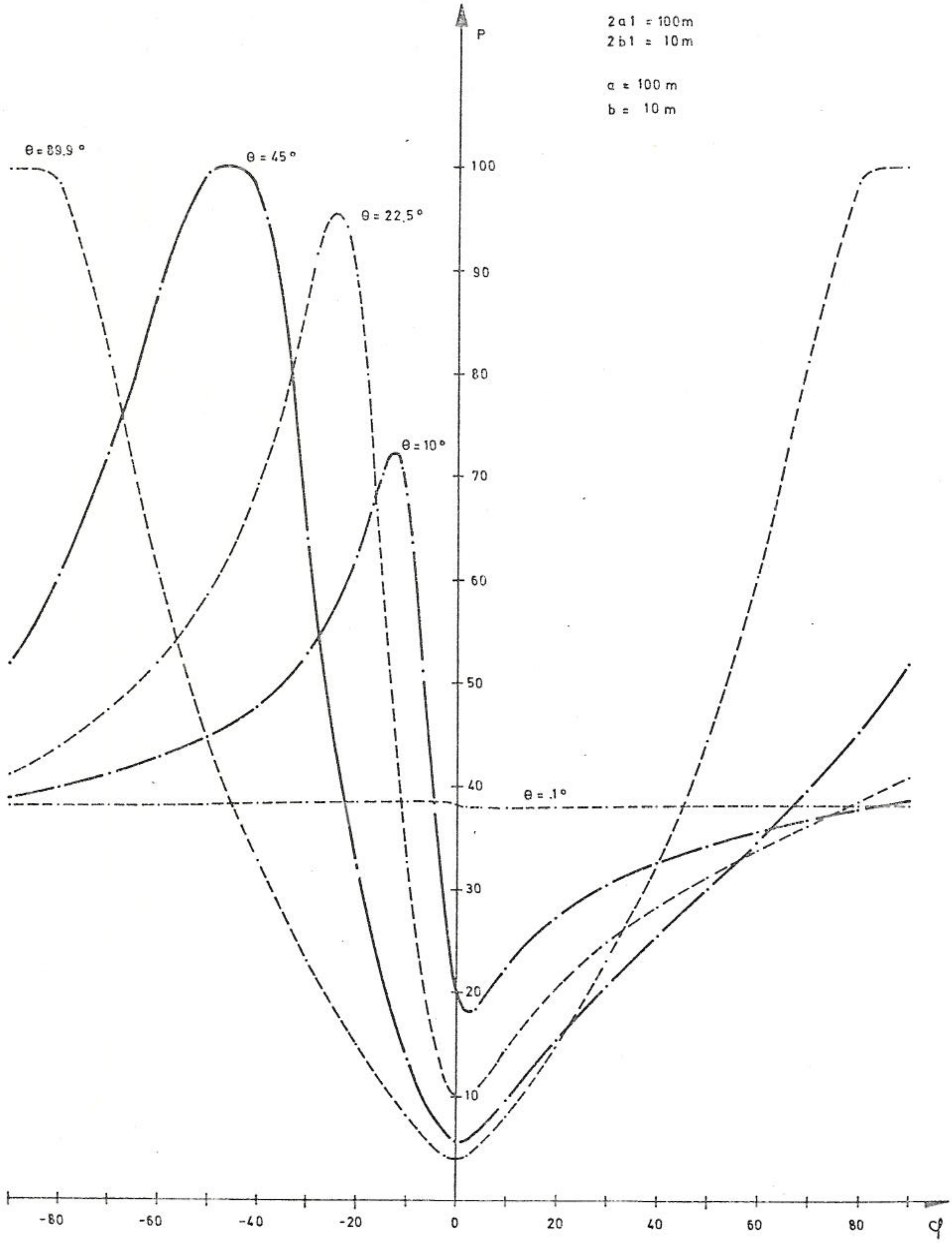


Figure 2.7 Hit-probability vs corridor angle Φ, a = 100 m, b = 10 m

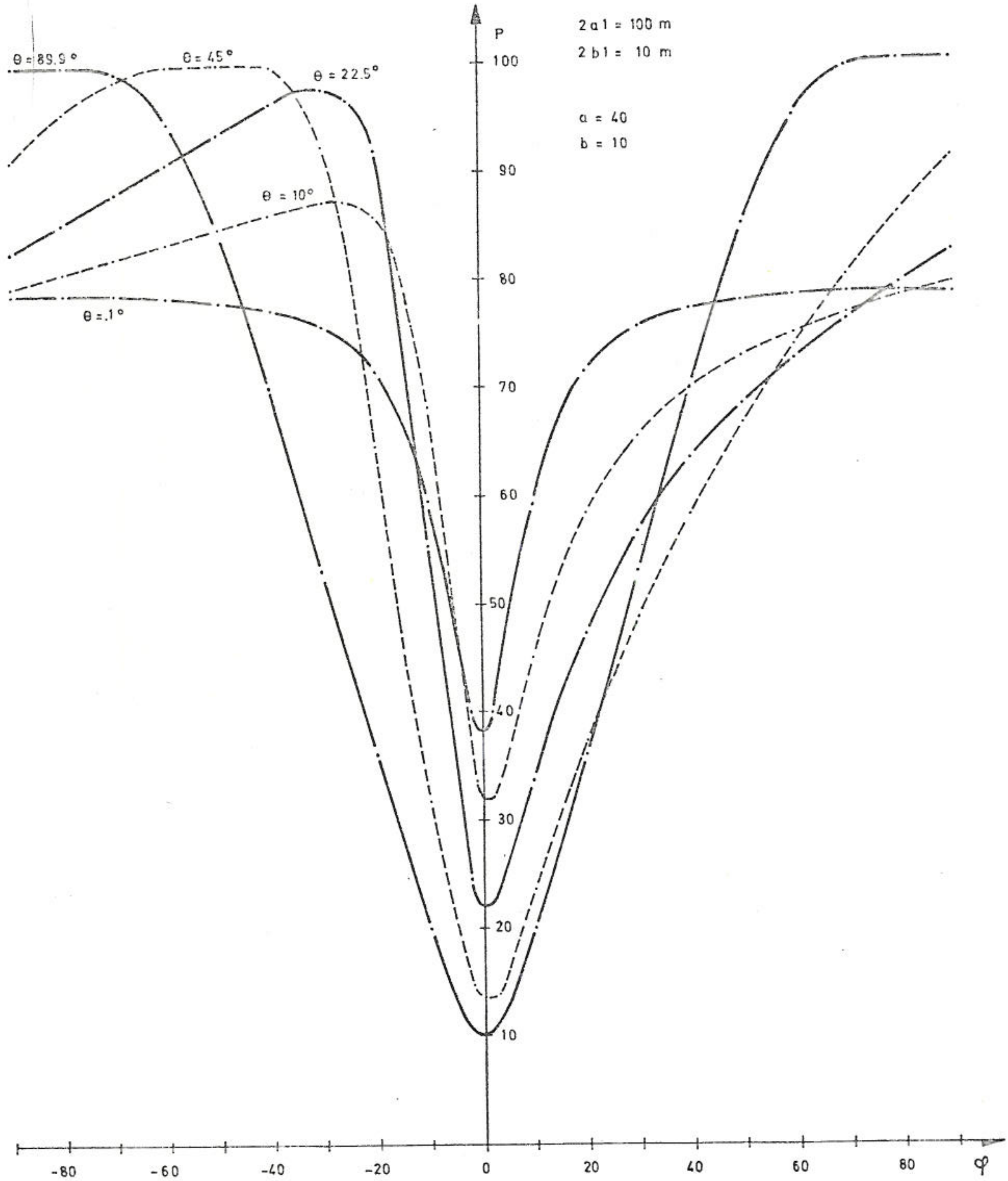


Figure 2.8 Hit-probability vs corridor angle φ, a = 40 m, b = 10 m

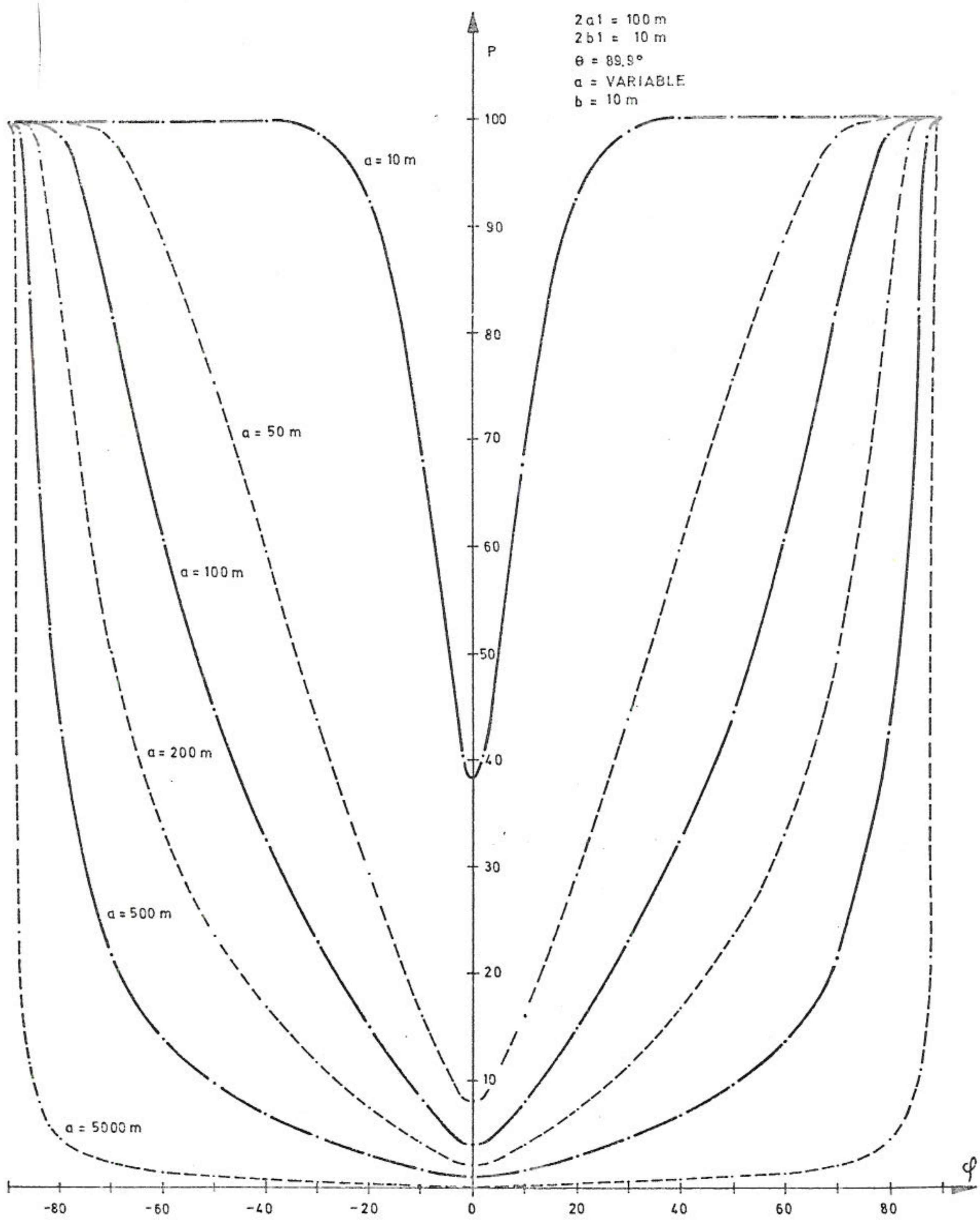


Figure 2.9 Hit-probability vs corridor angle Φ, a = variable, b = 10 m, θ = 89.9°

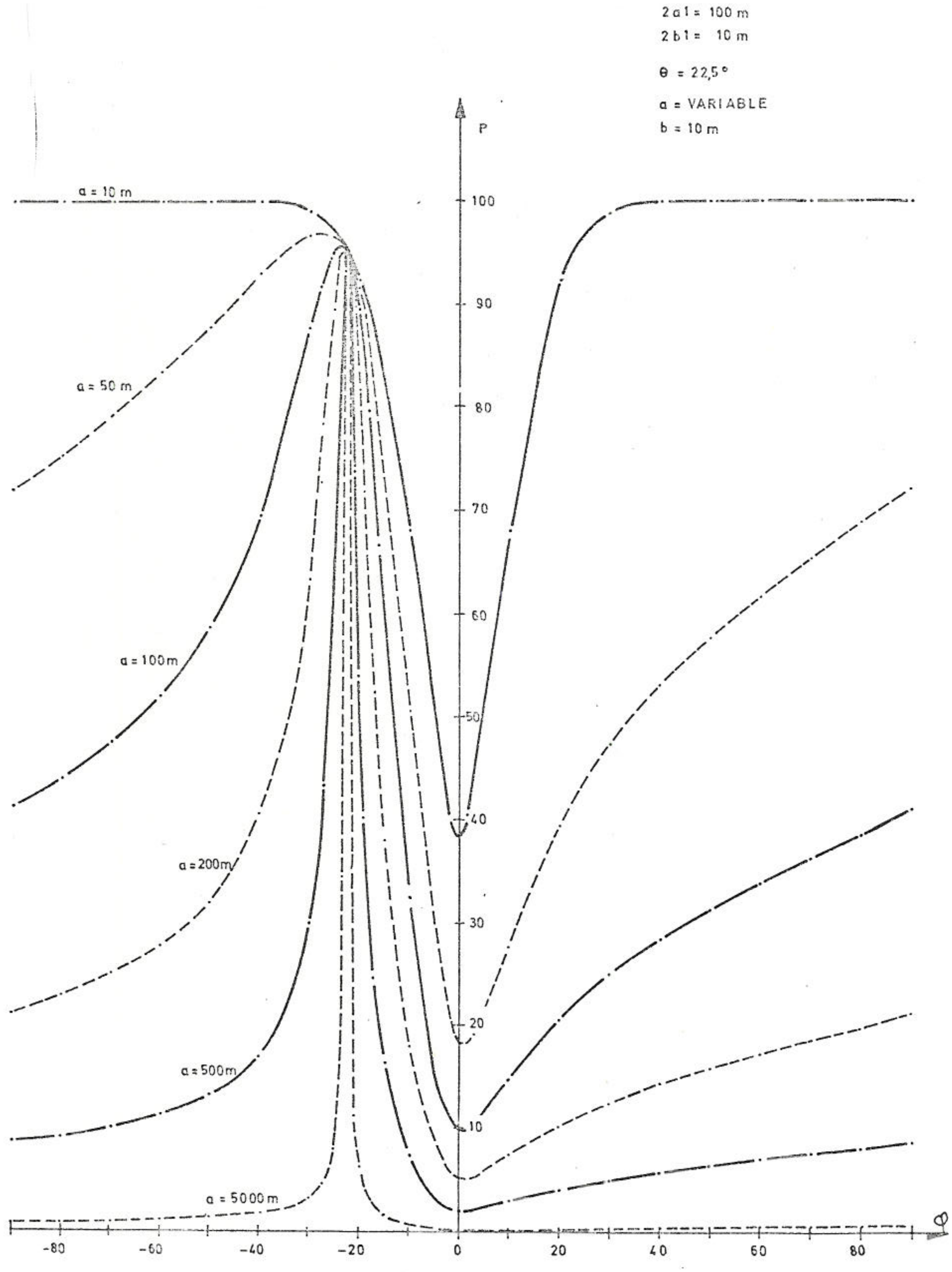


Figure 2.10 Hit-probability vs corridor angle Φ, a = variable, b = 10 m, θ = 22,5°

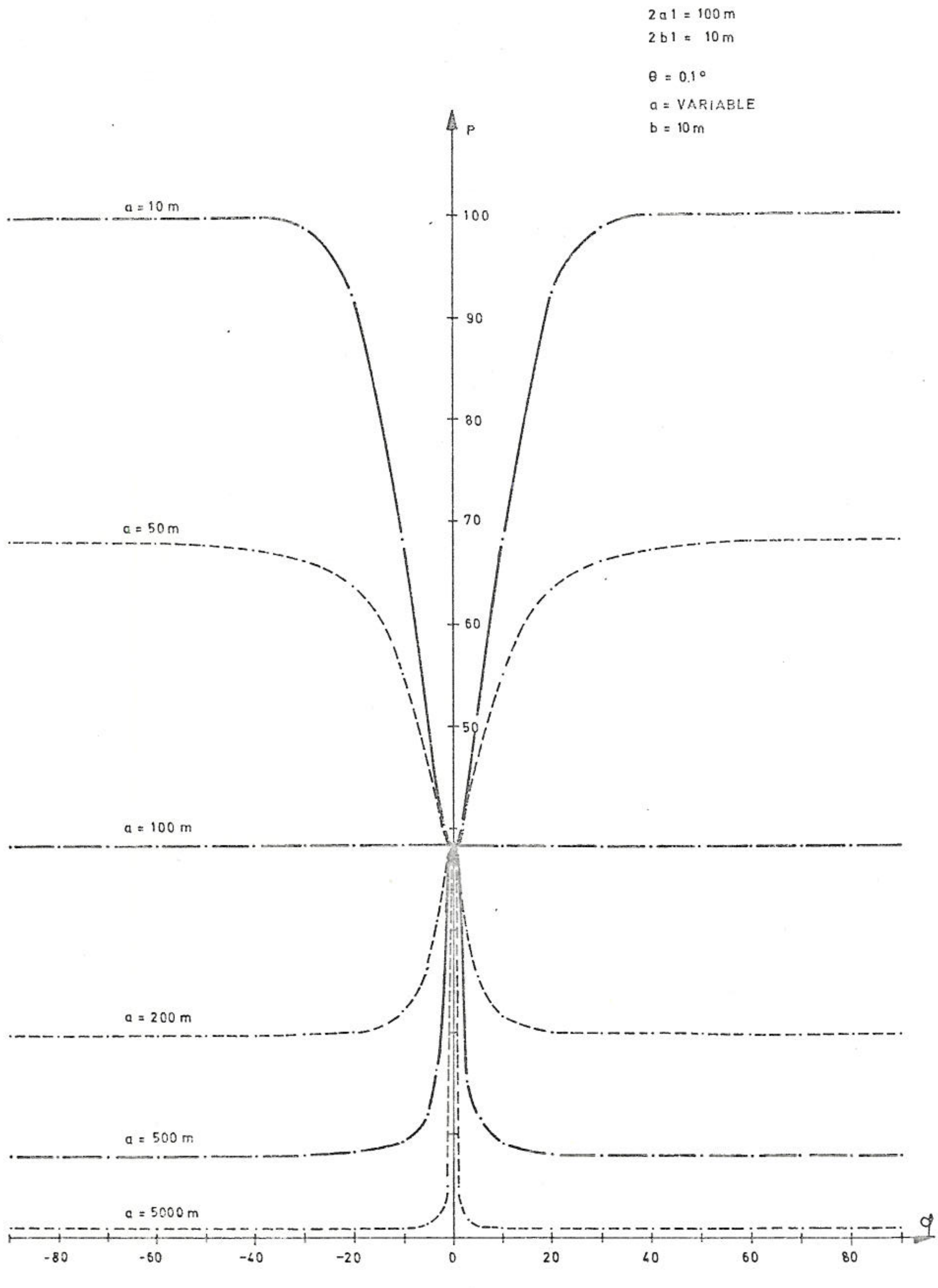


Figure 2.11 Hit-probability vs corridor angle Φ, a = variable b = 10 m, θ = 0,1°

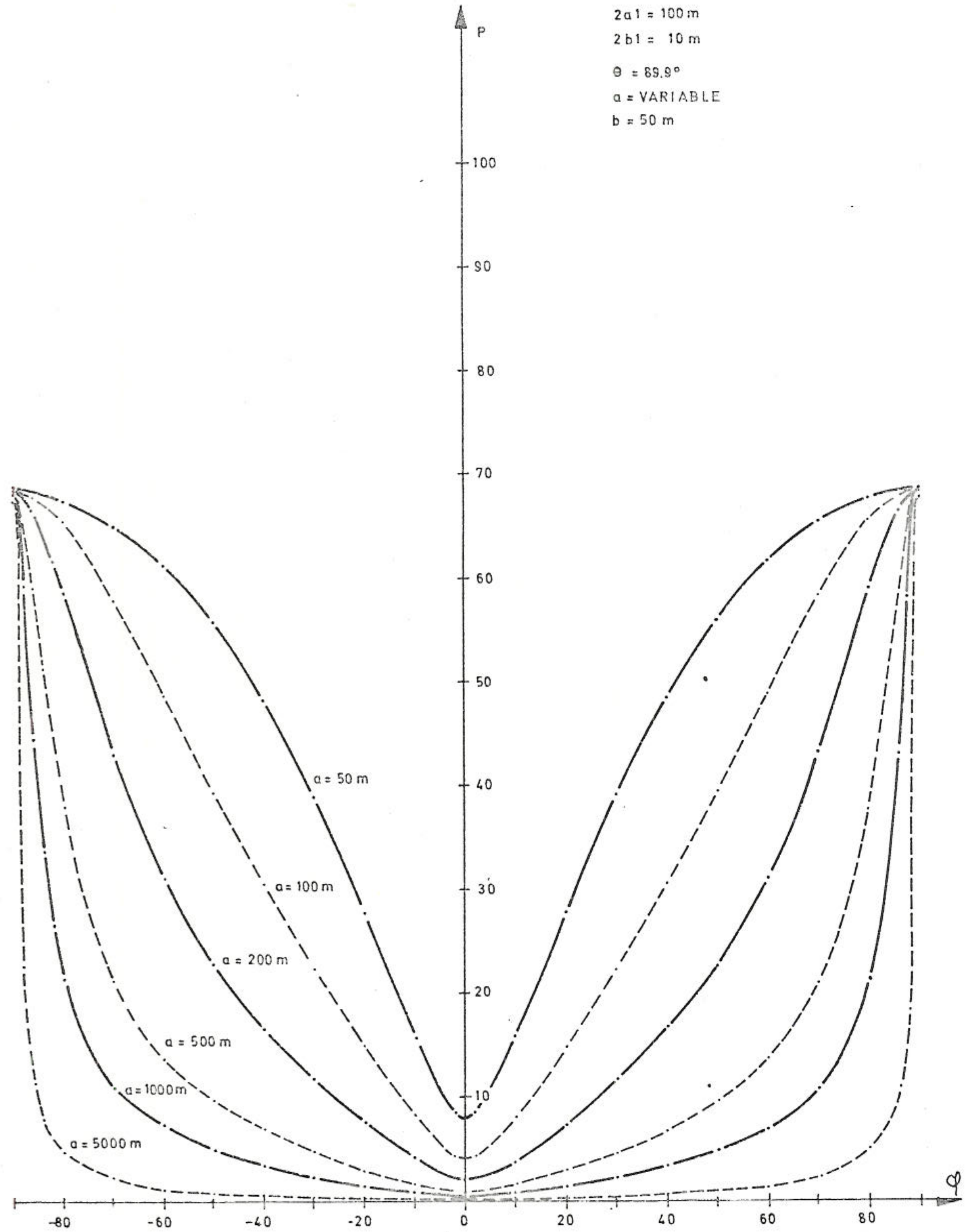


Figure 2.12 Hit-probability vs corridor angle Φ, a = variable, b = 50 m, θ = 89.9°

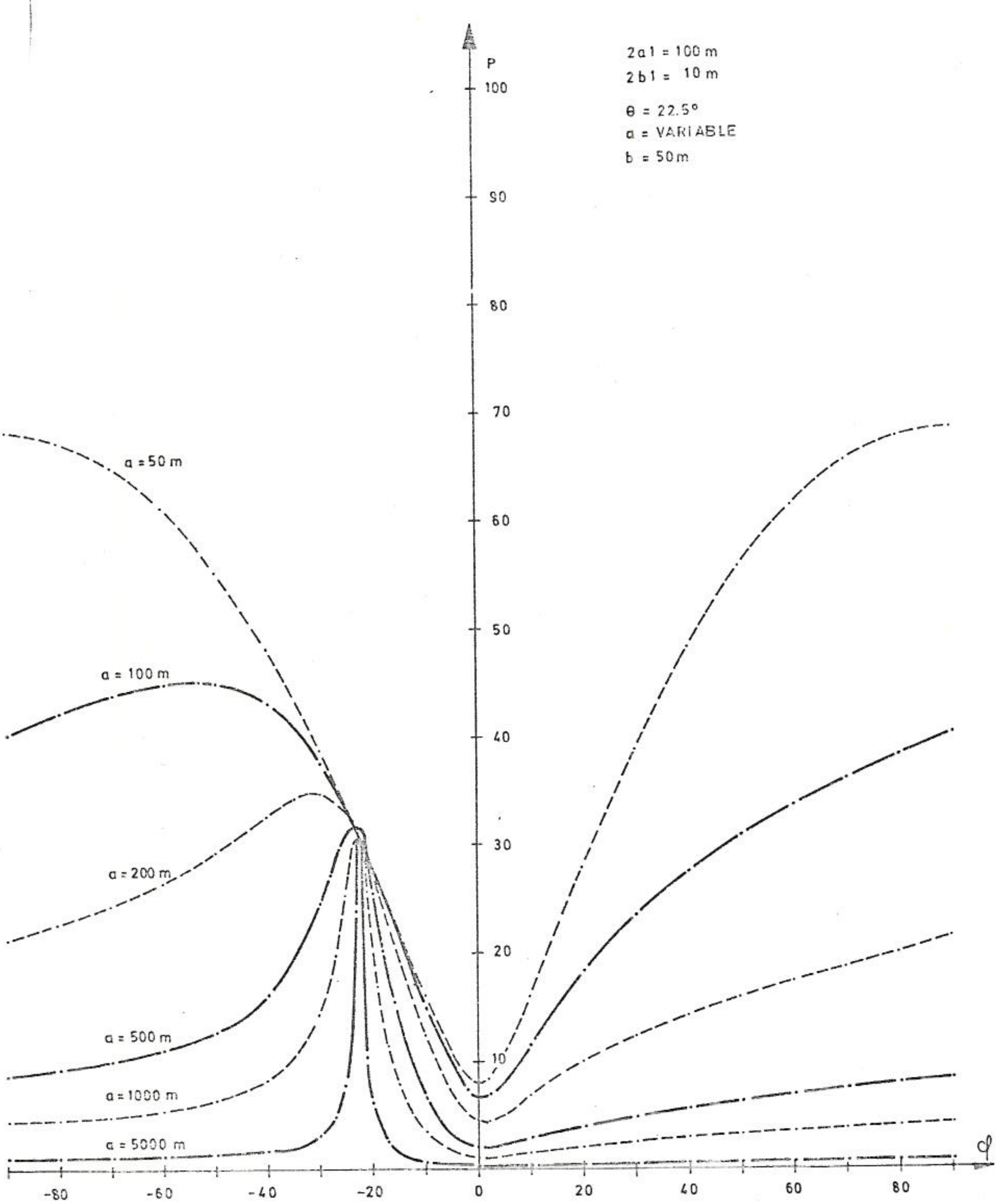


Figure 2.13 Hit-probability vs corridor angle φ, a = variable, b = 50 m, θ = 22,5°

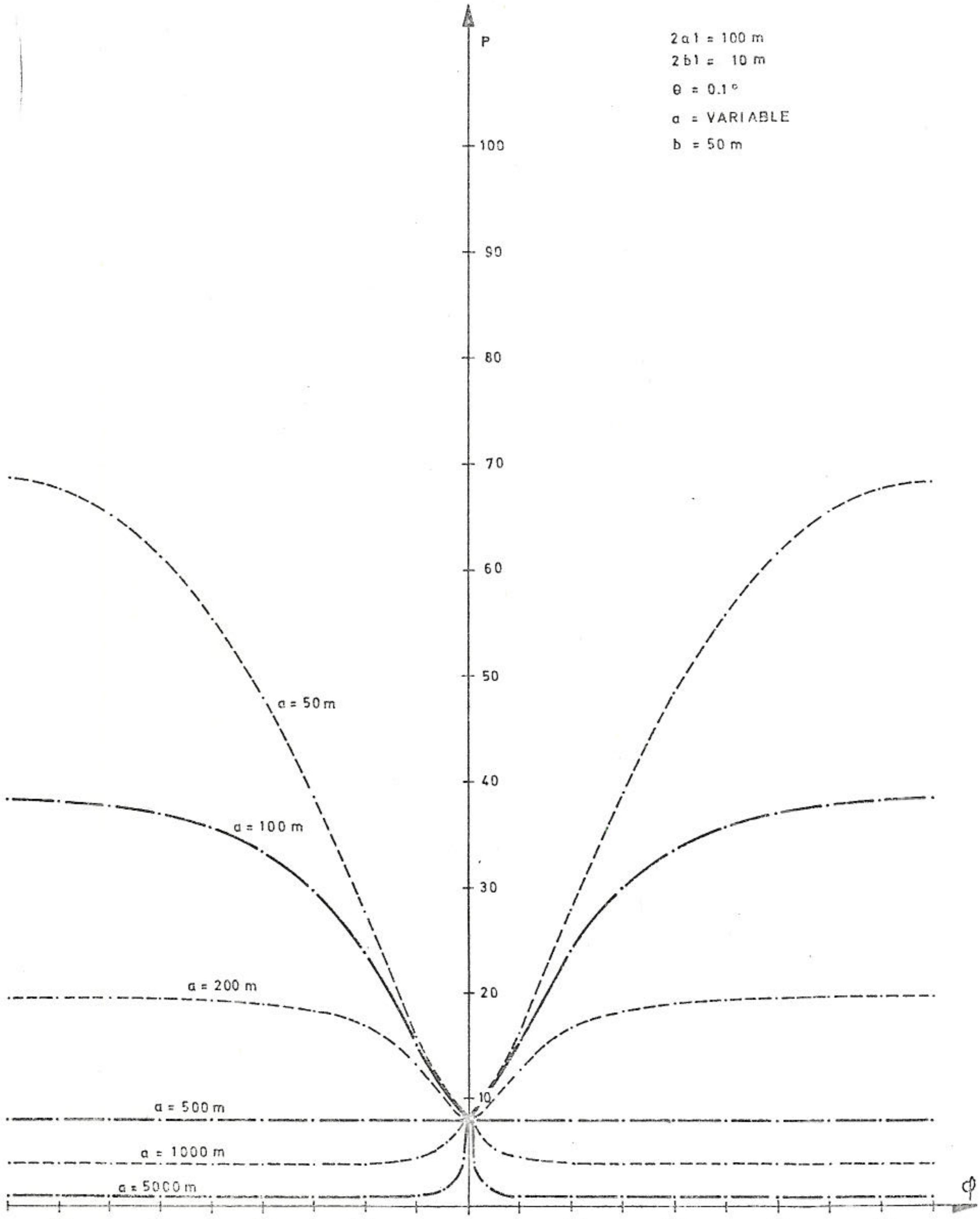


Figure 2.14 Hit-probability vs corridor angle φ, a = variable, b = 50 m, θ = 0,1°

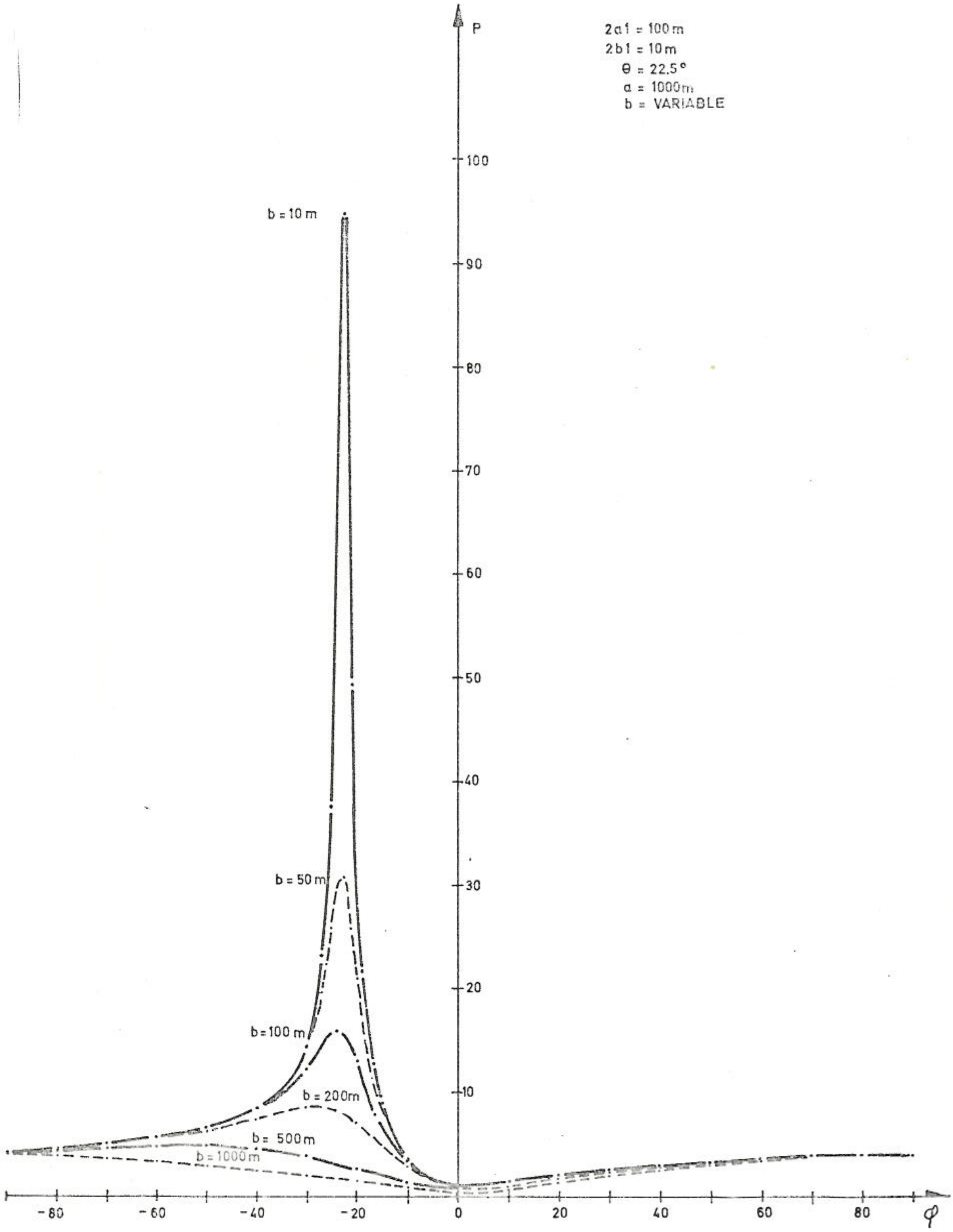


Figure 2.15 Hit-probability vs corridor angle φ, a = 1000 m, b = variable, θ = 22,5°

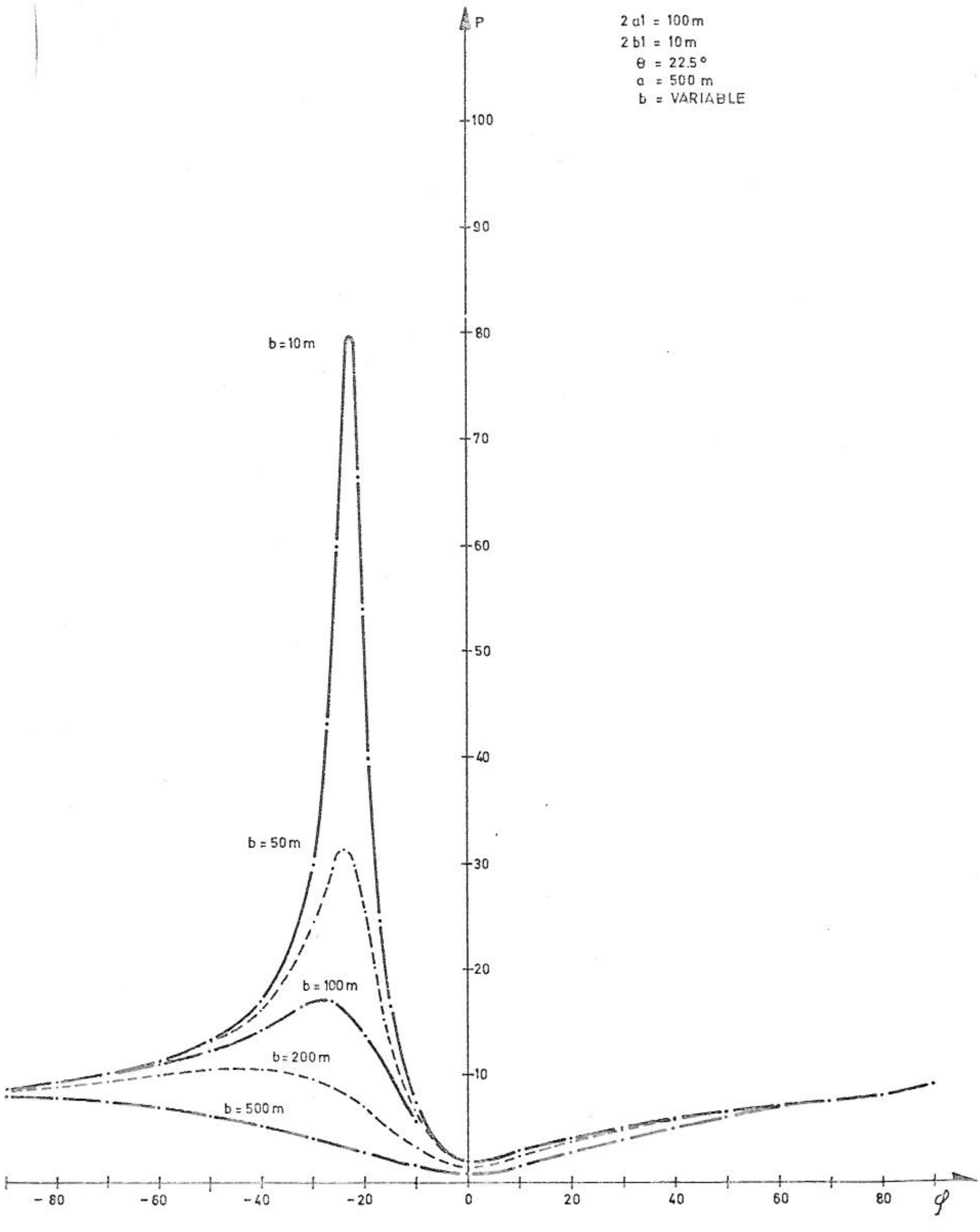


Figure 2.16 Hit-probability vs corridor angle φ, a = 500 m, b = variable, θ = 22,5°

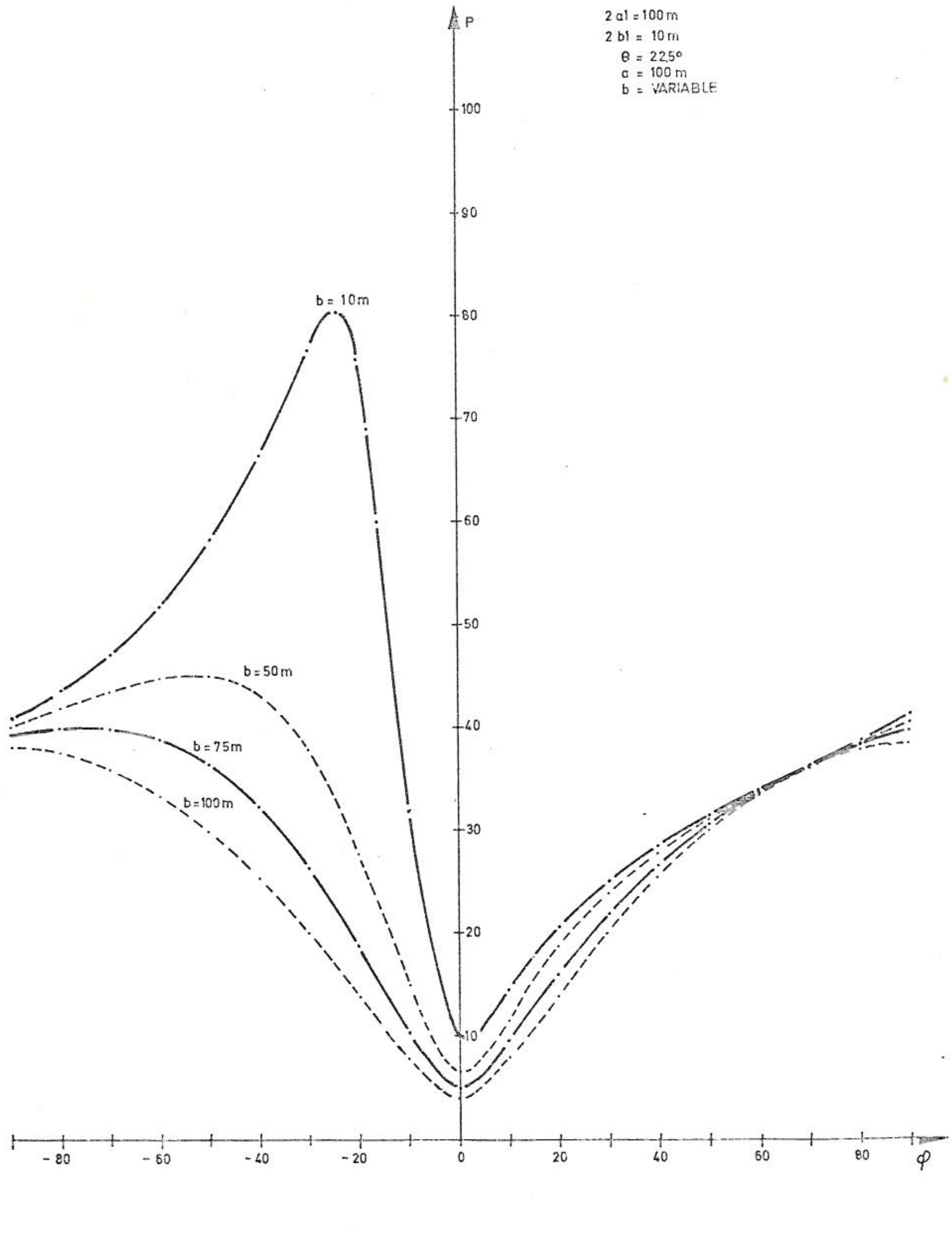


Figure 2.17 Hit-probability vs corridor angle Φ, a = 100 m, b = variable, θ = 22,5°

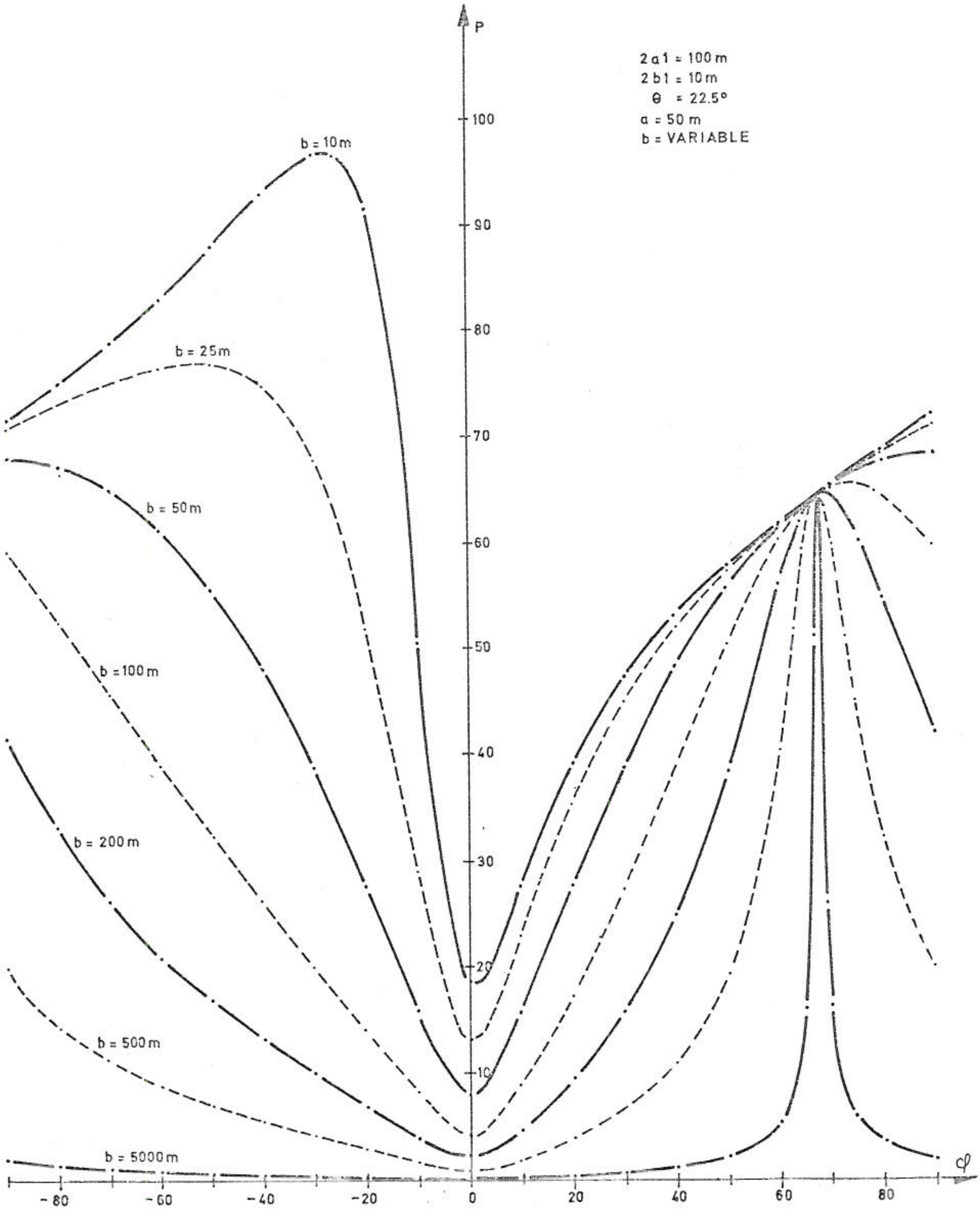


Figure 2.18 Hit-probability vs corridor angle ϕ, a = 50 m, b = variable, θ = 22,5°

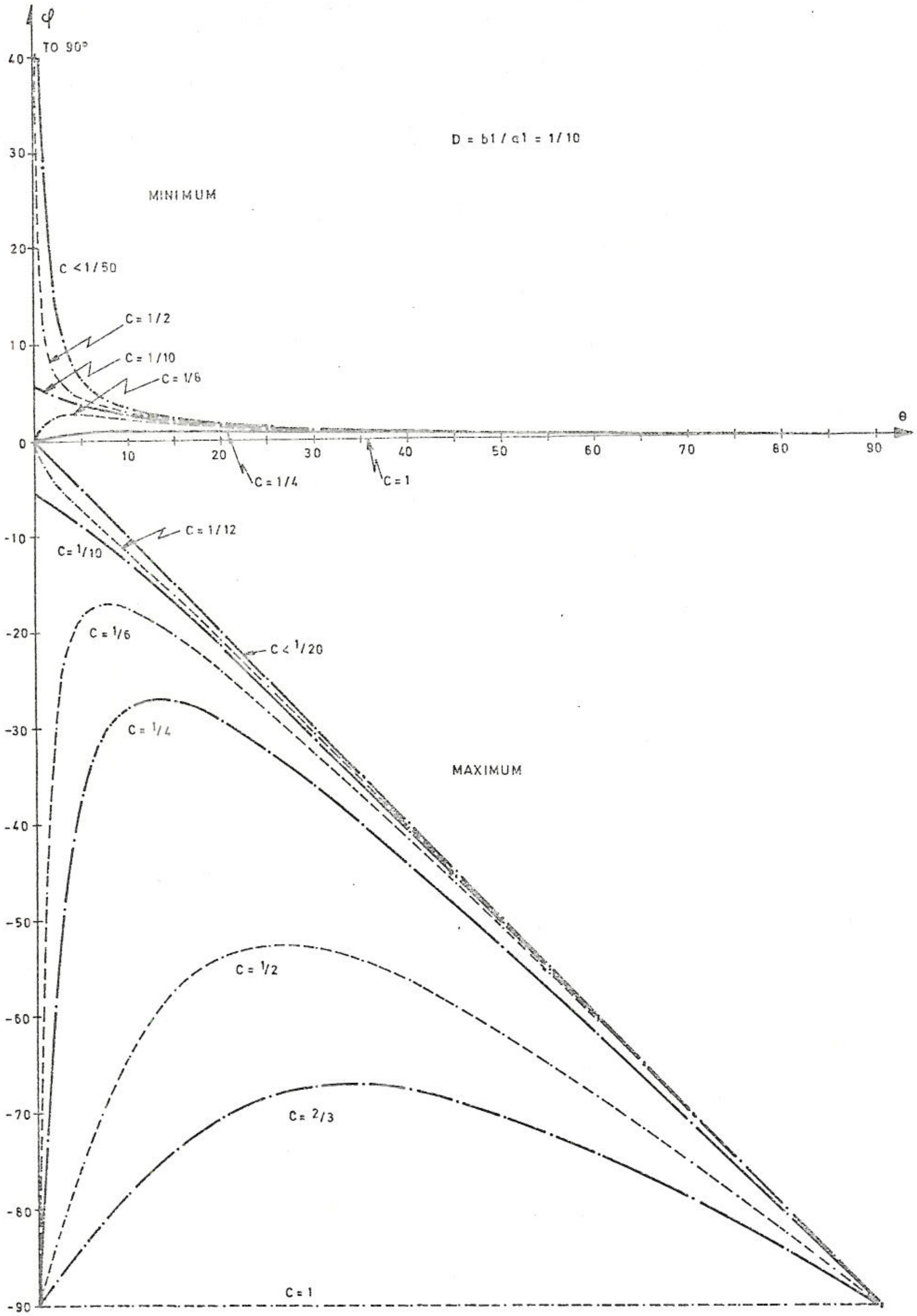


Figure 2.19 Corridor angle φ yielding maximum and minimum probability of hit vs uncertainty-ellipse tilt angle θ

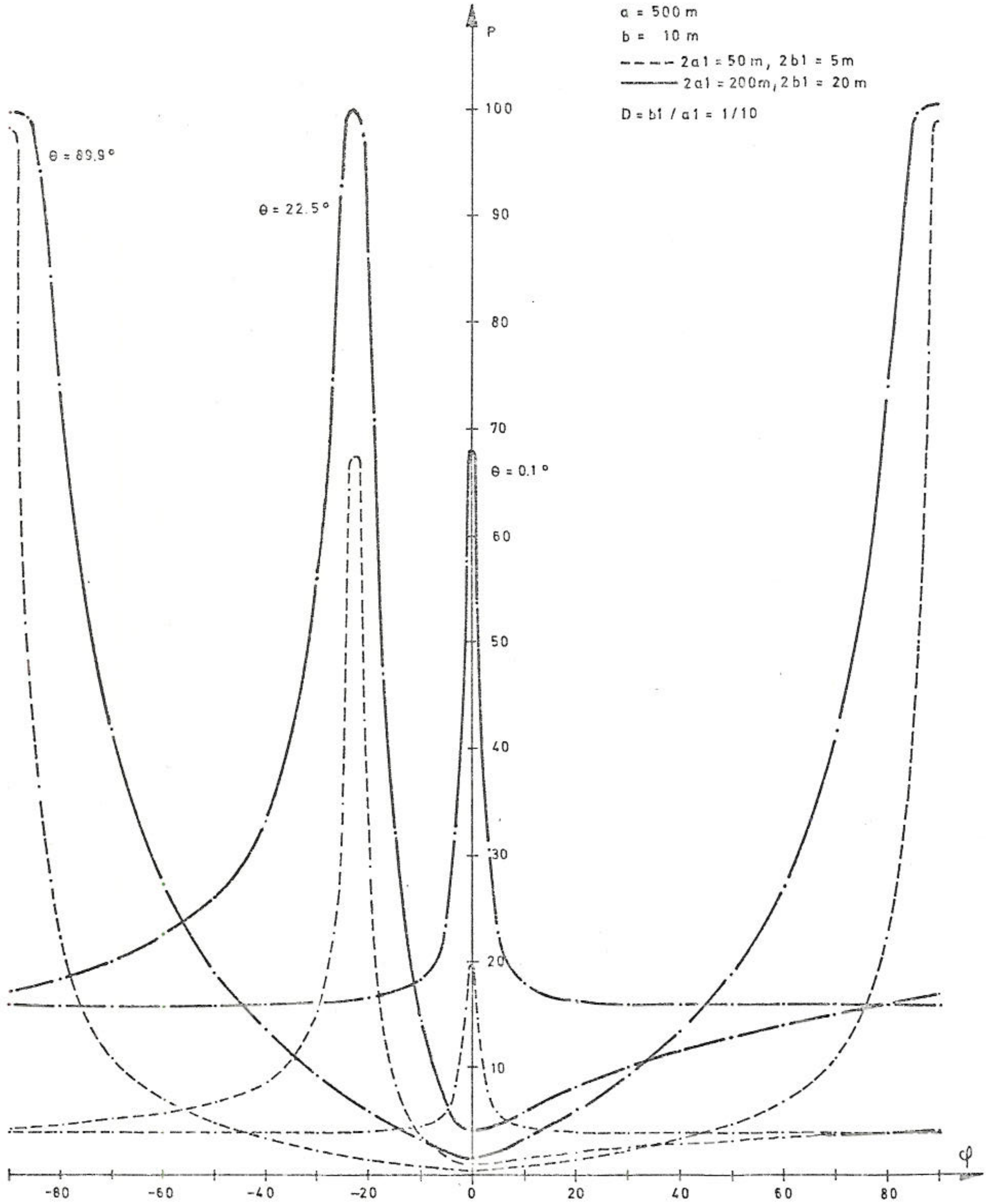


Figure 2.20 Hit-probability vs corridor angle Φ, for 2 different sizes of target-ellipses

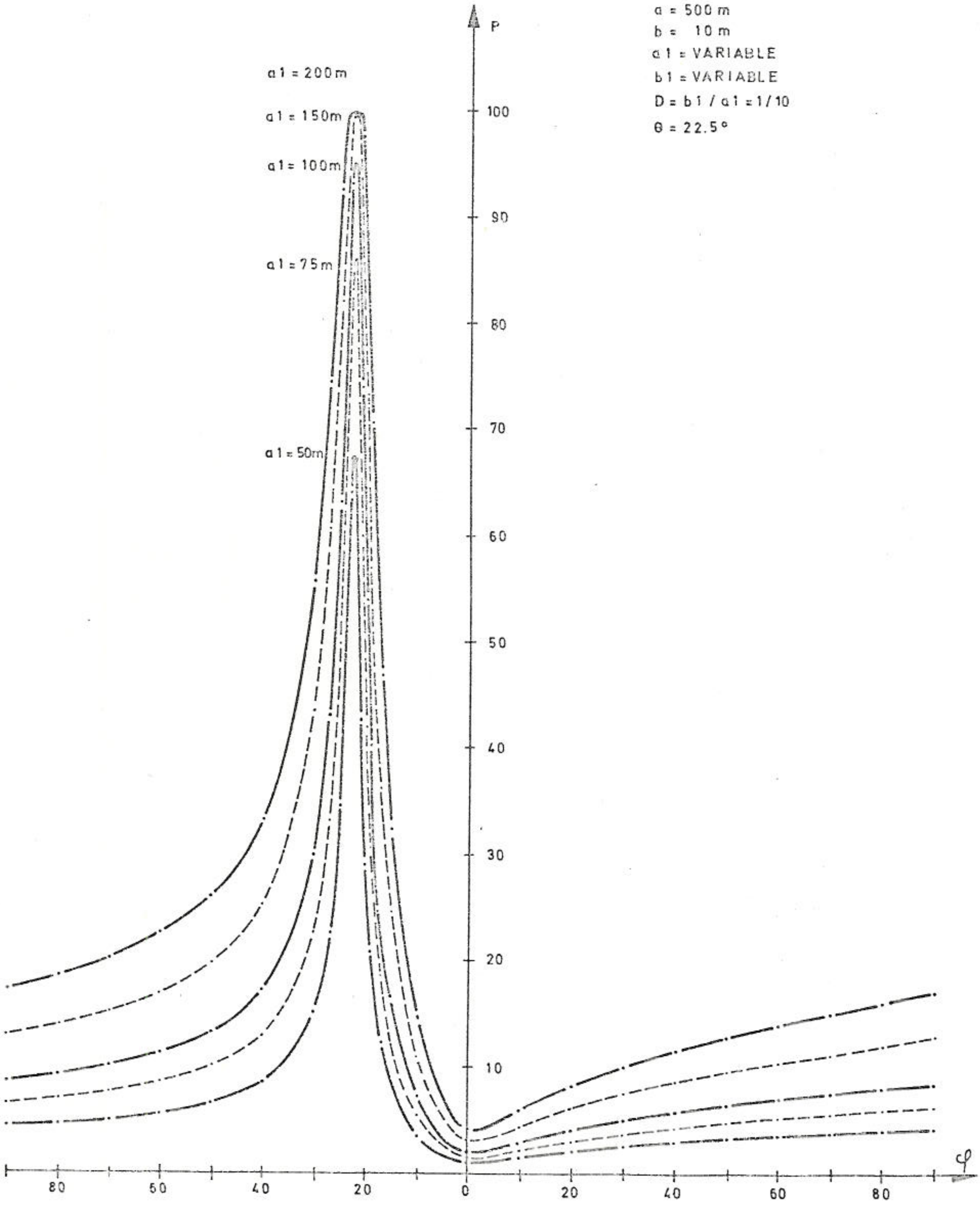


Figure 2.21 Hit-probability vs corridor angle ϕ, a = 500 m, b = 10 m, $\theta = 22.5^{\circ}$, $a_1 = 10 \cdot b_1$ = variable

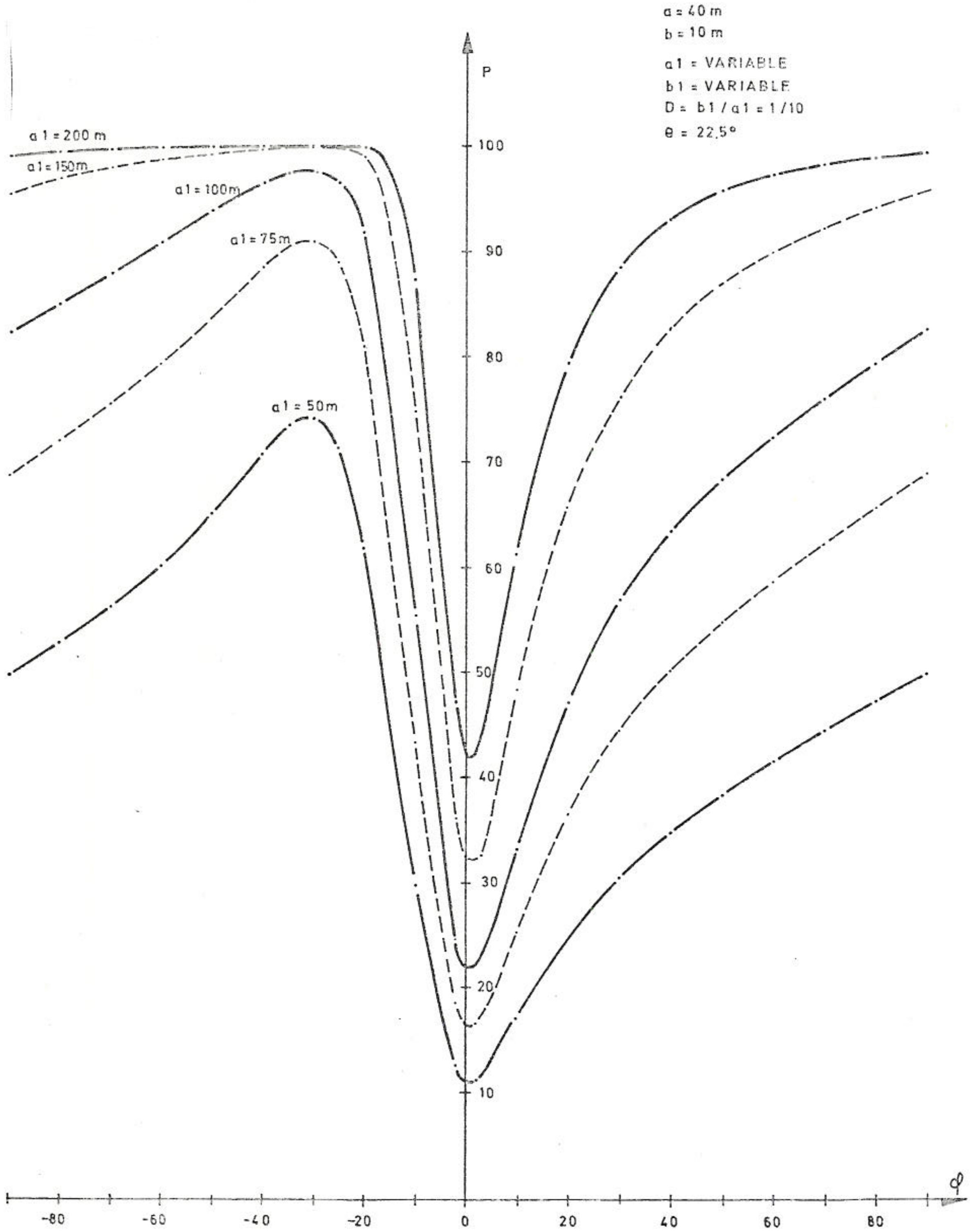


Figure 2.22 Hit-probability vs corridor angle ϕ $a = 40$ m, $b = 10$ m, $\theta = 22.5^{\circ}$, $a_1 = 10 \cdot b_1$ = variable

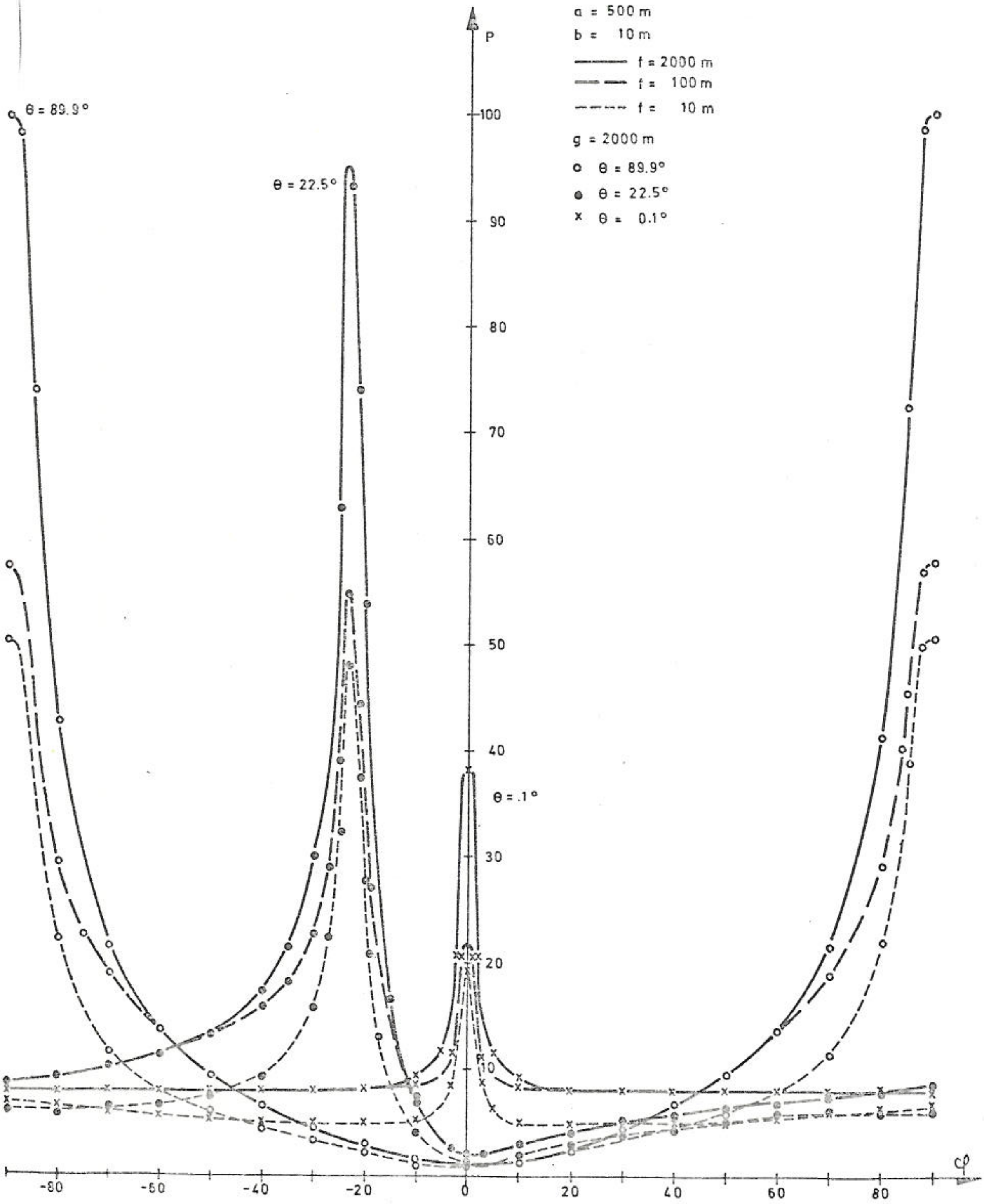


Figure 2.23 Hit-probability vs corridor angle φ, a = 500 m, b = 10 m, f = variable

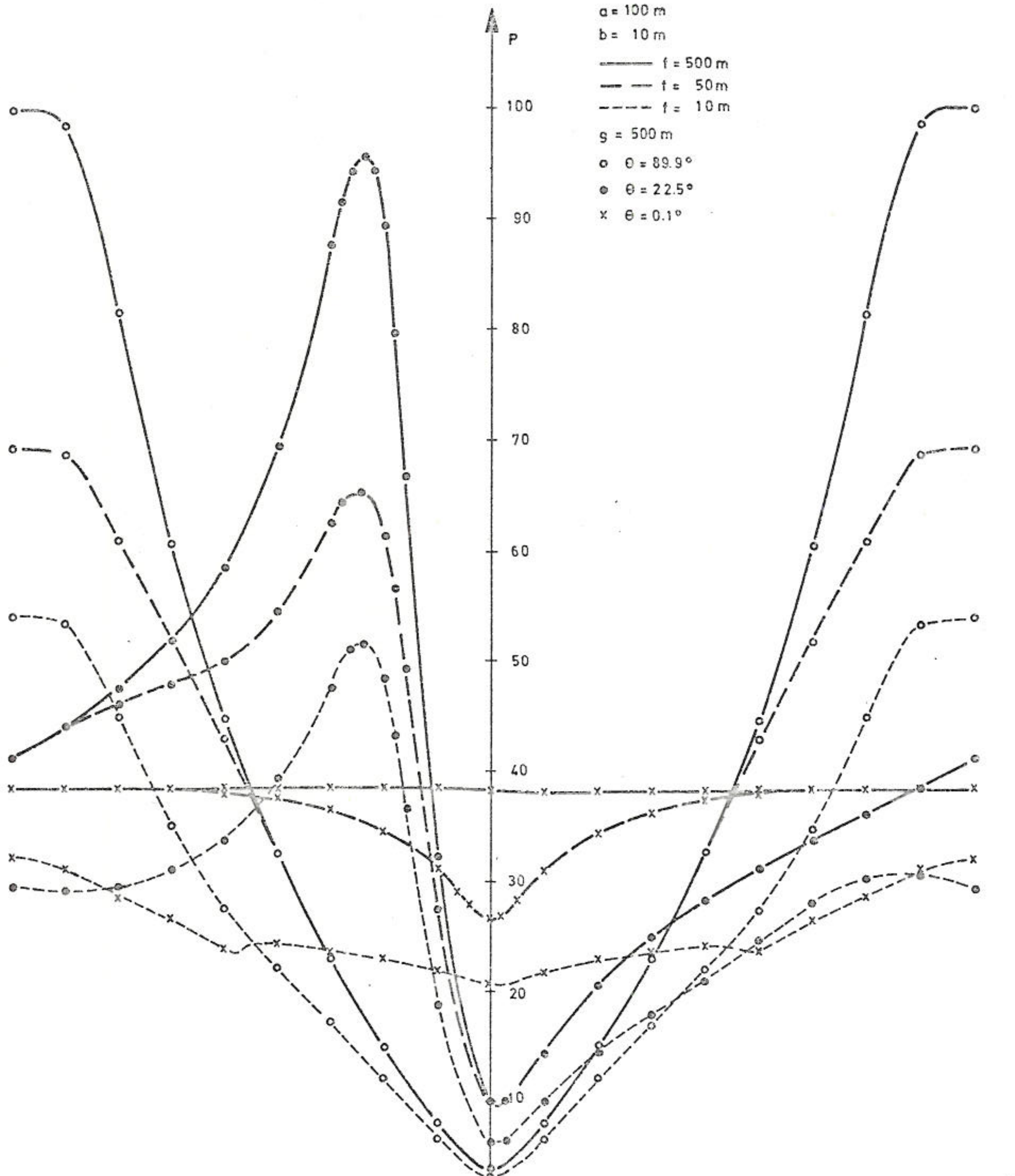


Figure 2.24 Hit-probability vs corridor angle φ, a = 100 m, b = 10 m, f = variable

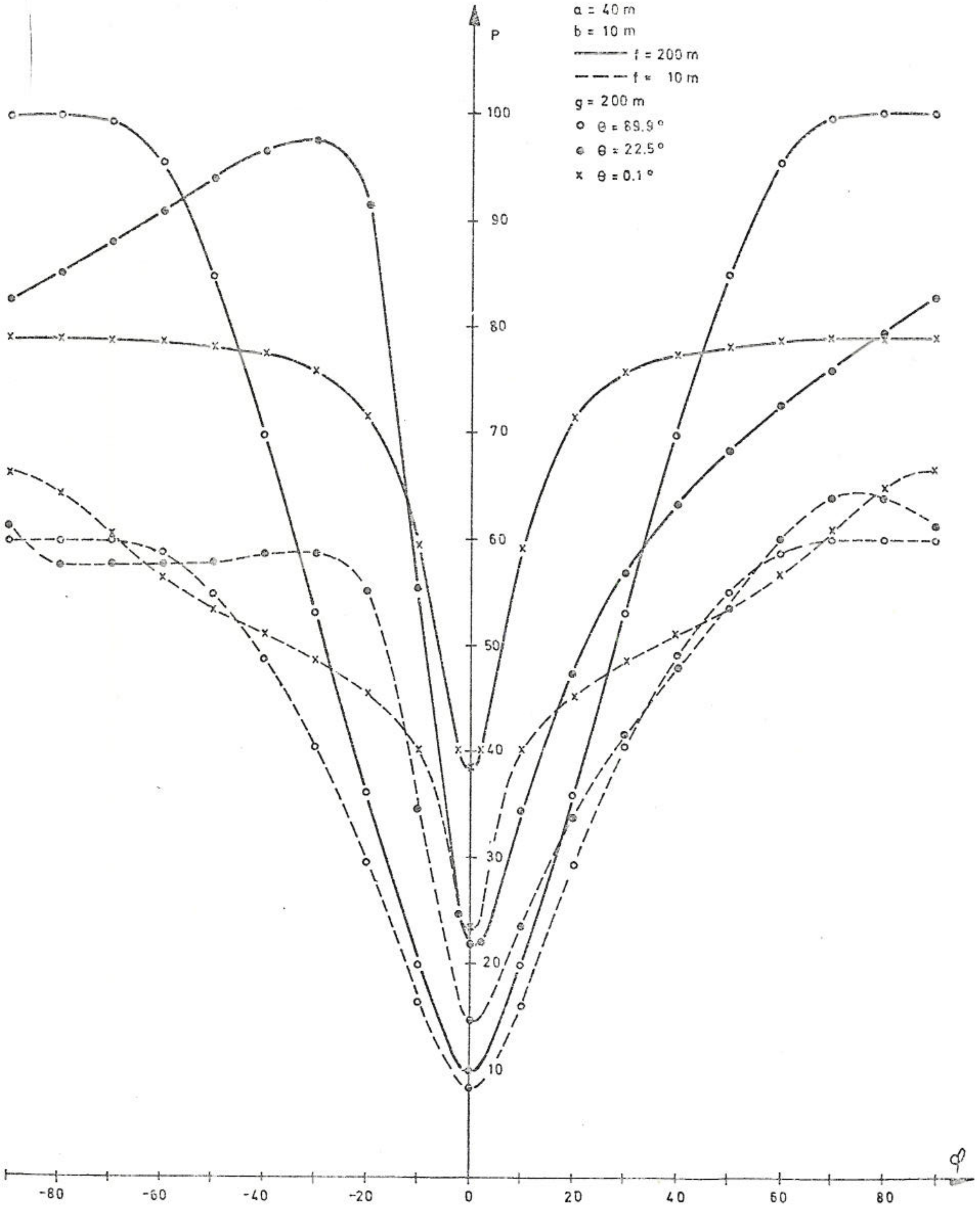


Figure 2.25 Hit-probability vs corridor angle φ, a = 40 m, b = 10 m, f = variable

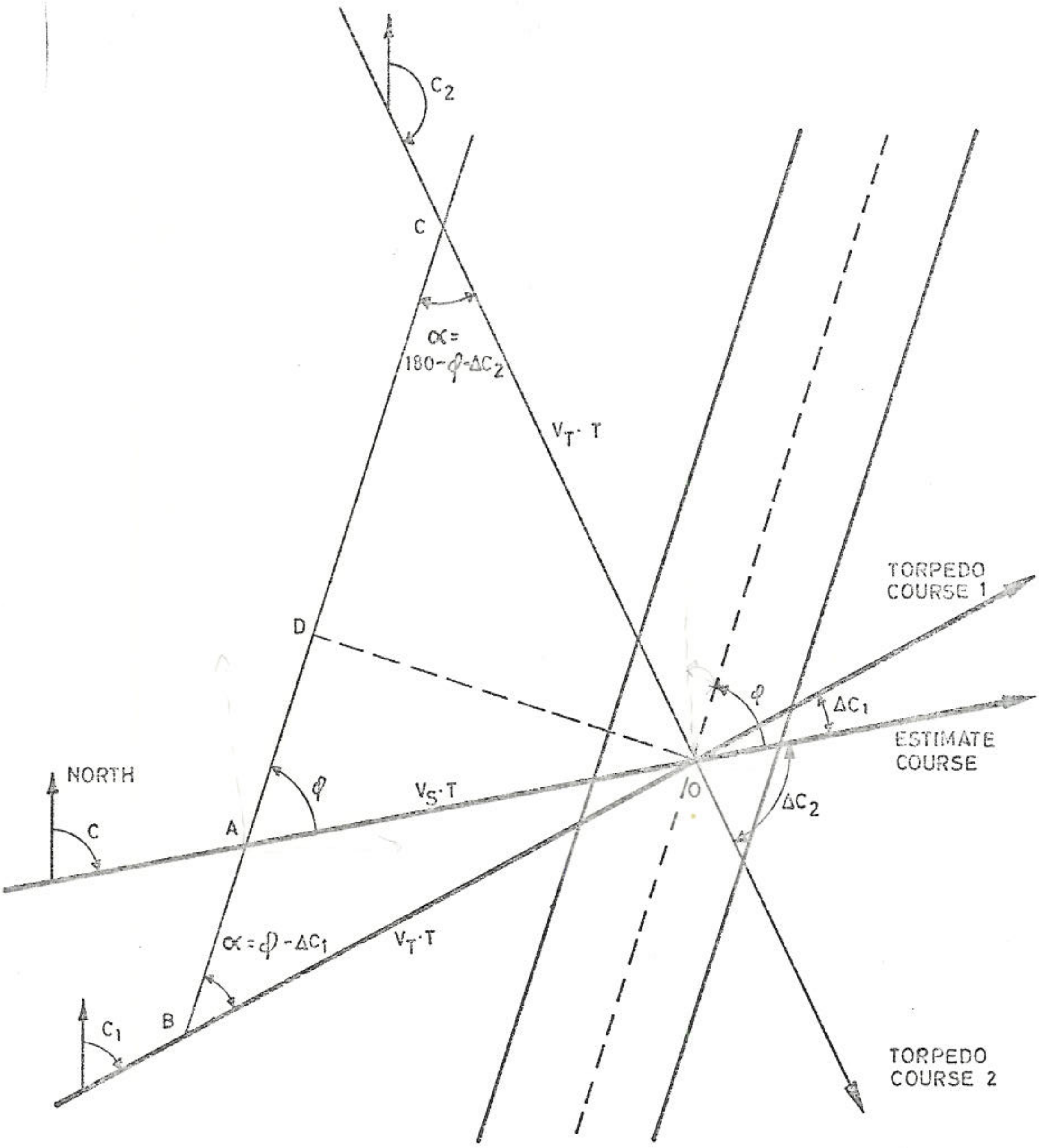


Figure 2.26 Simultaneous positions of the estimate (A) and torpedo (B and C), which can be used to find the relations between corridor angle ϕ and torpedo courses C_1 and C_2

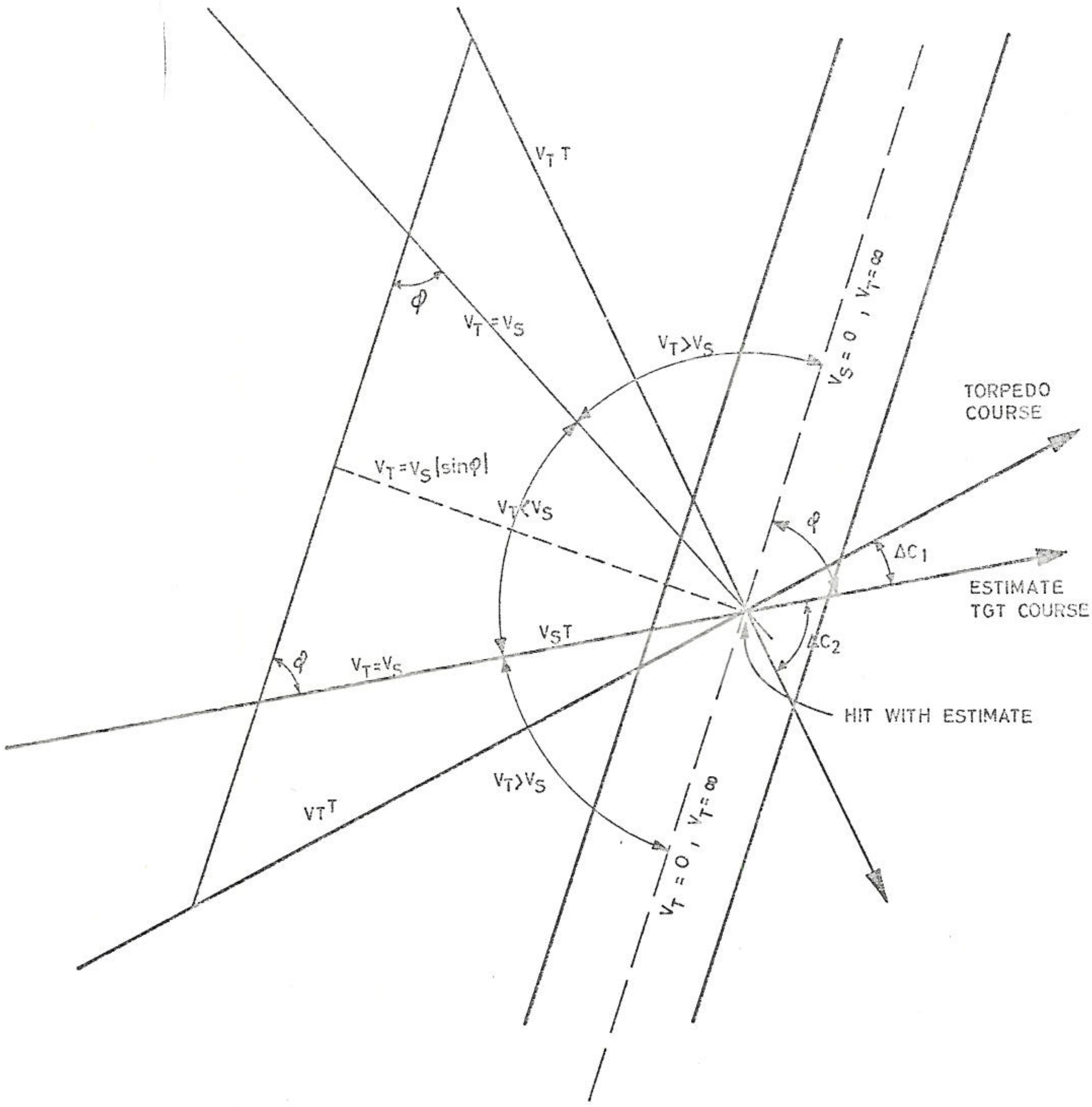


Figure 2.27 Possible solutions of torpedo courses for a given hit-corridor angle φ, when the ratio between target- and torpedo-speed is varied

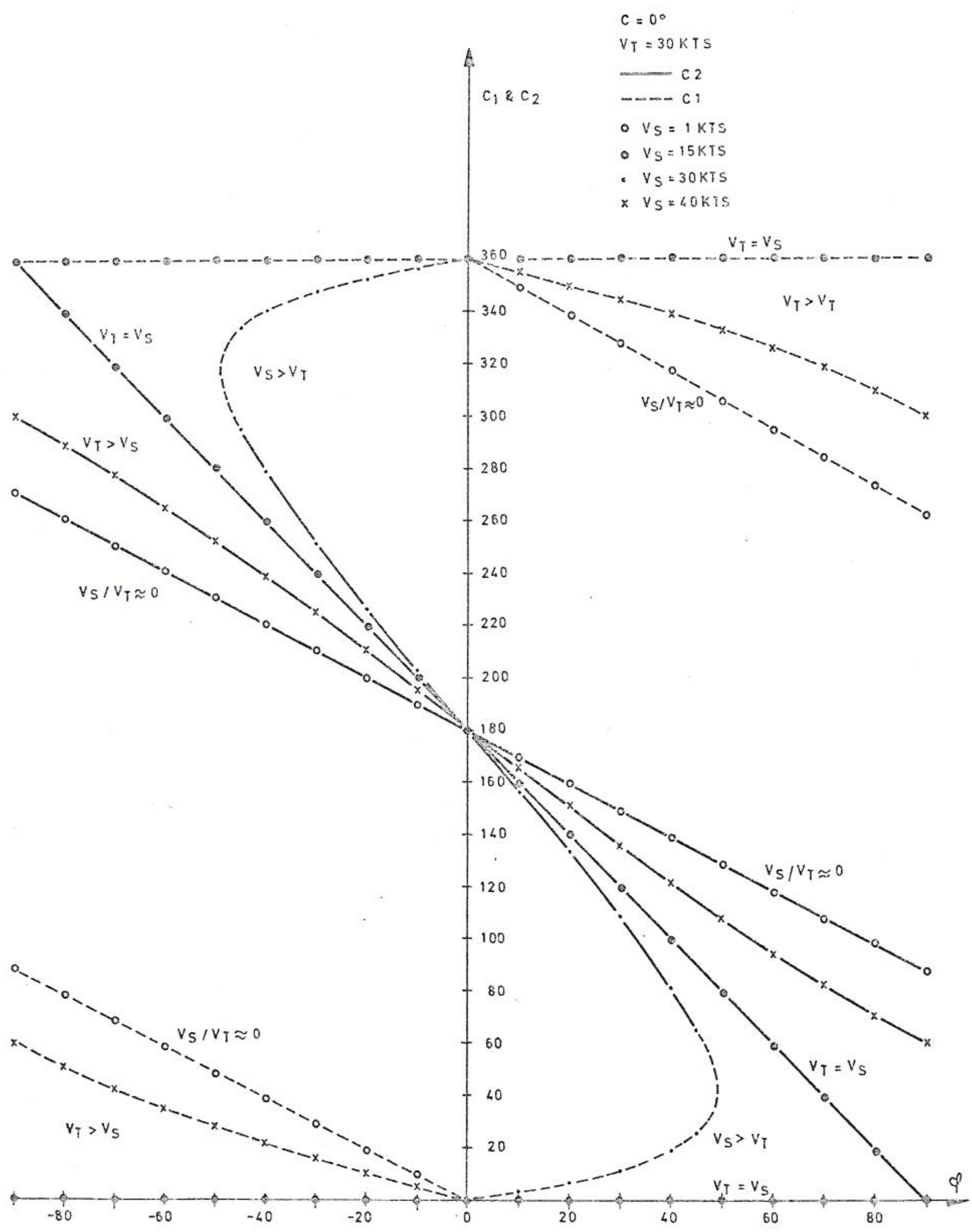


Figure 2.28 Two possible torpedo-courses vs corridor angle φ, with ratio of torpedo and target speeds as parameters. Estimated target course C = 0°

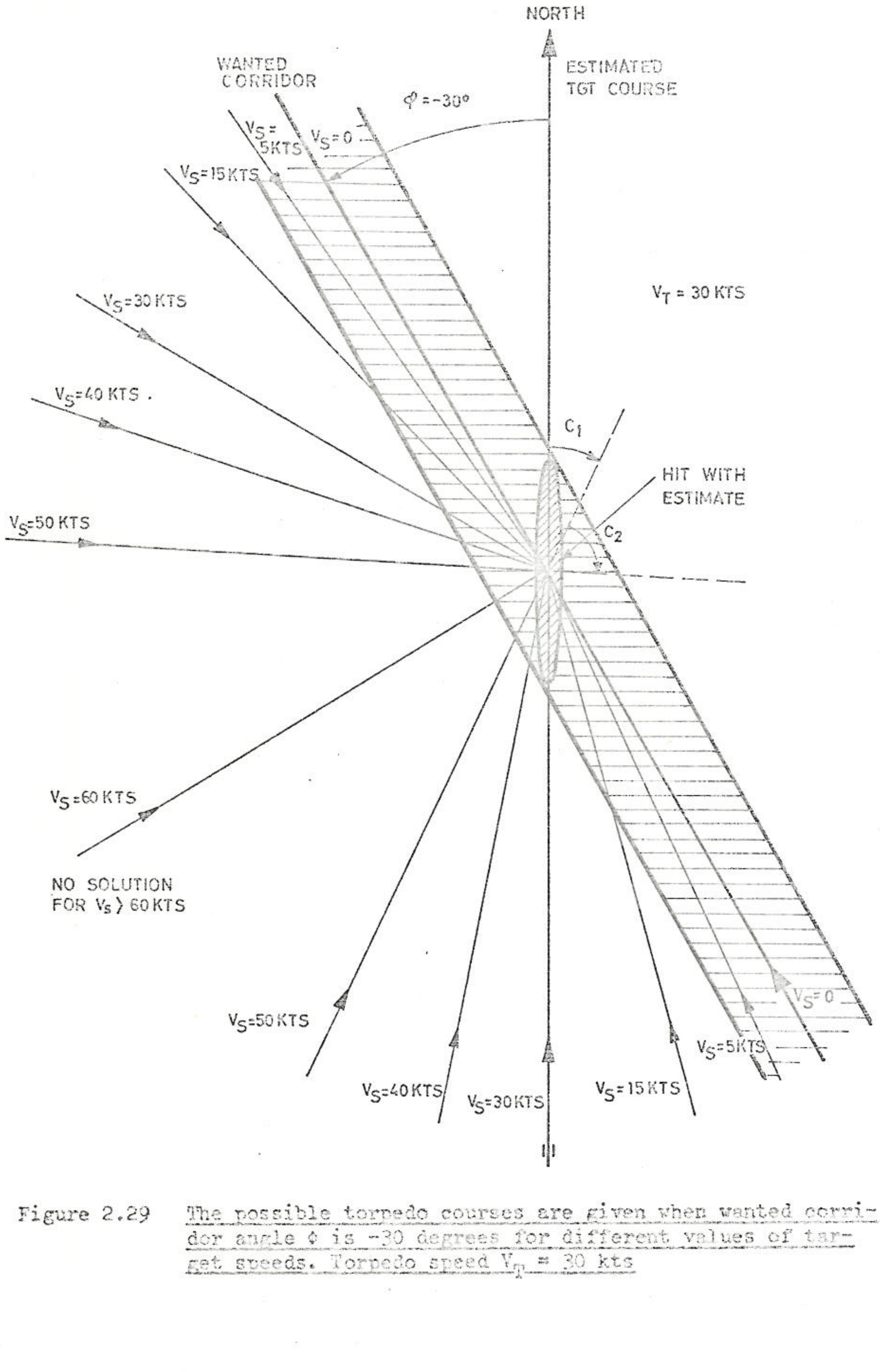


Figure 2.29 The possible torpedo courses are given when wanted corridor angle φ is -30 degrees for different values of target speeds. Torpedo speed V_T = 30 kts

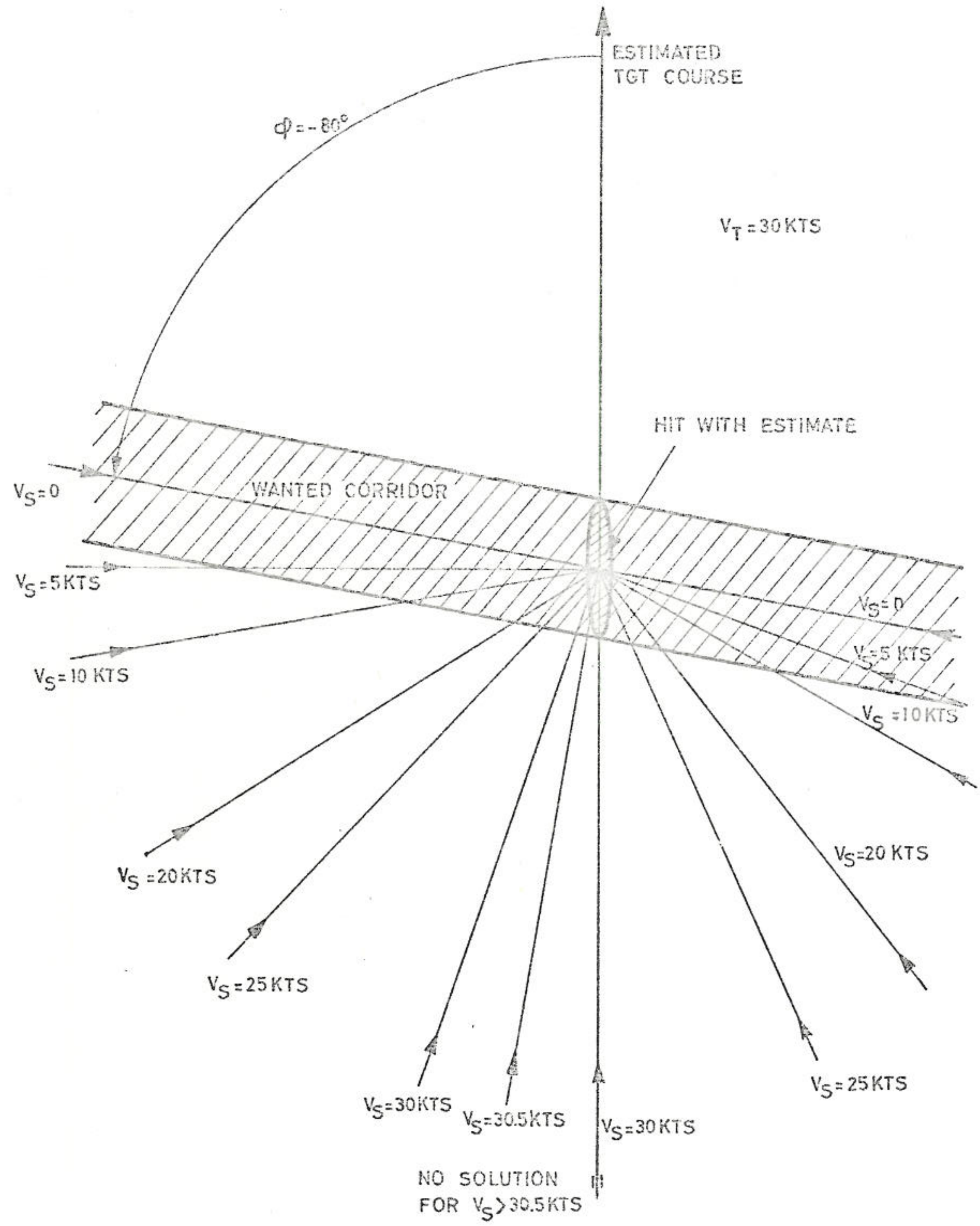


Figure 2.30 The possible torpedo courses are given when wanted corridor angle φ is -80 degrees for different values of target speeds. Torpedo speed = 30 kts

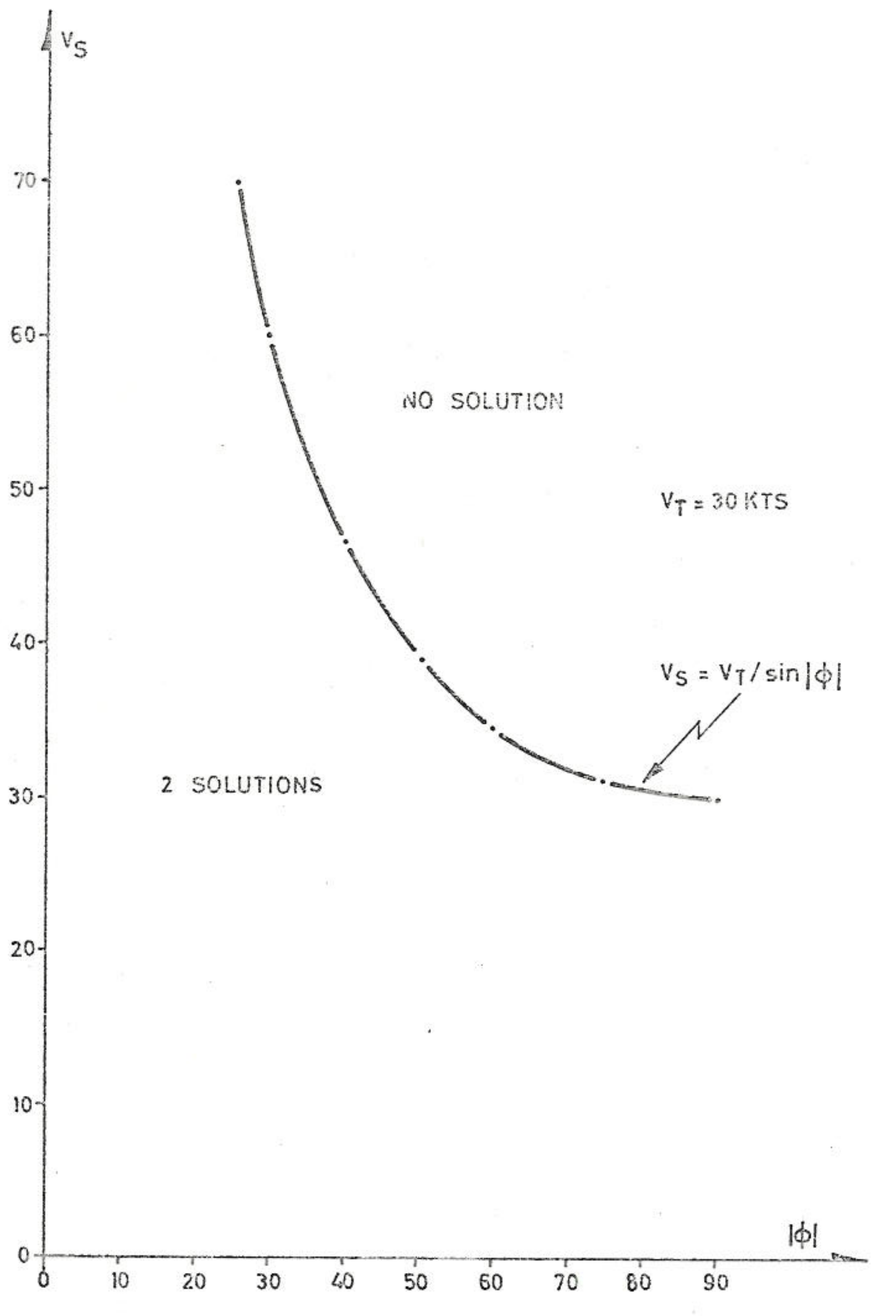


Figure 2.31 Graph showing which target velocities which will yield solutions for a given corridor angle ϕ, and which will not

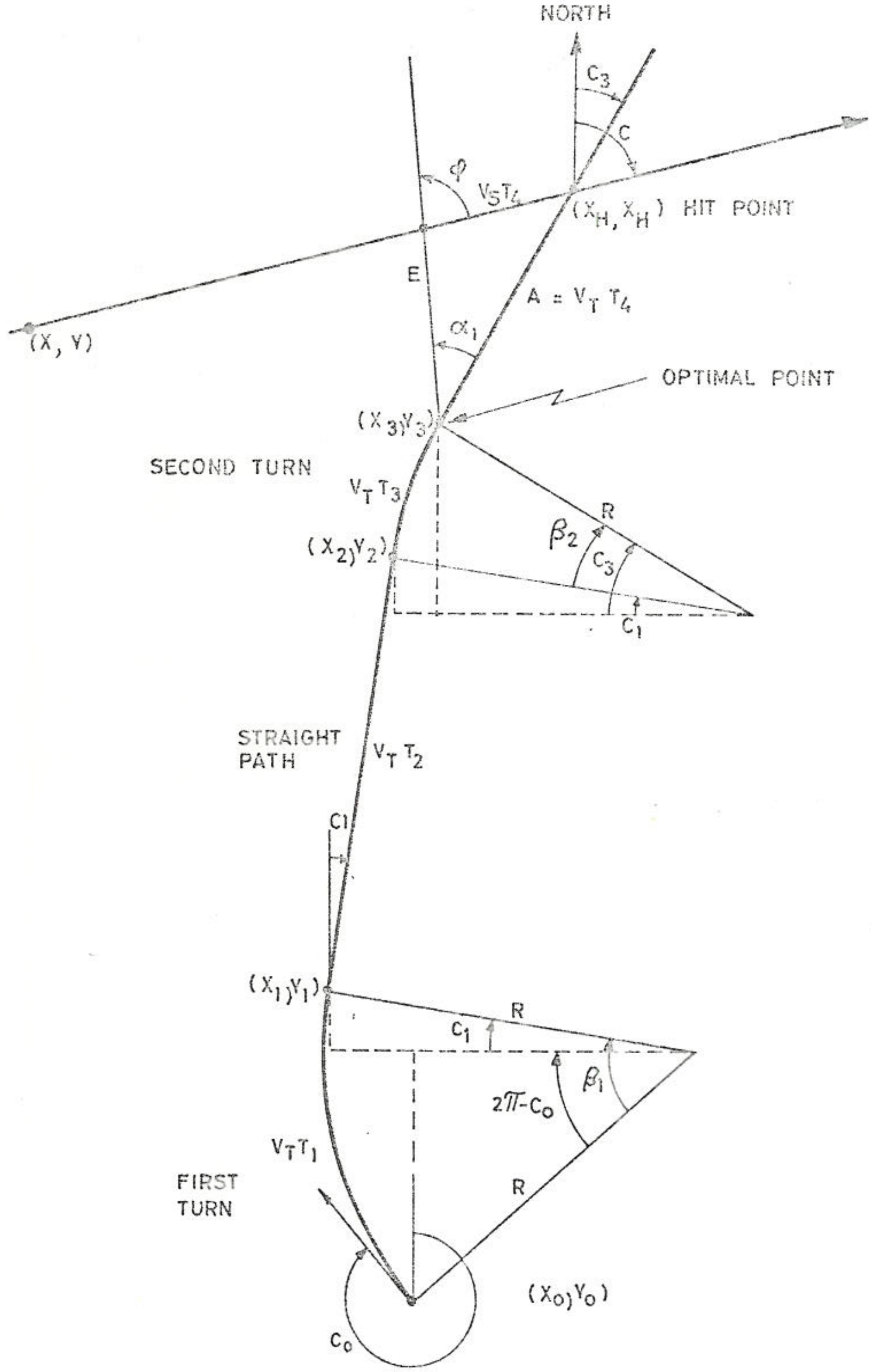


Figure 3.1 The fundamental torpedo trajectory employed in optimal guidance

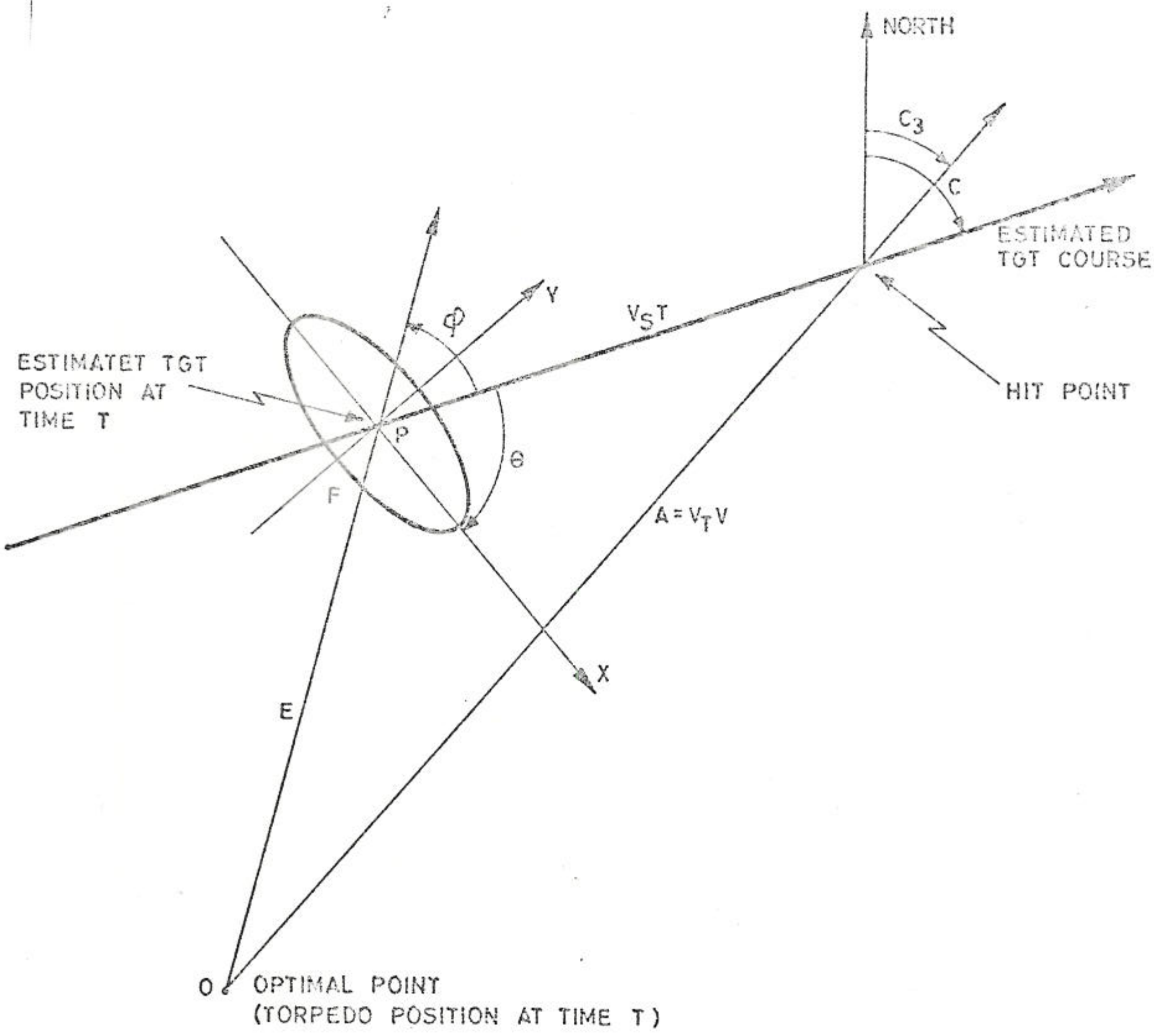


Figure 3.2 Relevant positions of the torpedo and the target estimate for calculation of the optimal distance E

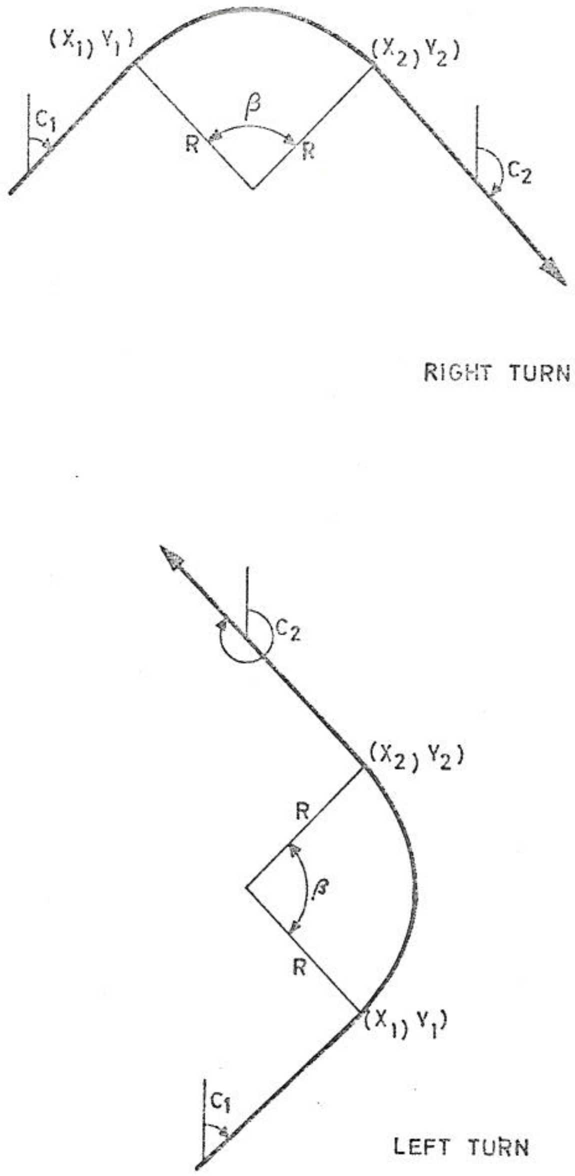


Figure 3.3 Right and left turn

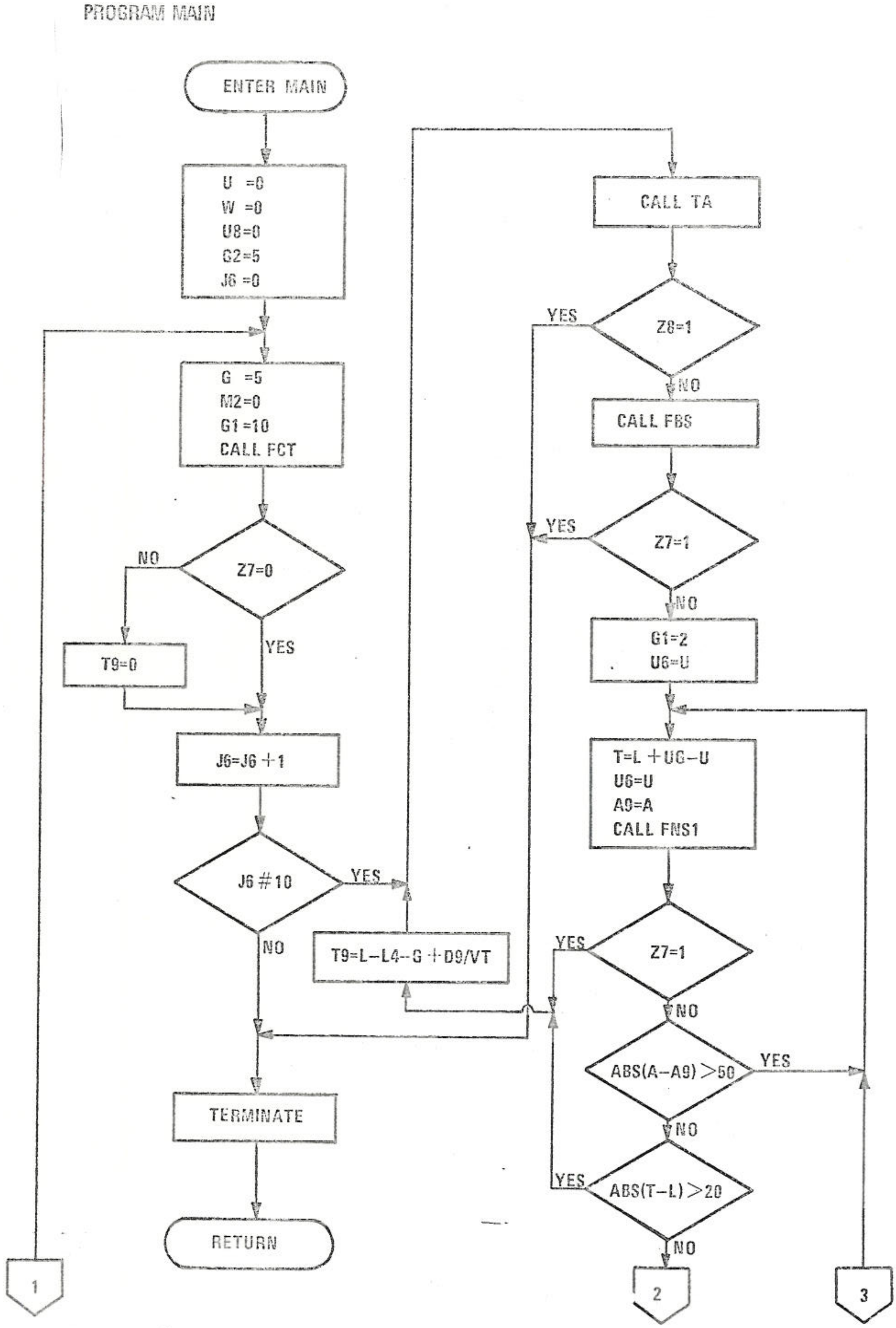


Figure 3.4 Flow chart of MODE 1 and MODE 2 calculations finding the best of all possible solutions

PROGRAM MAIN continued

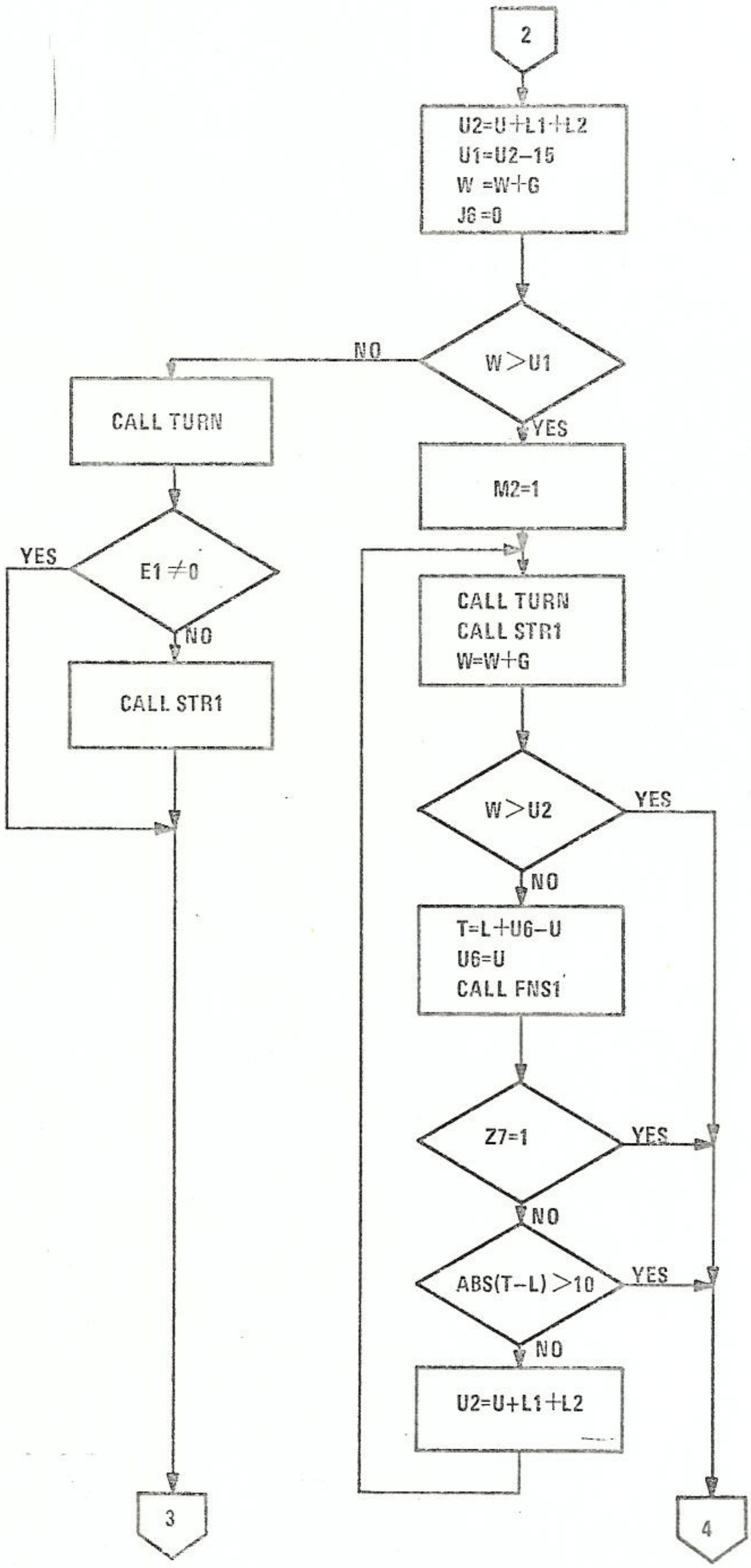


Figure 3.5 Flow chart of STAGE 1 guidance guiding the torpedo towards the optimal point

PROGRAM MAIN continued

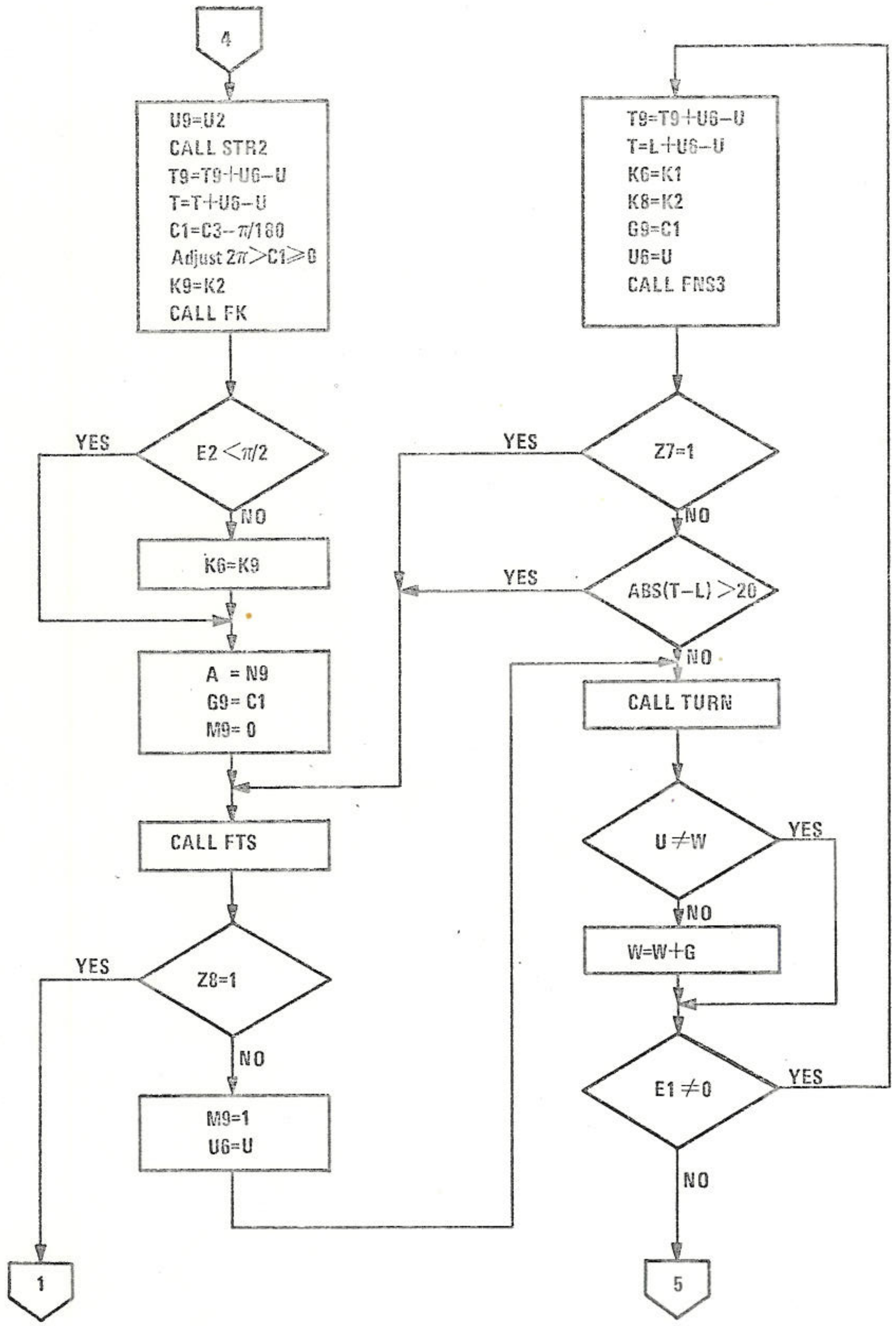


Figure 3.6 Flow chart of STAGE 2 guidance guiding the torpedo until reaching the optimal point

PROGRAM MAIN continued

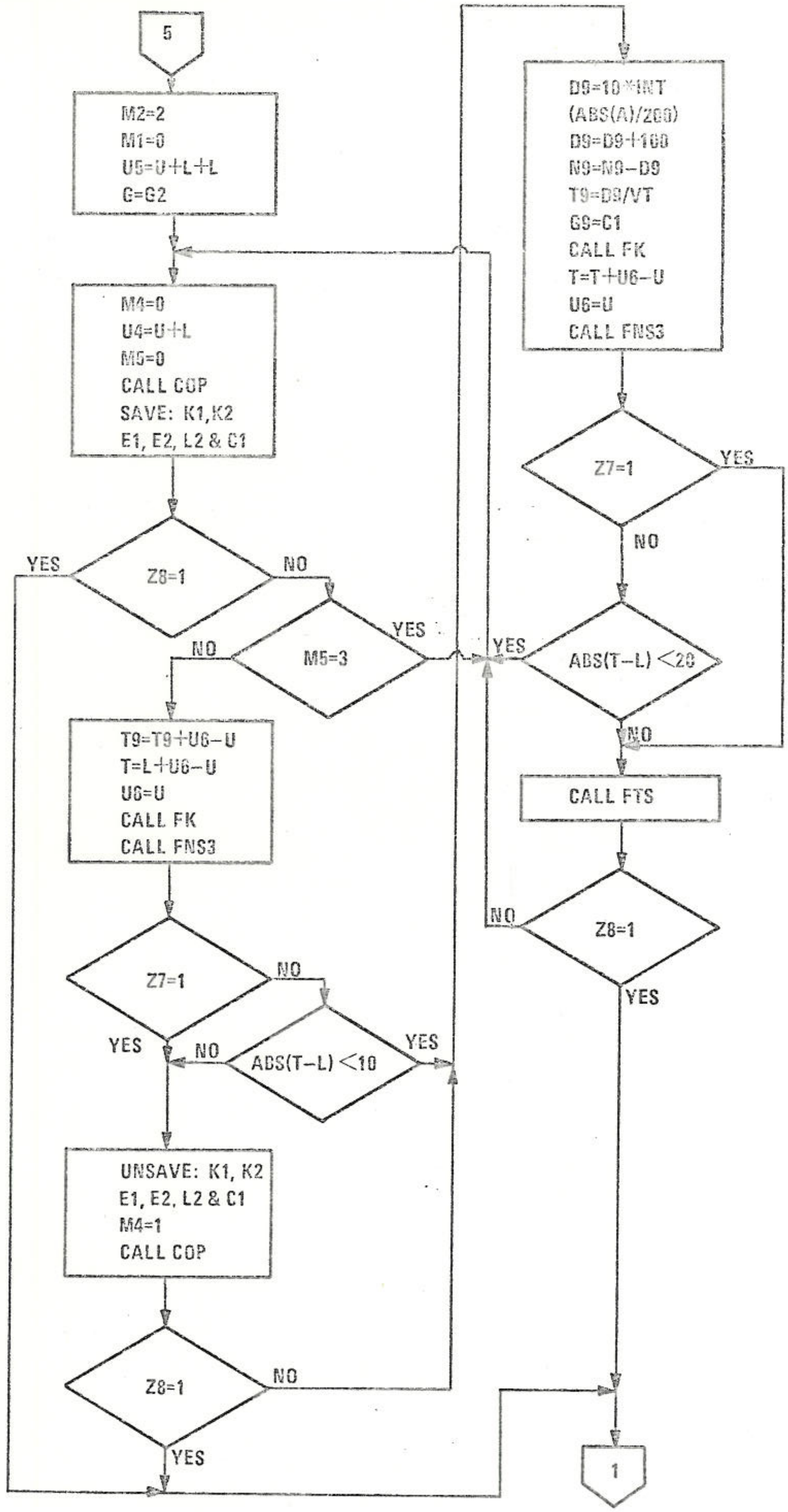


Figure 3.7 Flow chart of STAGE 3 guidance guiding the torpedo until the start of the next multipass

PROGRAM CCT

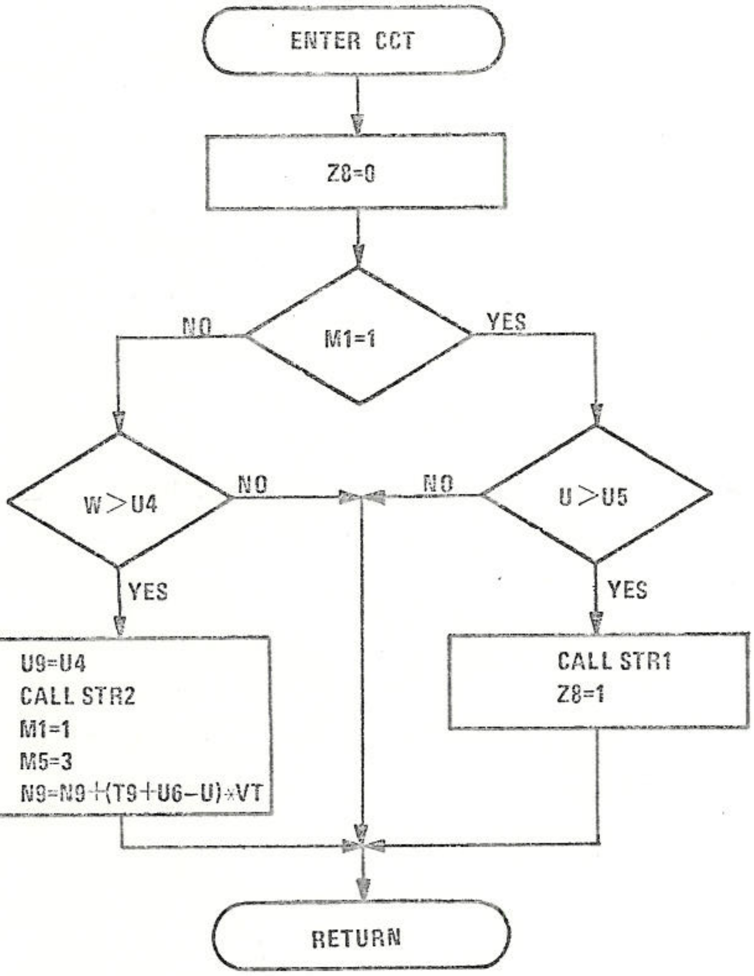


Figure 3.8 Flow chart of Program CCT which checks if the time for collision or start of multipass will be exceeded

PROGRAM COP

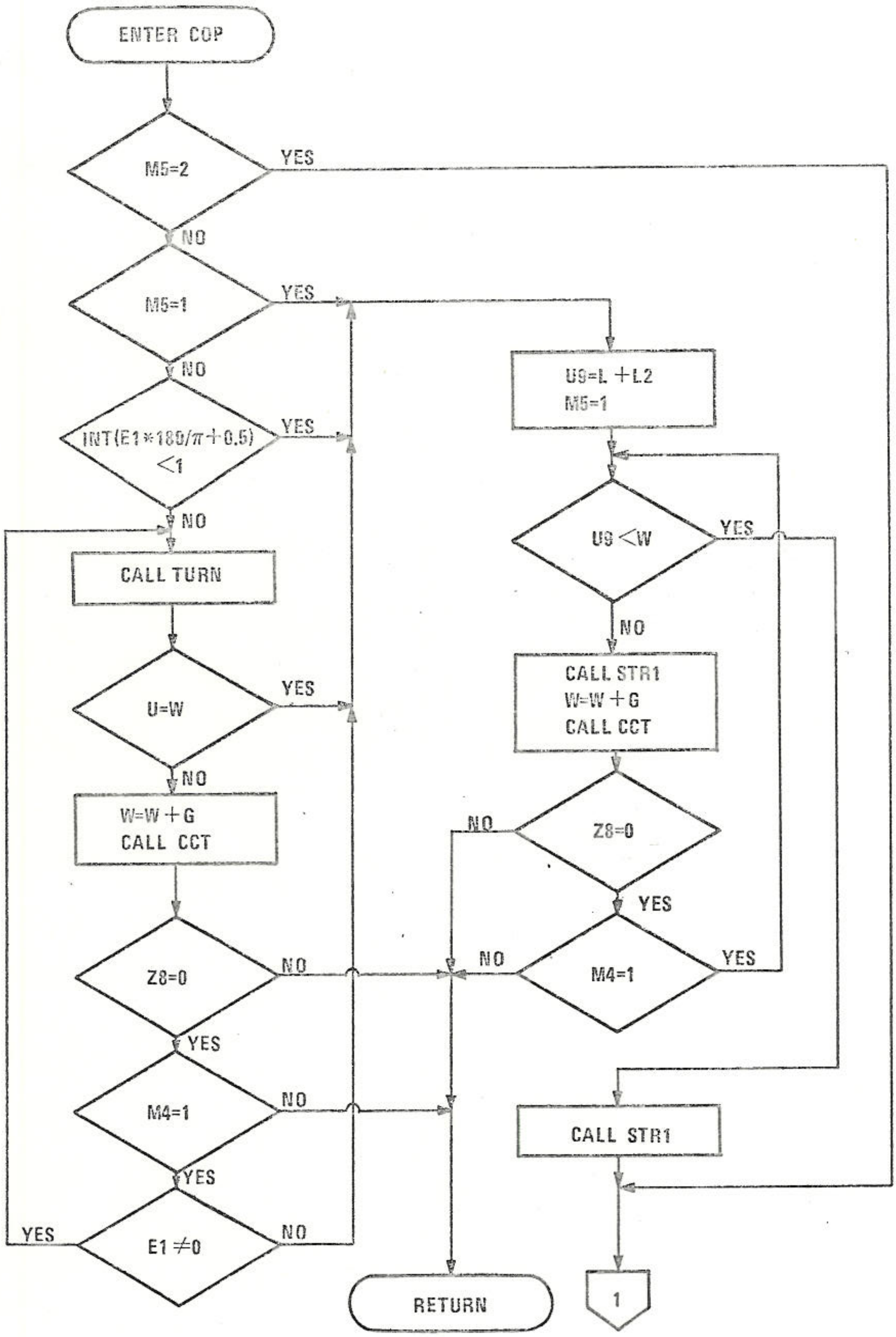


Figure 3.9 Flow chart of Program COP which will guide the torpedo according to plan until next calculation time when M4 = 0 and until second turn is finished if M4 = 1

PROGRAM COP continued

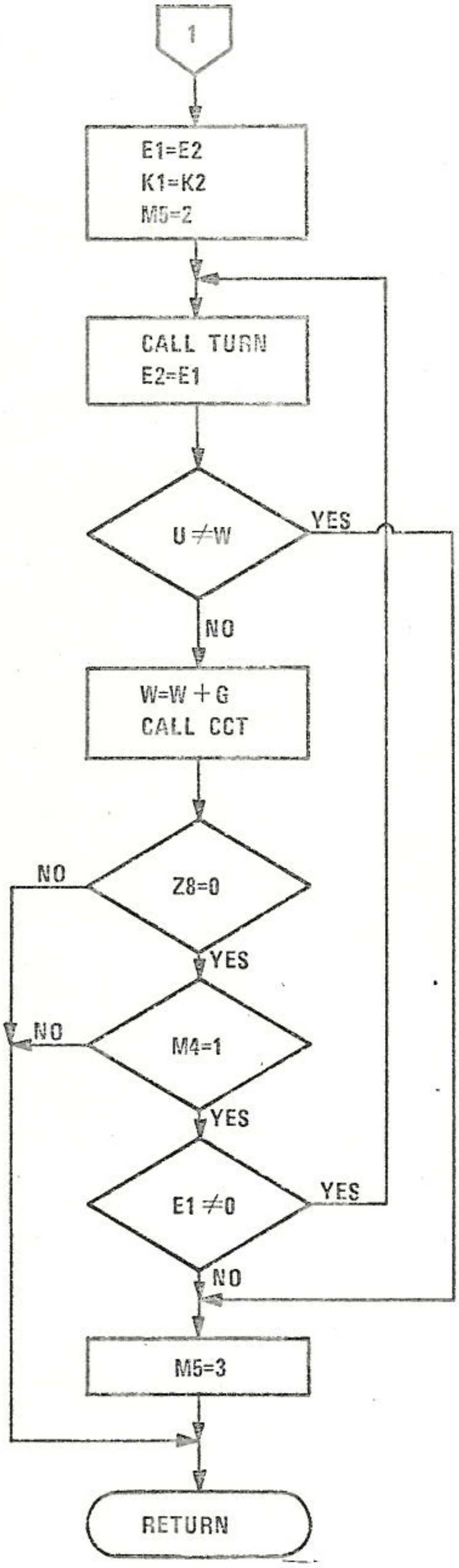


Figure 3.10 Flow chart of Program COP continued

PROGRAM CT

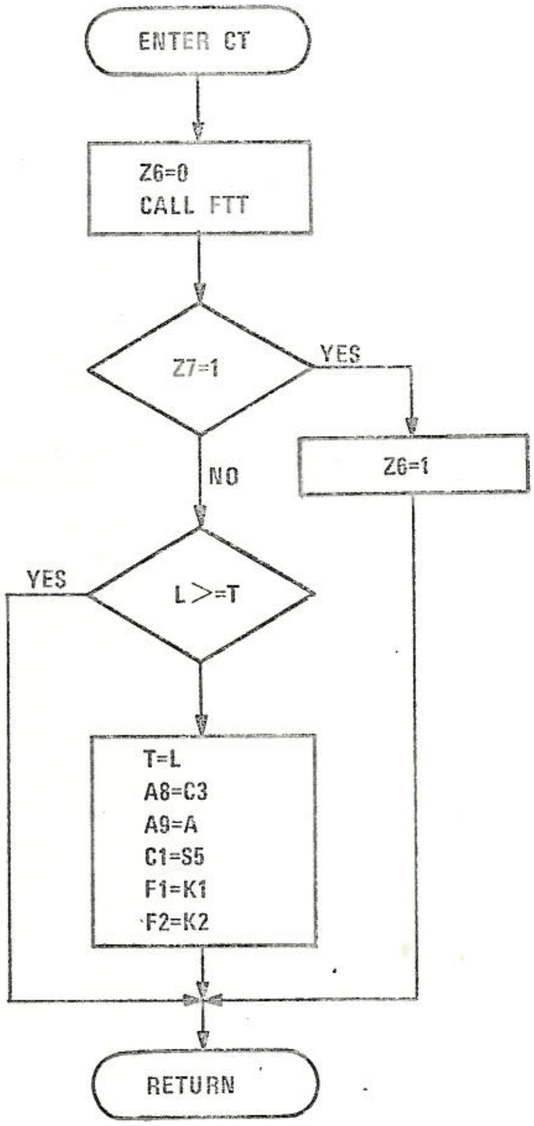


Figure 3.11 Flow chart of Program CT testing if the new solution is better than the former best solution

PROGRAM FA

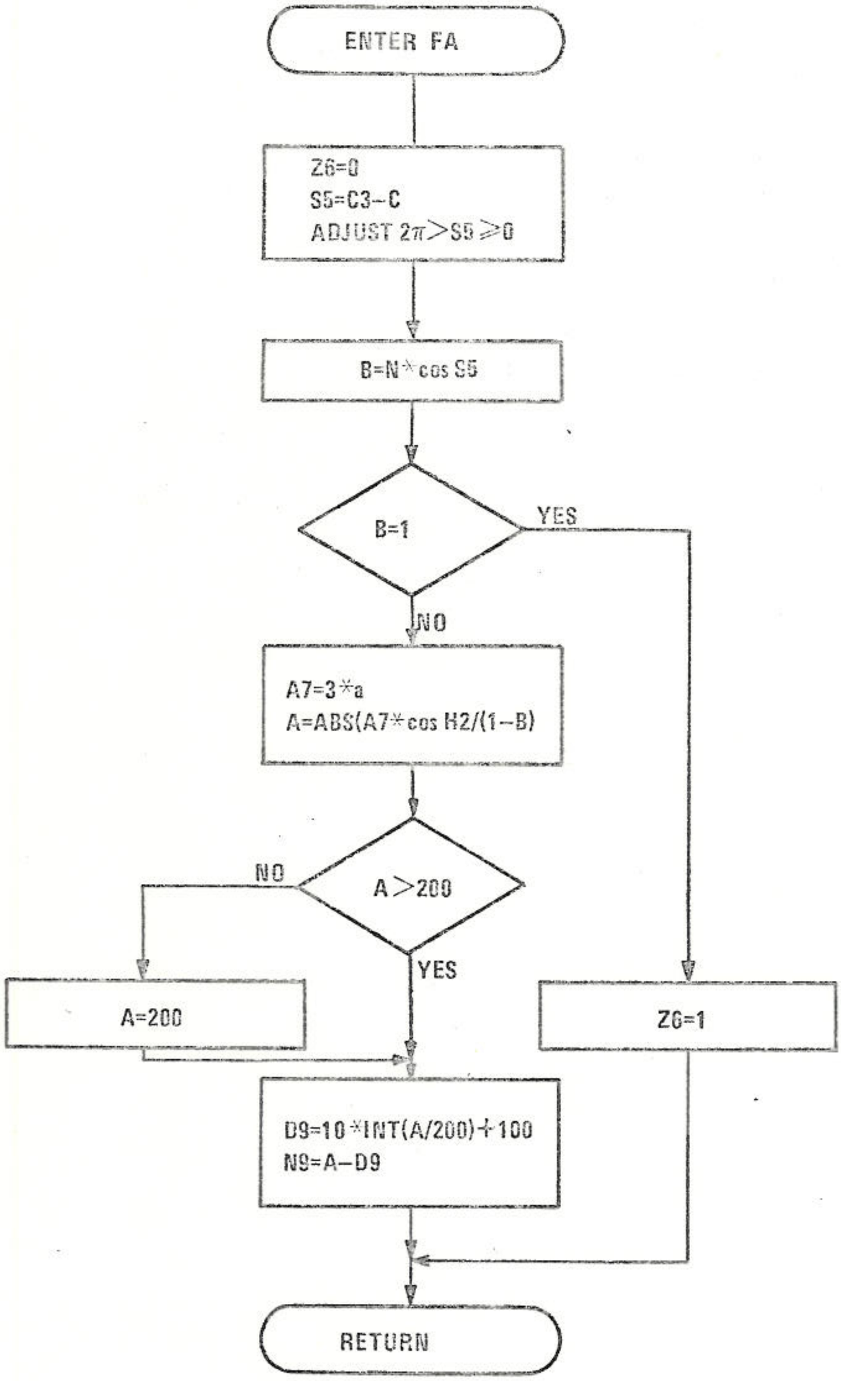


Figure 3.12 <u>Flow chart of Program FA computing the distance "A" between the optimal point and the hit-point</u>

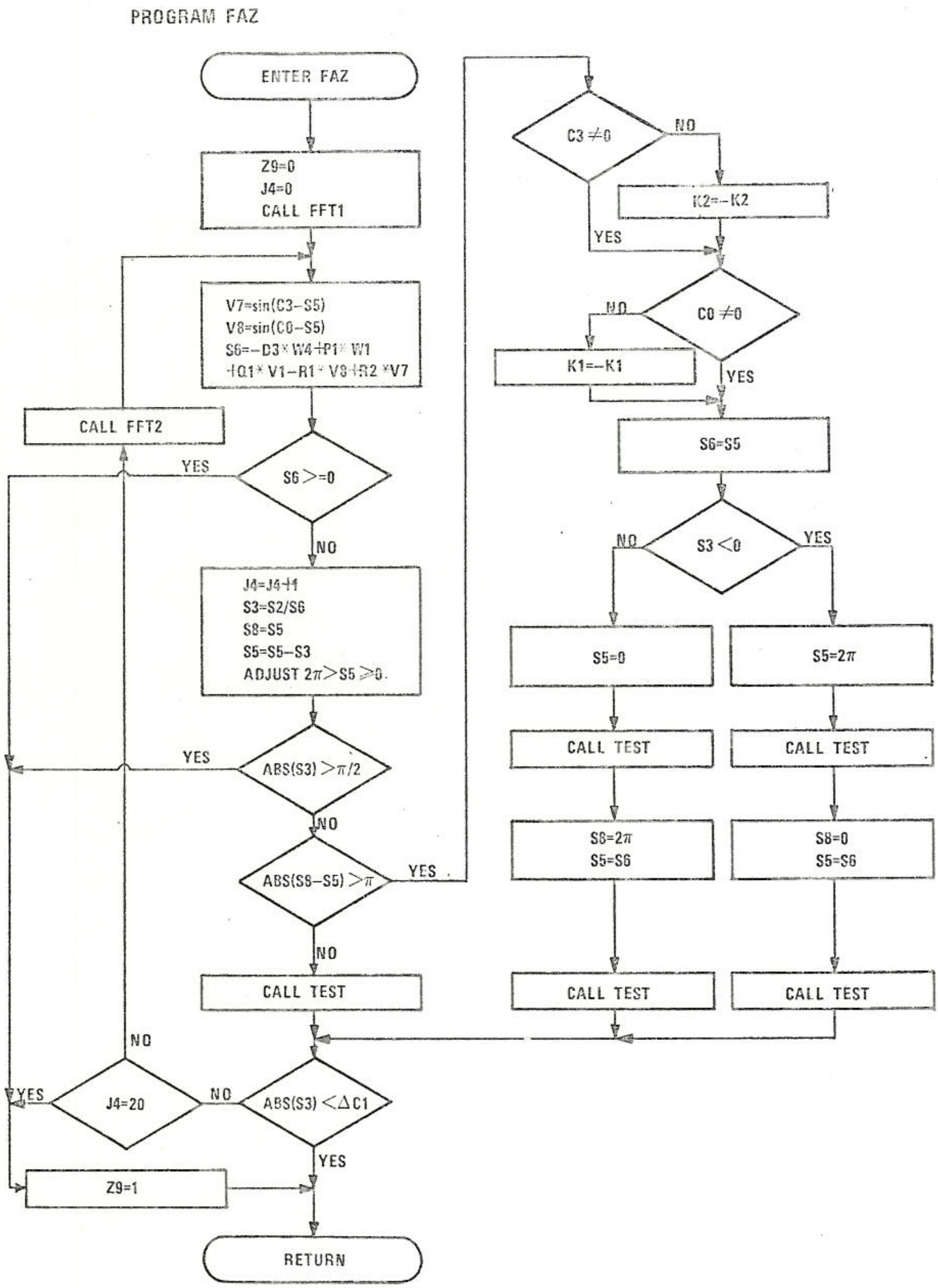


Figure 3.13 Flow chart of Program FAZ, finding S5 making F(S5) = 0

PROGRAM FBA

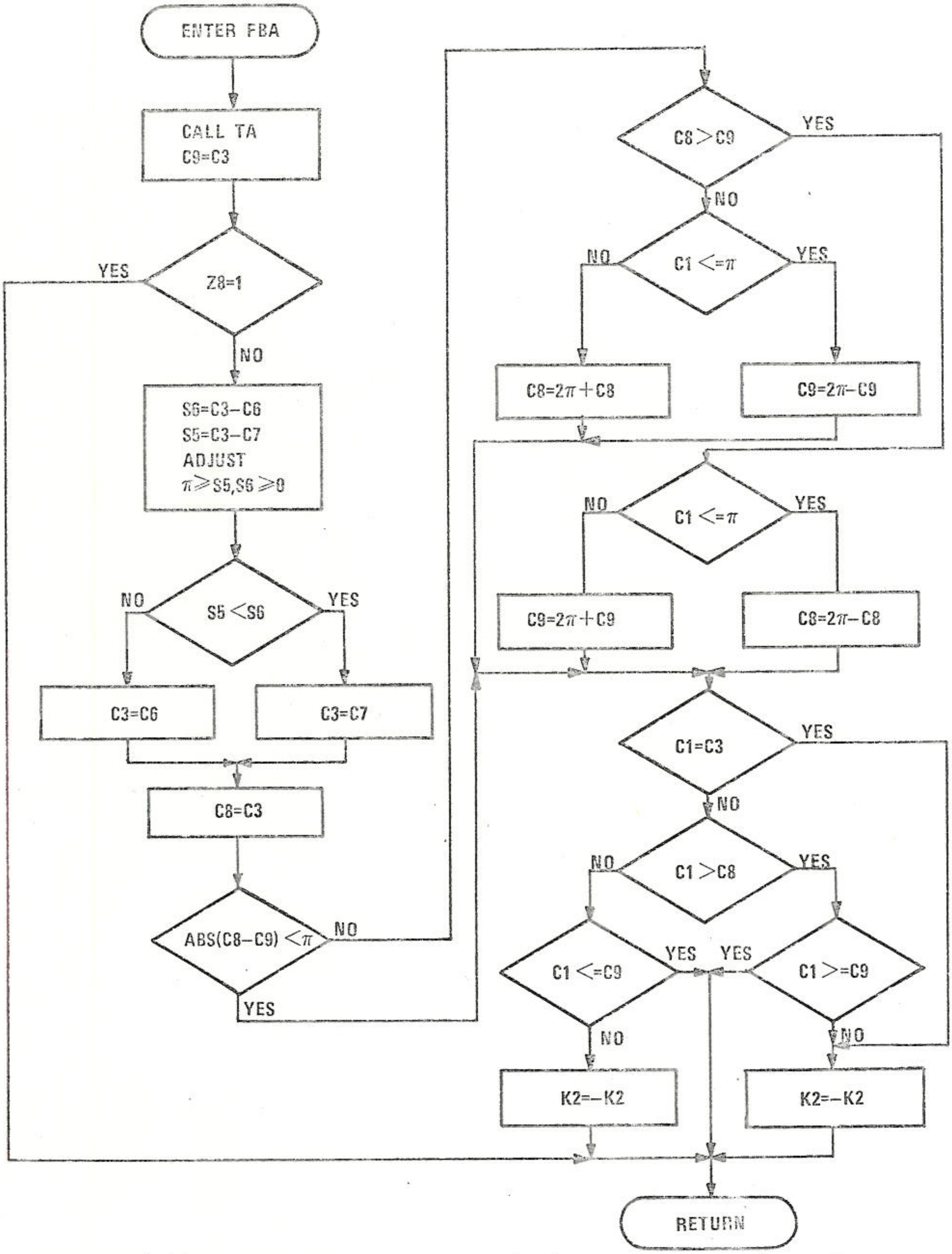


Figure 3.14 Flow chart of Program FBA finding the new torpedo angle which is closest to the former value

PROGRAM FBS

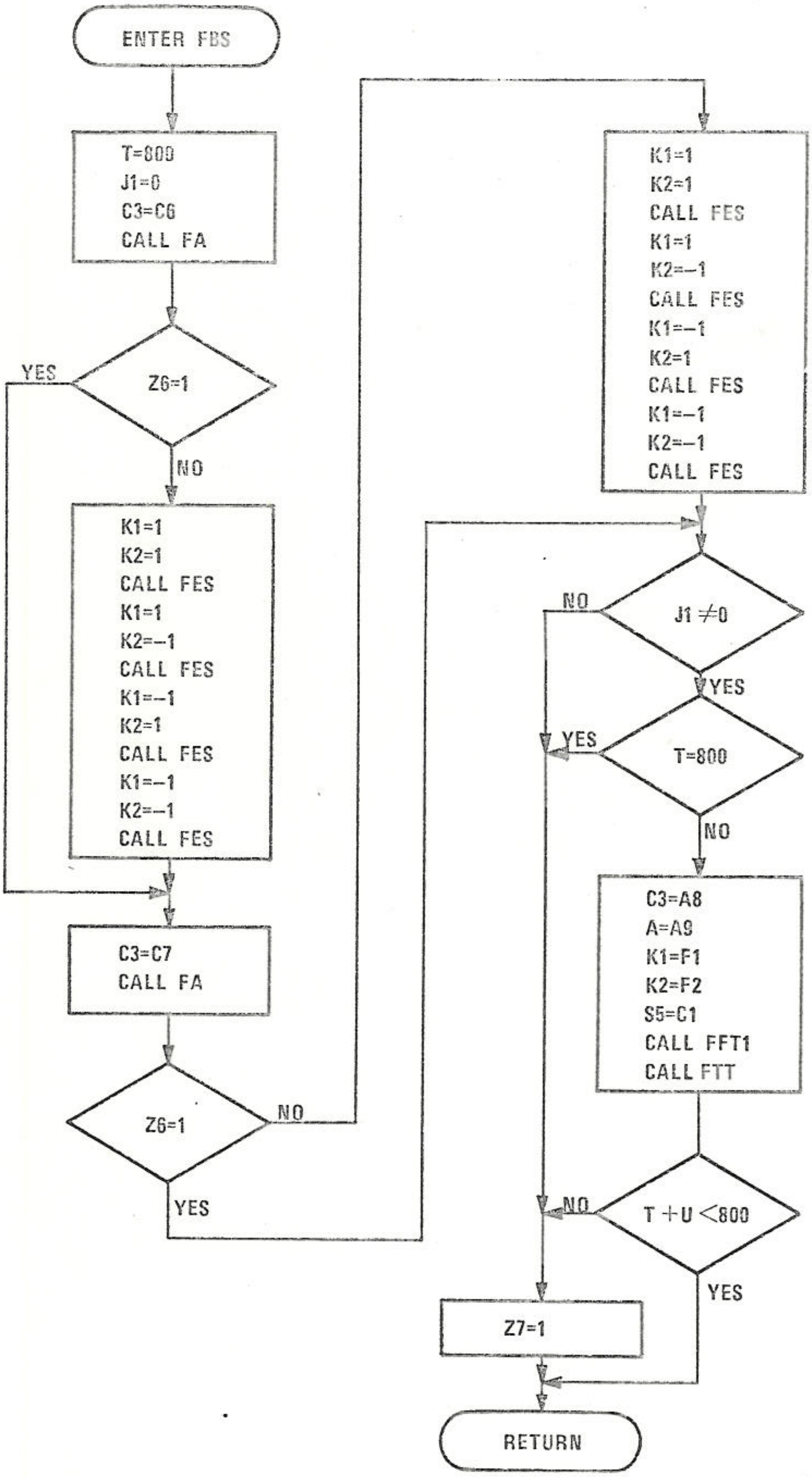


Figure 3.15 Flow chart of Program FBS finding the best of all possible solution by mode 1 calculations

PROGRAM FCA

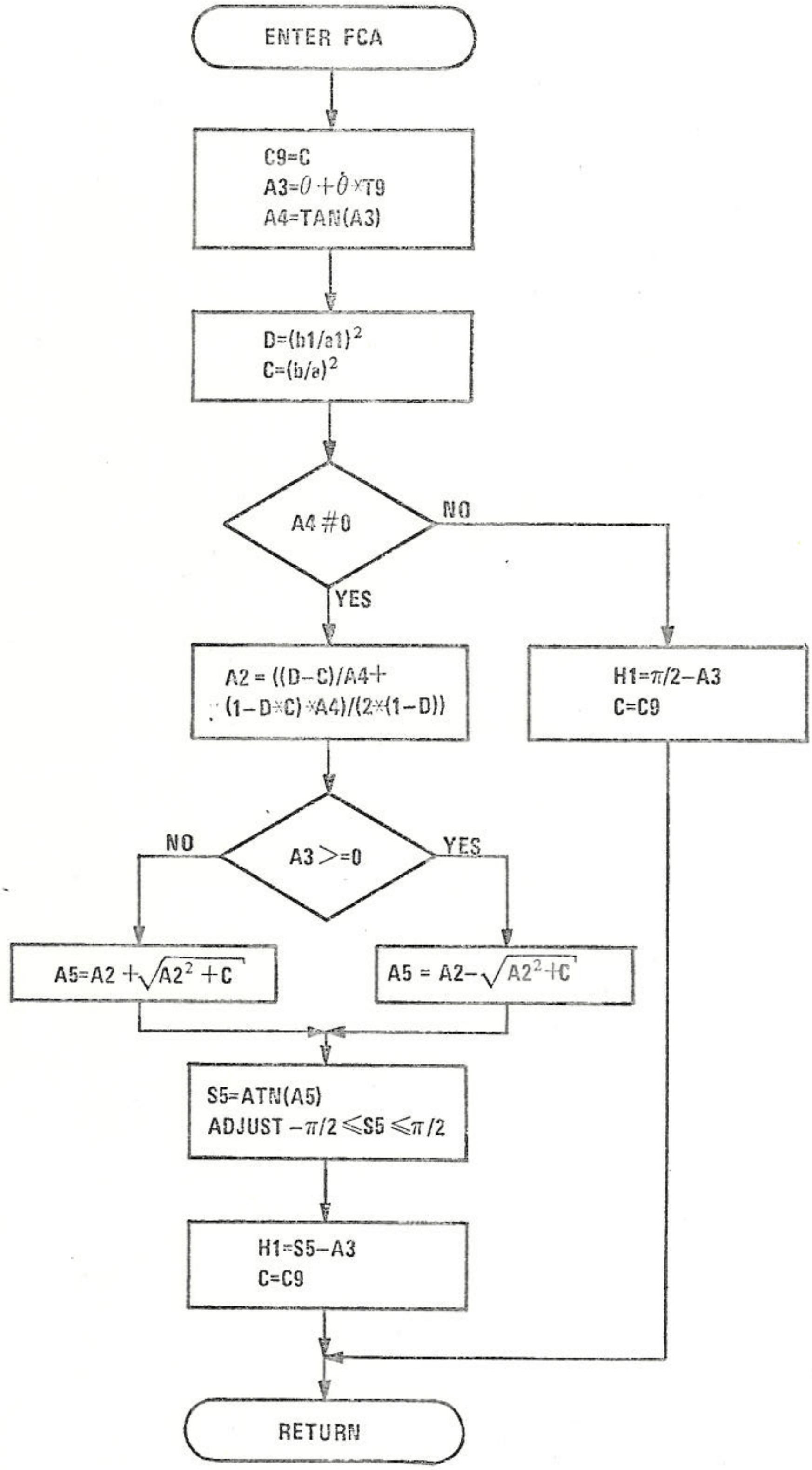


Figure 3.16 Flow chart of Program FCA calculating the wanted corridor angle H1

PROGRAM FCT

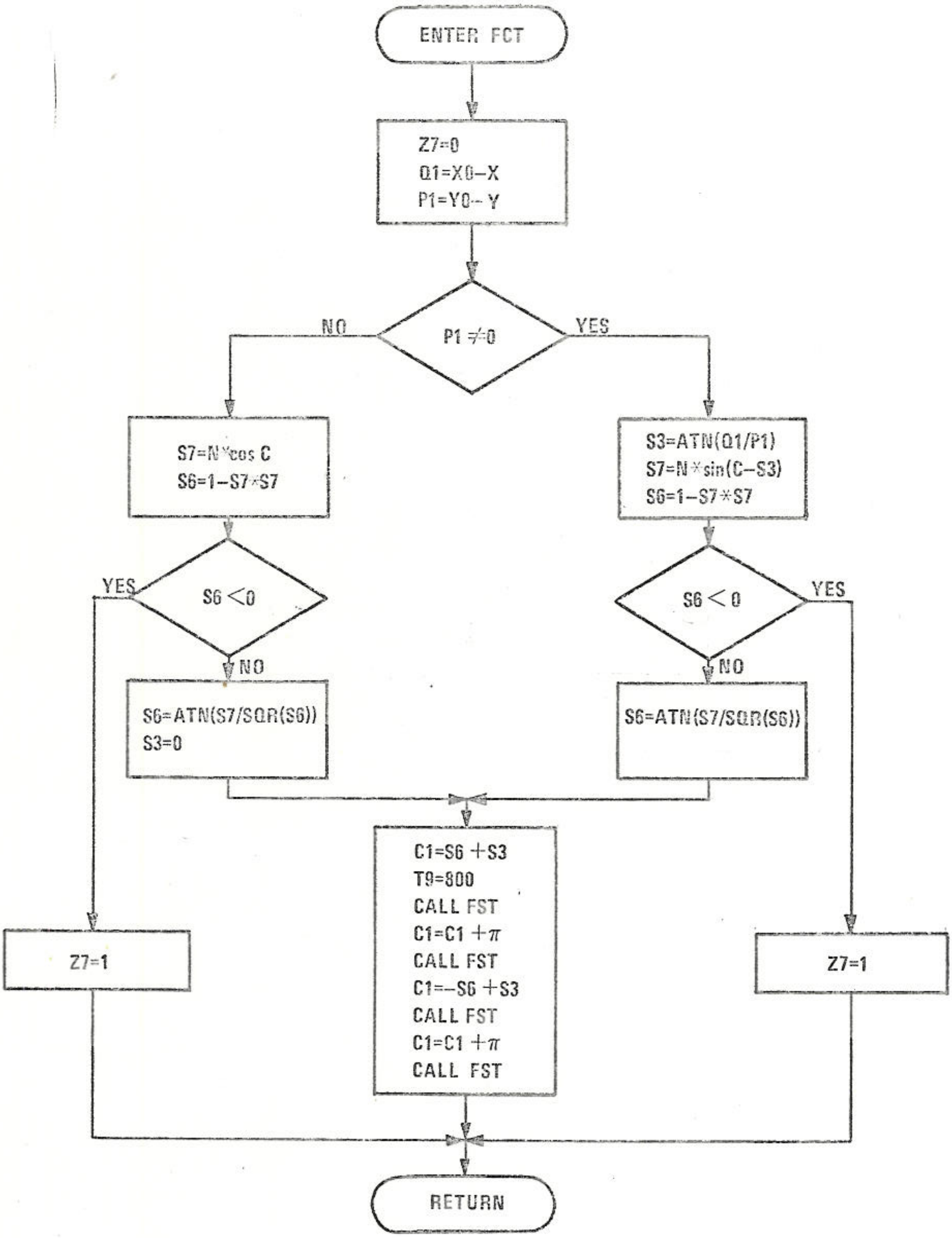


Figure 3.17 Flow chart of Program FCT finding the least time to collision point when no turns are considered

PROGRAM FES

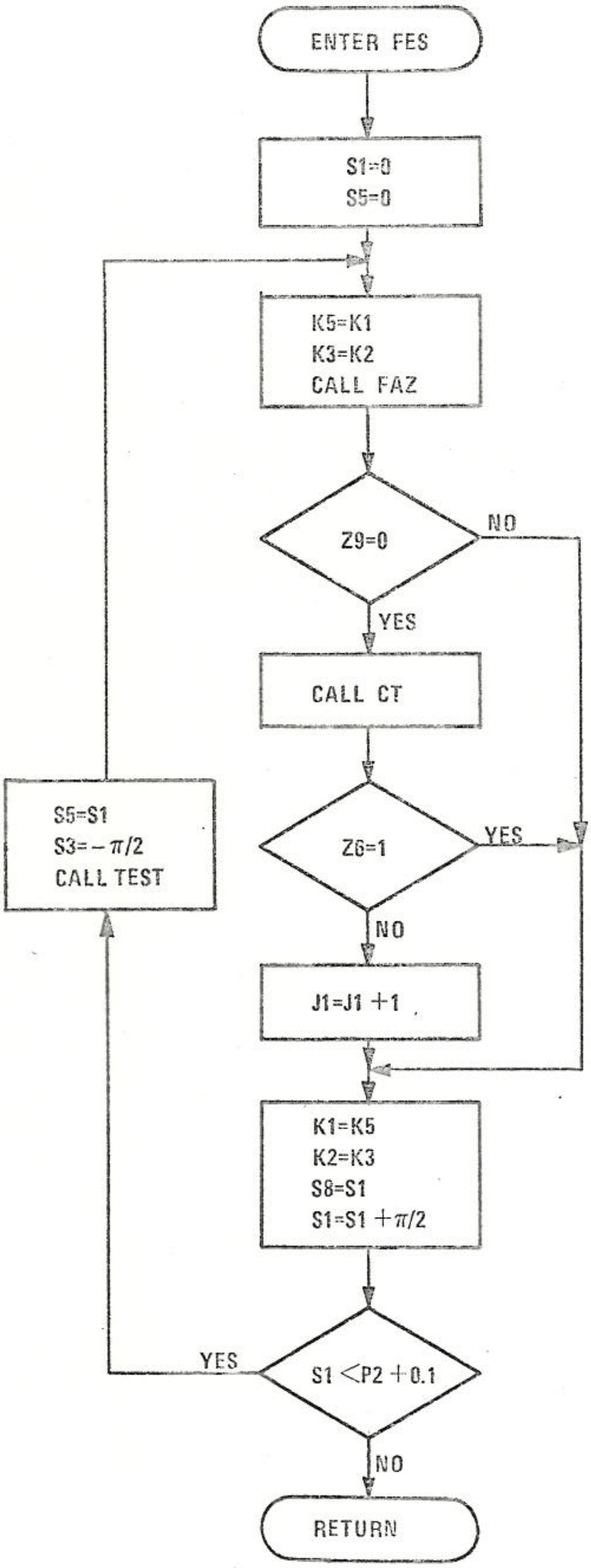


Figure 3.18 Flow chart of Program FES, finding the best solution for given values of C_3, K_1 and K_2

PROGRAM FFT

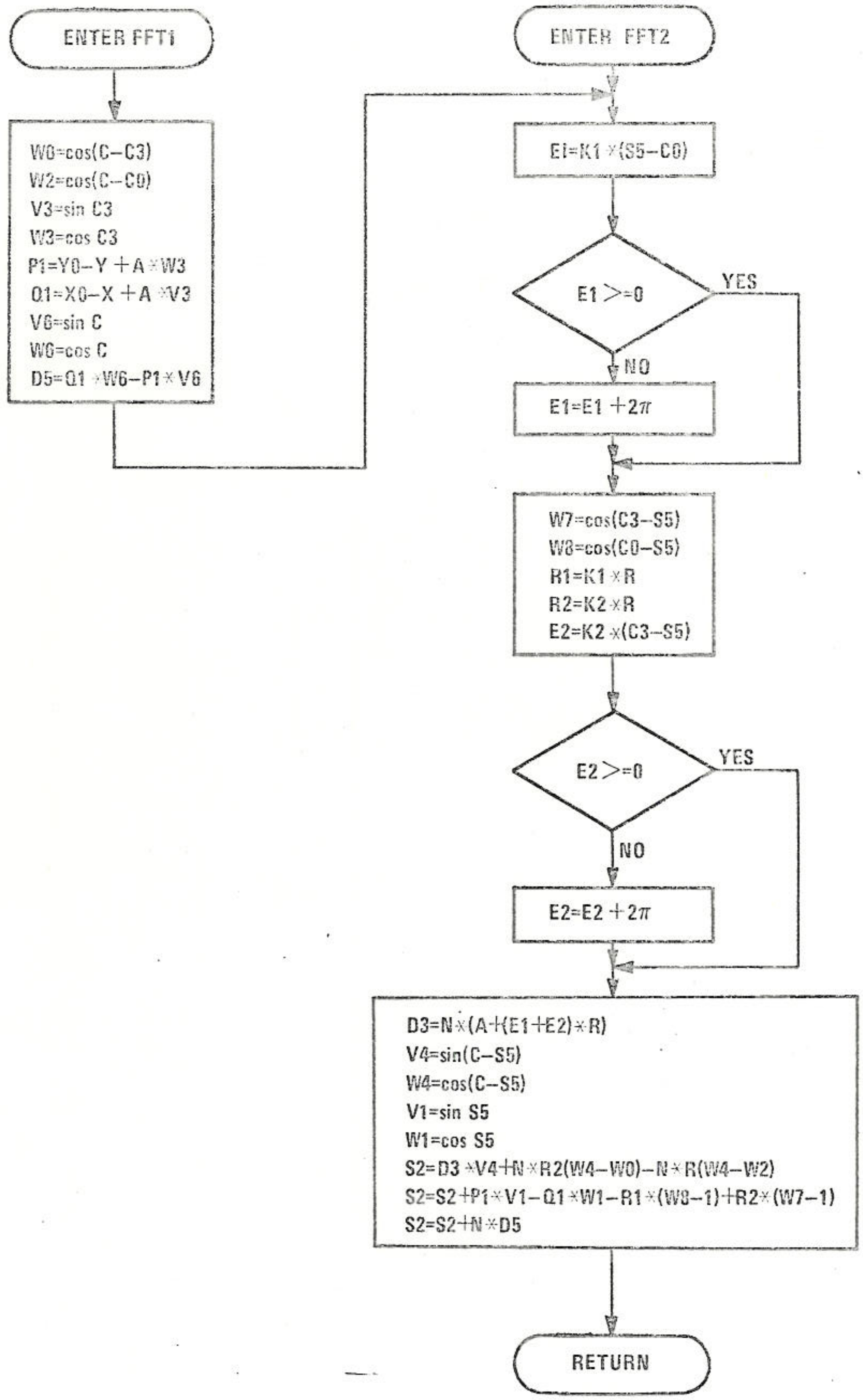


Figure 3.19 Flow chart of Program FFT finding F(S5) and the first and second turn

PROGRAM FK

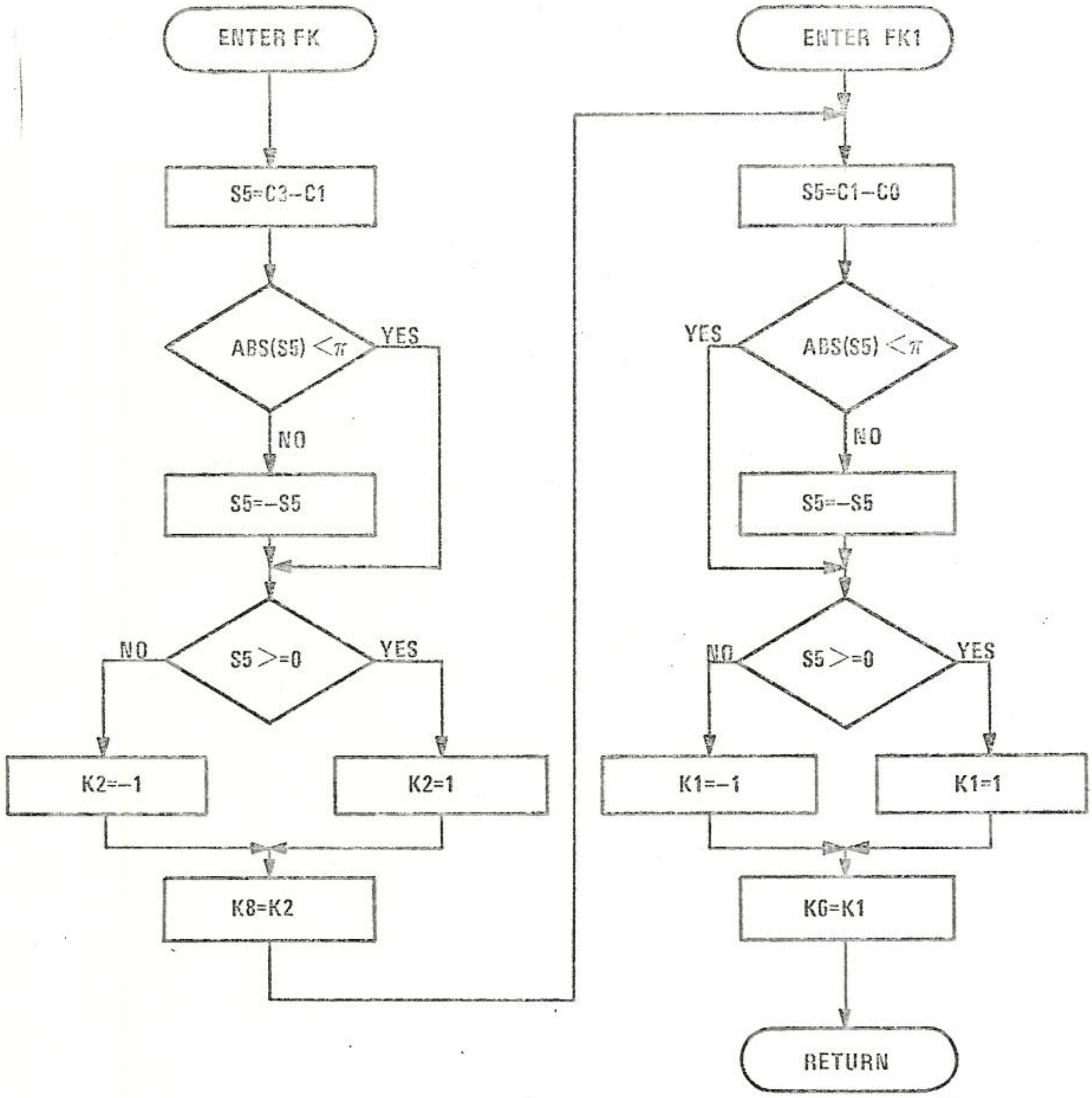


Figure 3.20 Flow chart of Program FK, finding the values of K_1 and K_2 when the turns are less than π radians

PROGRAM FNS

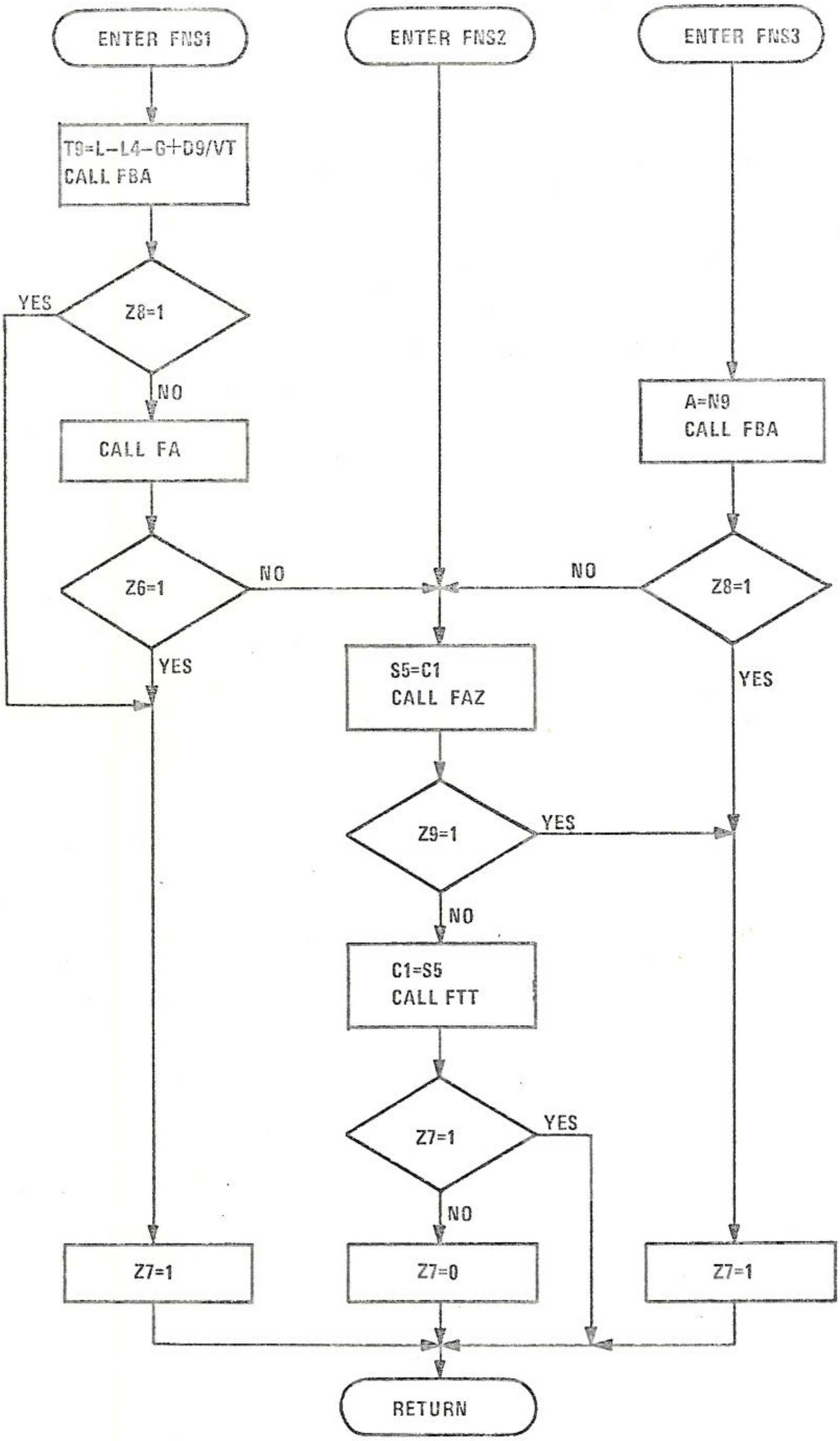


Figure 3.21 Flow chart of Program FNS finding a new solution of C_1 based on old values of C_1, by Mode 2 calculations

PROGRAM FST

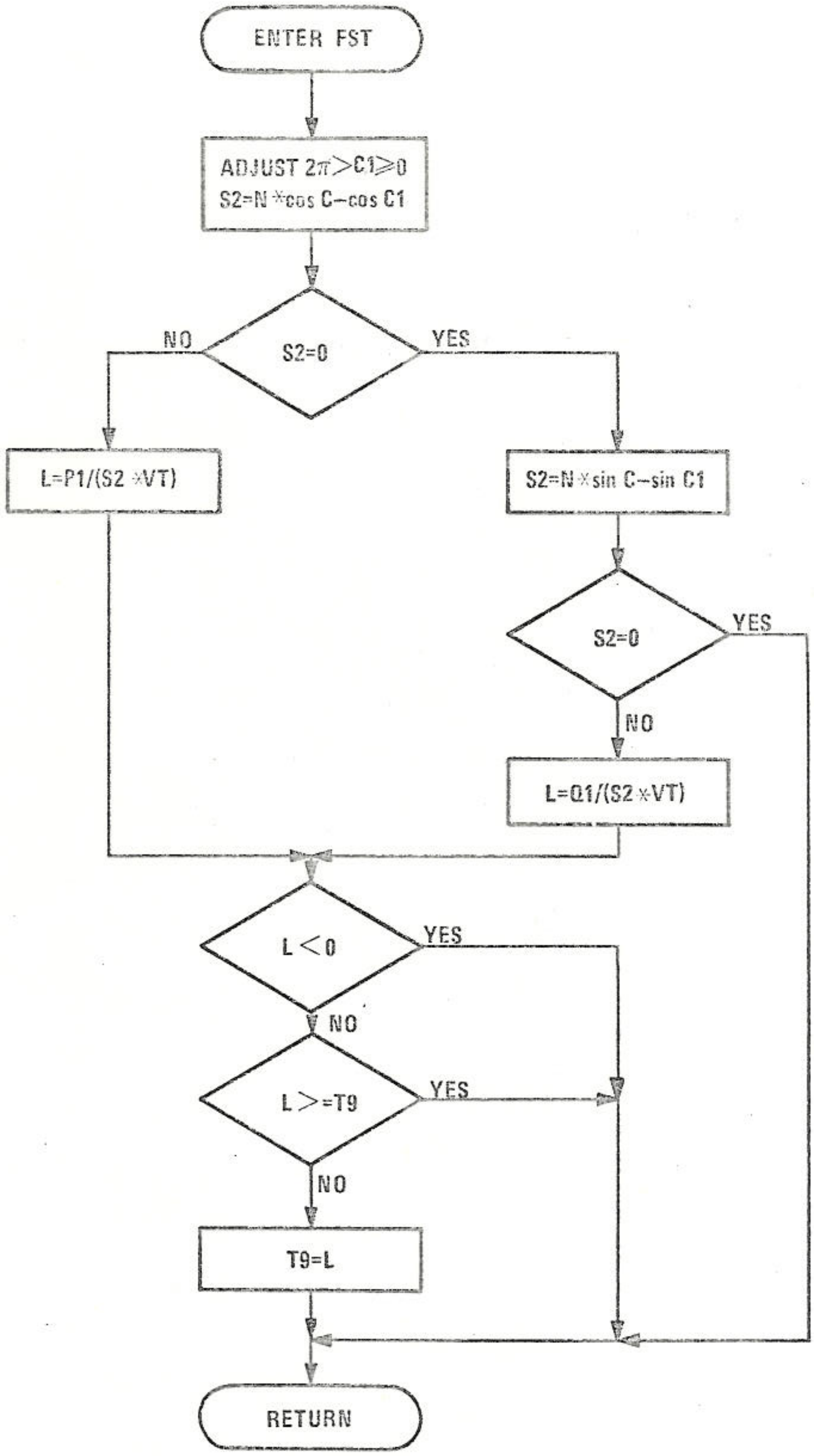


Figure 3.22 Flow chart of Program FST finding the shortest time to the collision point

PROGRAM FTS

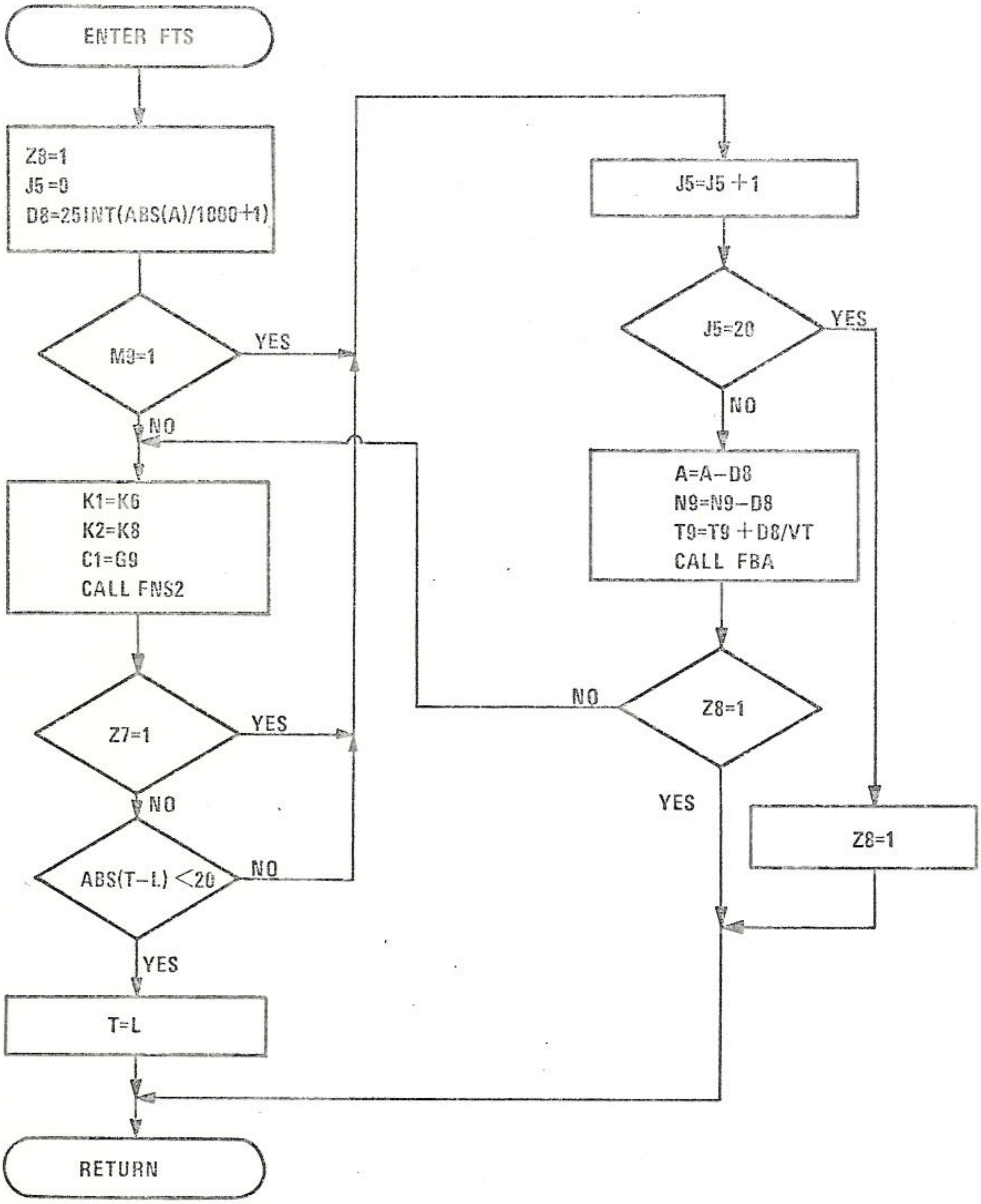


Figure 3.23 Flow chart of Program FTS, finding a new solution, transposed by D_8

PROGRAM FTT

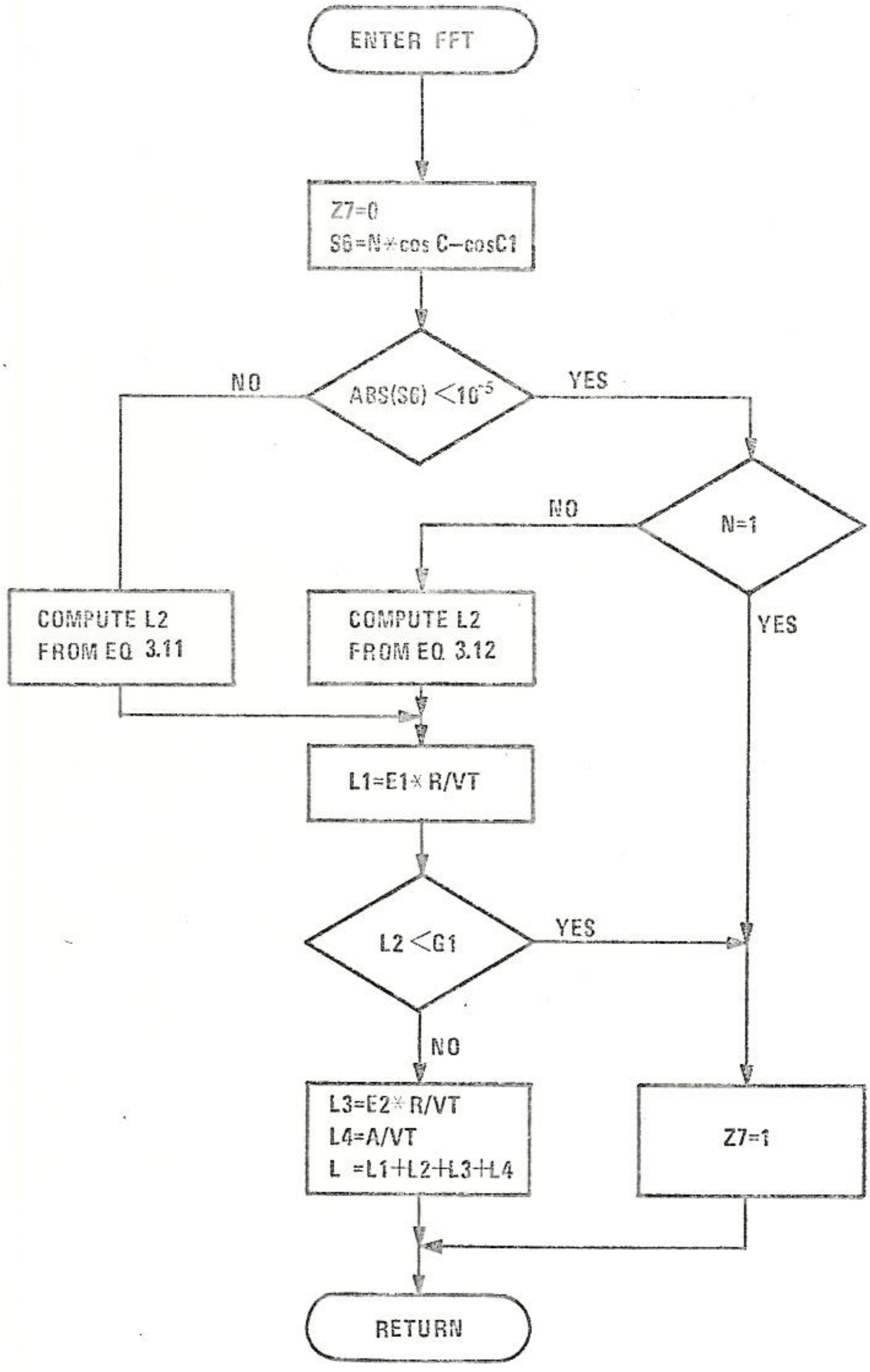


Figure 3.24 Flow chart of Program FTT finding total time to hit the estimate

PROGRAM NEP

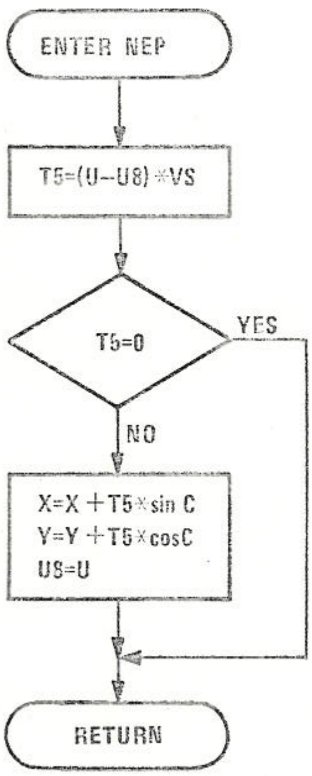


Figure 3.25 Flow chart of Program NEP simulating straight movement of estimate. Last updating is saved in U8, and U is current time.

PROGRAM NTS

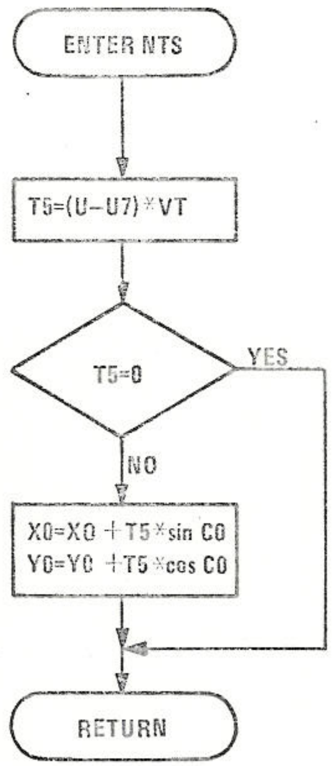


Figure 3.26 Flow chart of Program NTS simulating torpedo dead-reckoning in straight paths, from time U7 to current time U

PROGRAM NTT

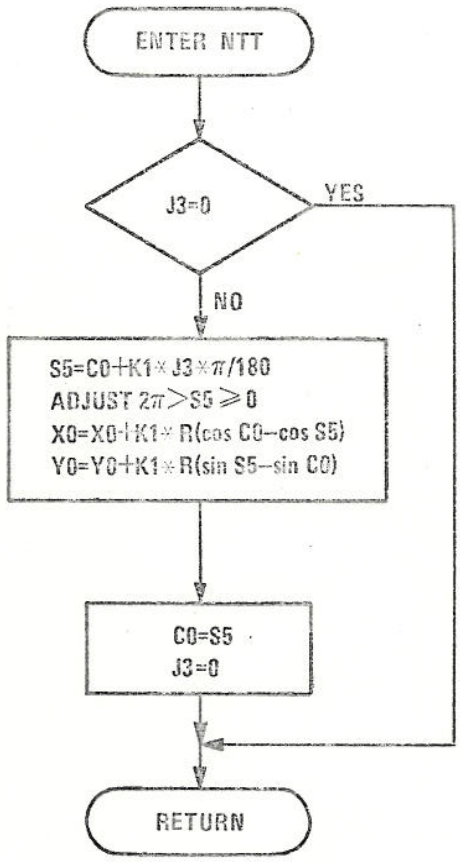


Figure 3.27 Flow chart of Program NTT simulating torpedo dead-reckoning in turns

PROGRAM STR

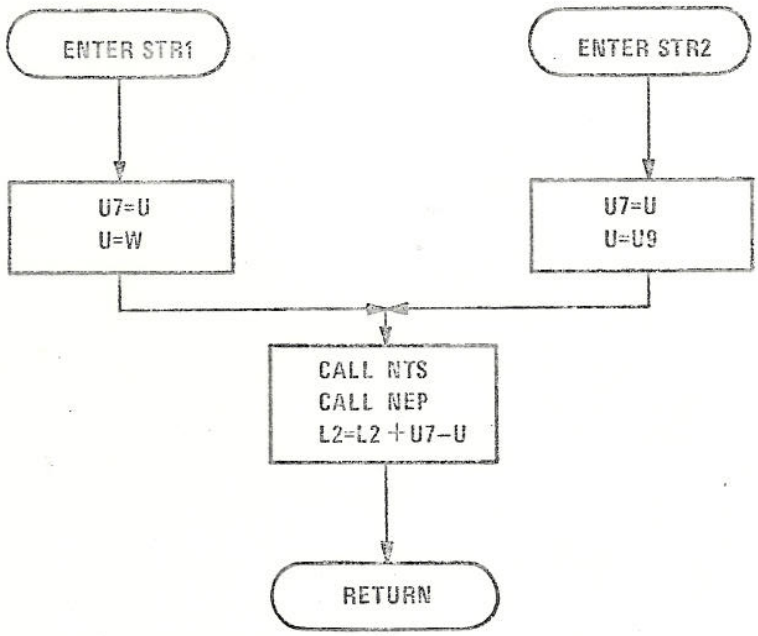


Figure 3.28 Flow chart of Program STR which will update the estimate and the torpedo positions to either time U9 or to next time for computations W

PROGRAM TA

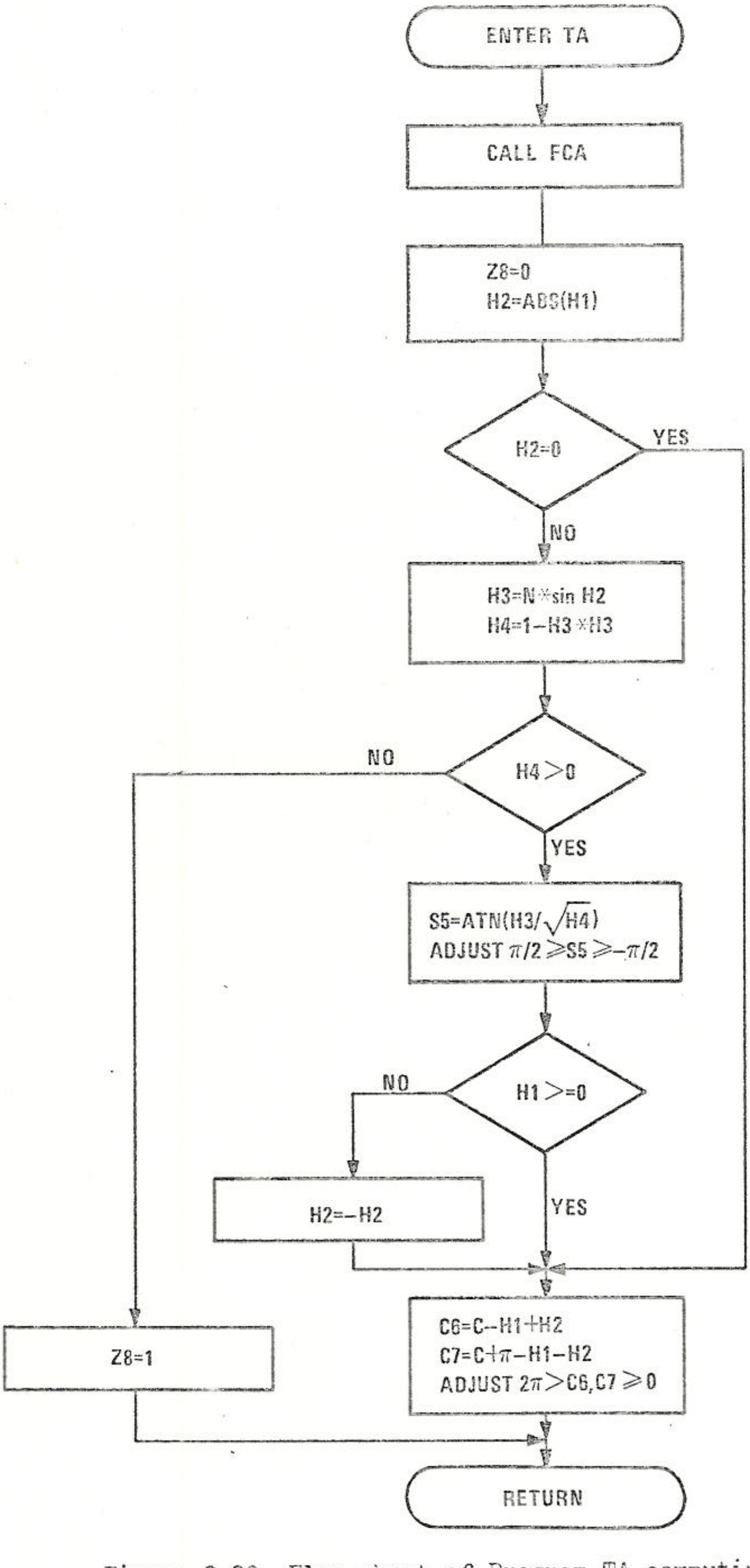


Figure 3.29 Flow chart of Program TA computing the two possible torpedo angles yielding the wanted hit-corridor

PROGRAM TEST

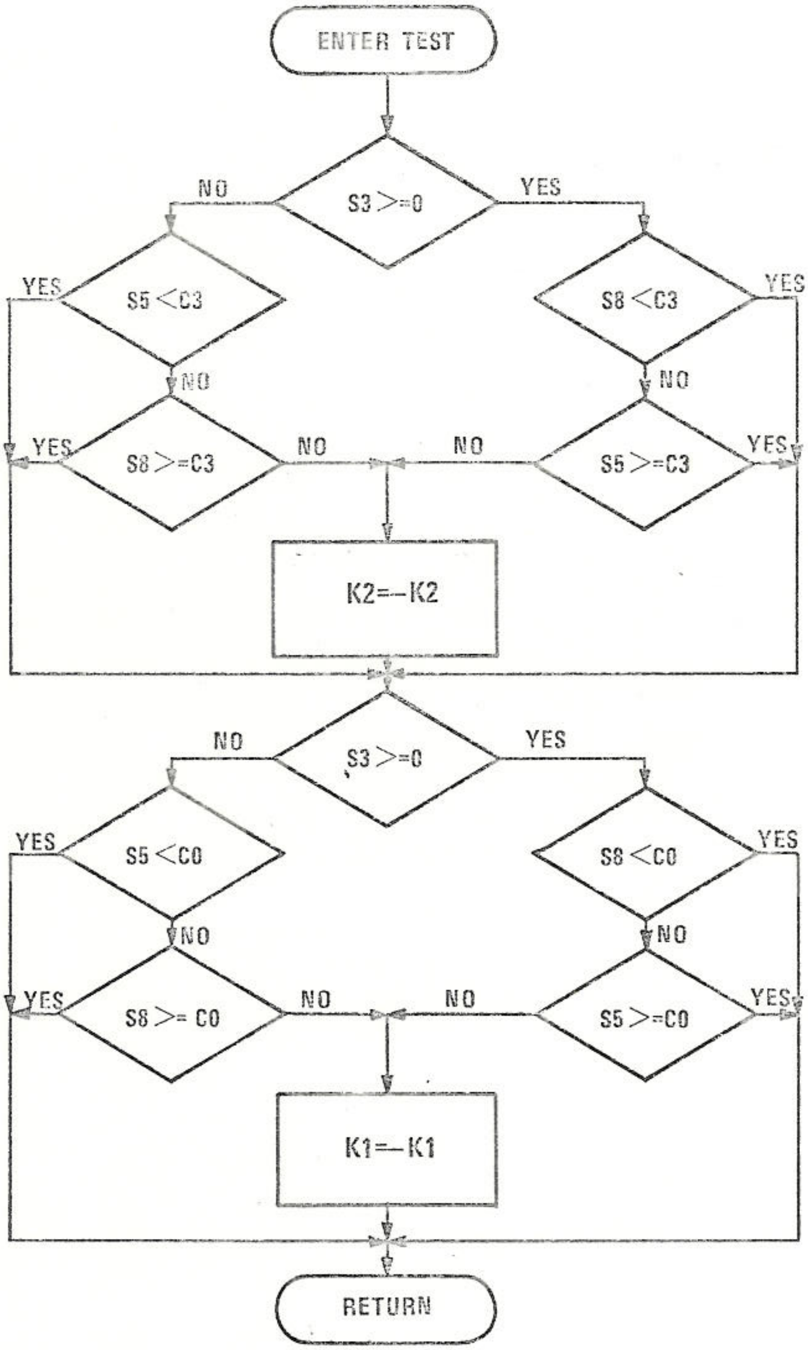


Figure 3.30 Flow chart of Program Test, testing if the iteration has produced angles requiring changing of the turn parameters

PROGRAM TURN

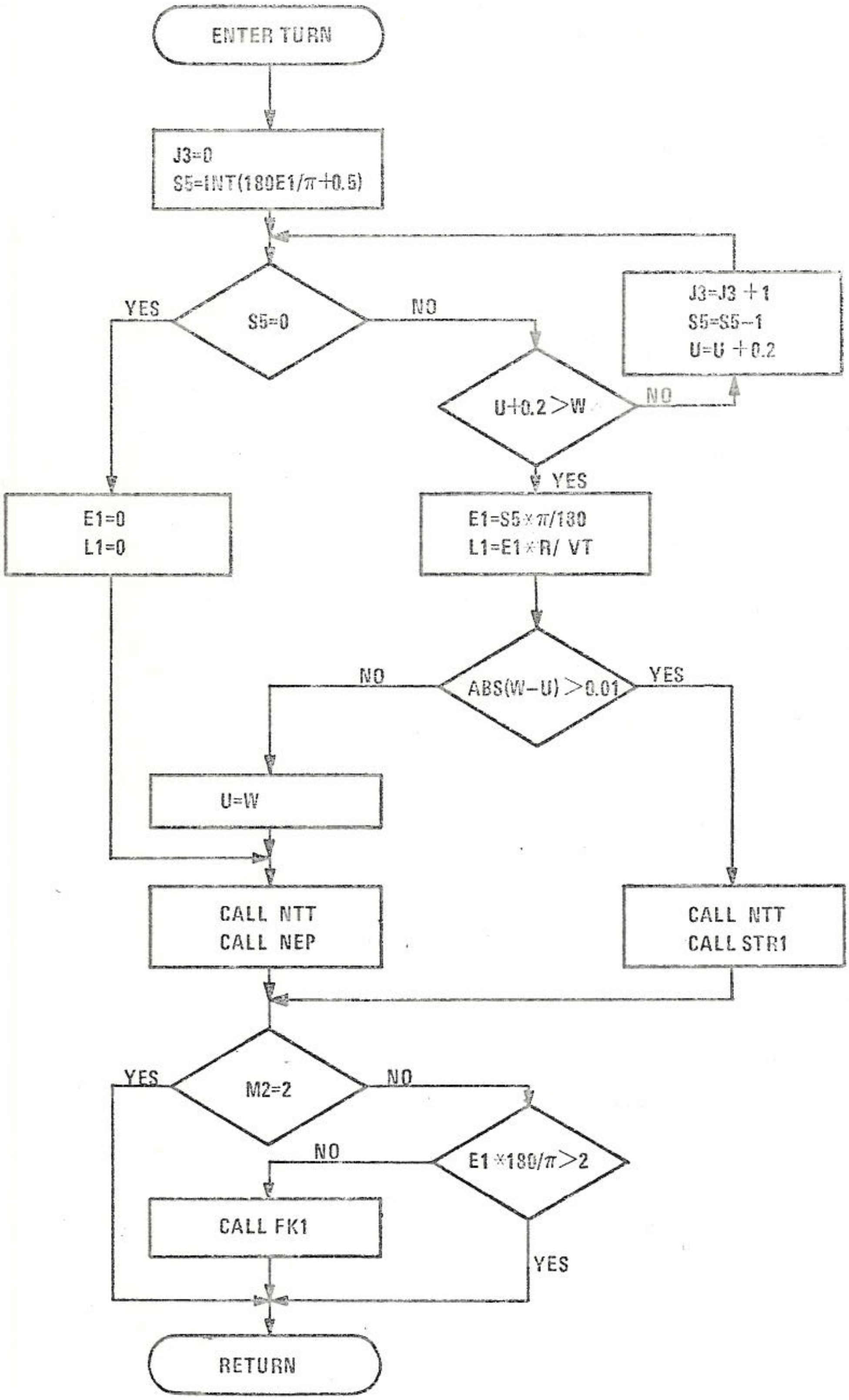


Figure 3.31 Flow chart of Program TURN simulating the turning of the torpedo until the turn angle is zero or until next calculation time

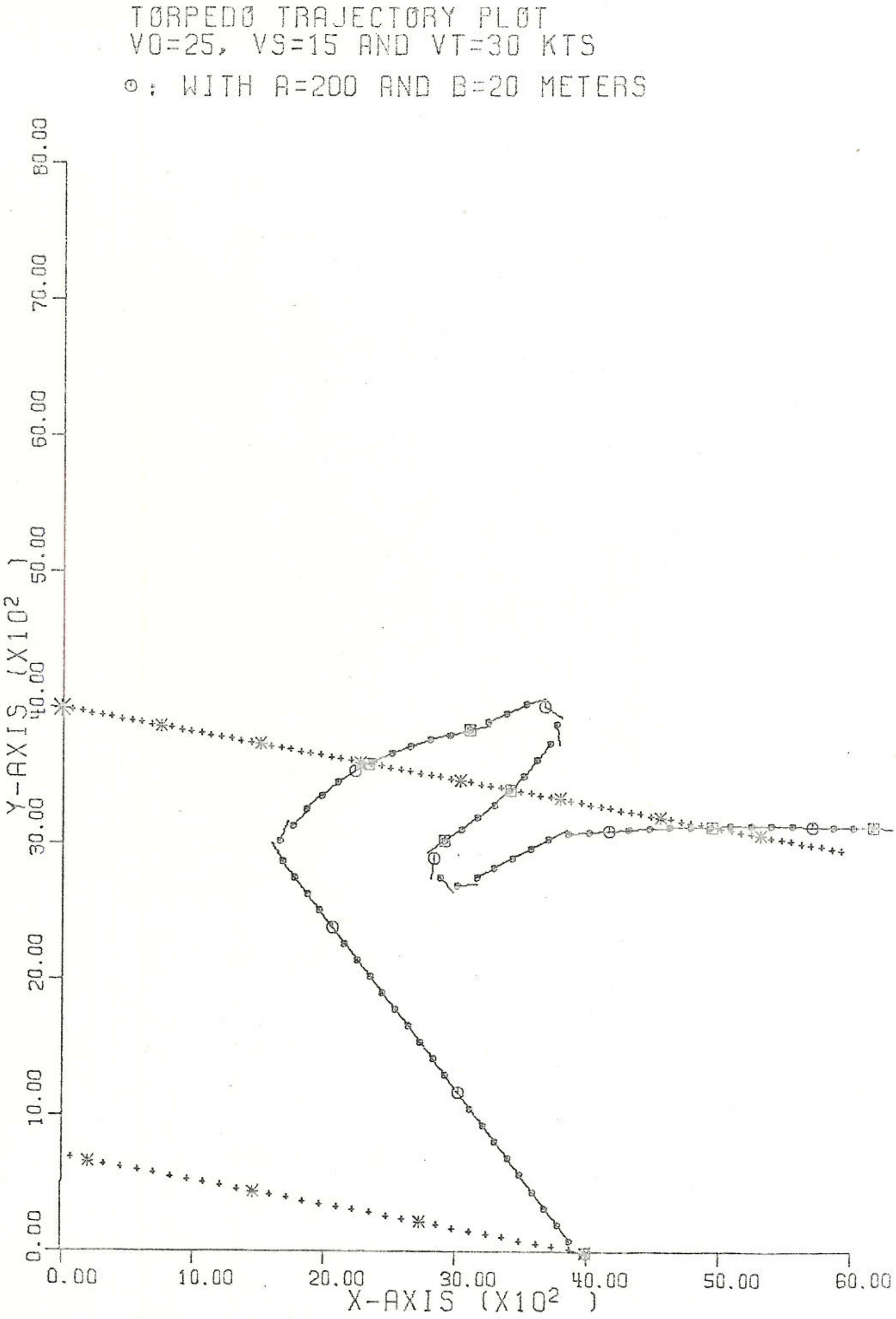


Figure 6.1 Trajectory plot

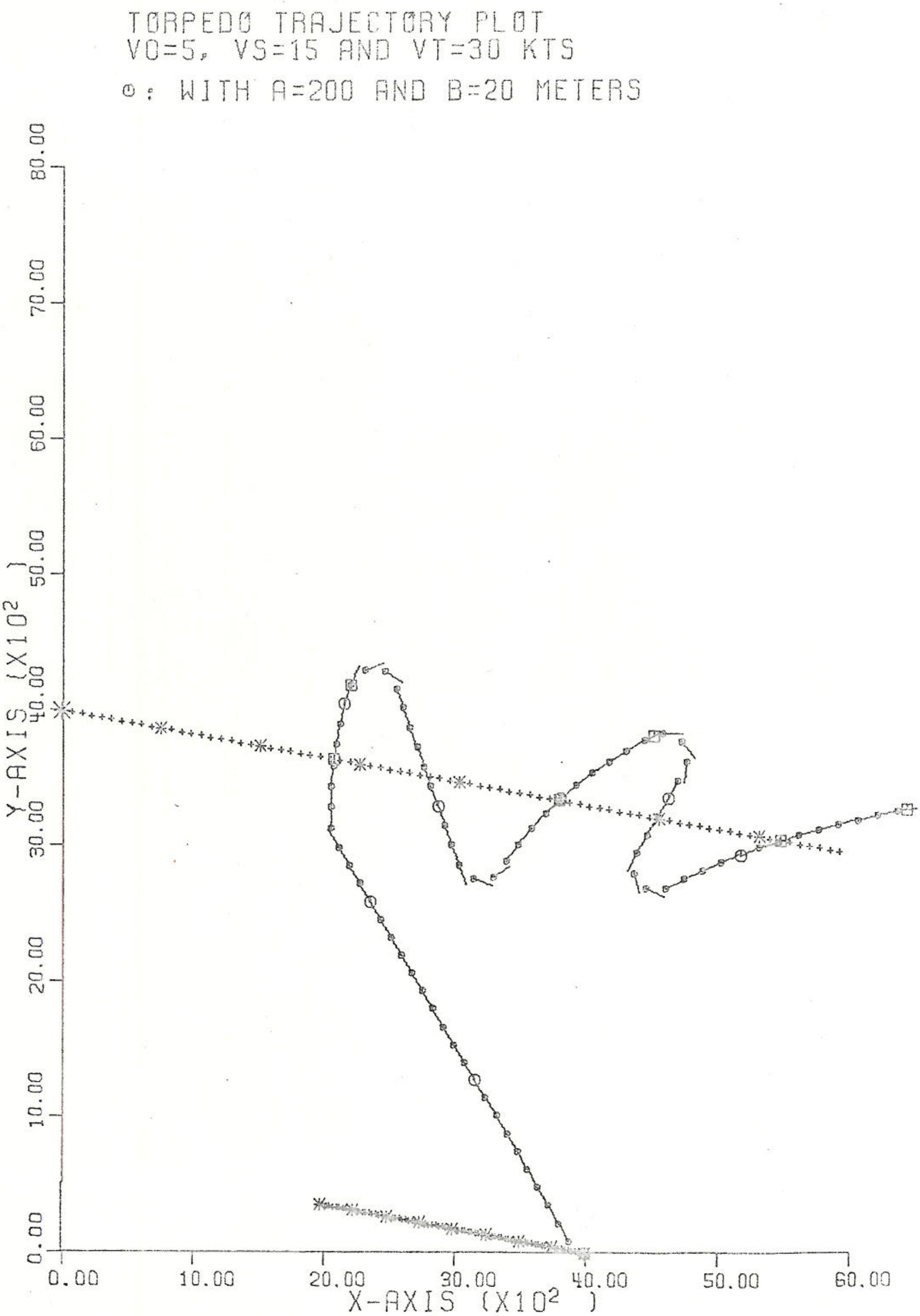


Figure 6.2 Trajectory plot

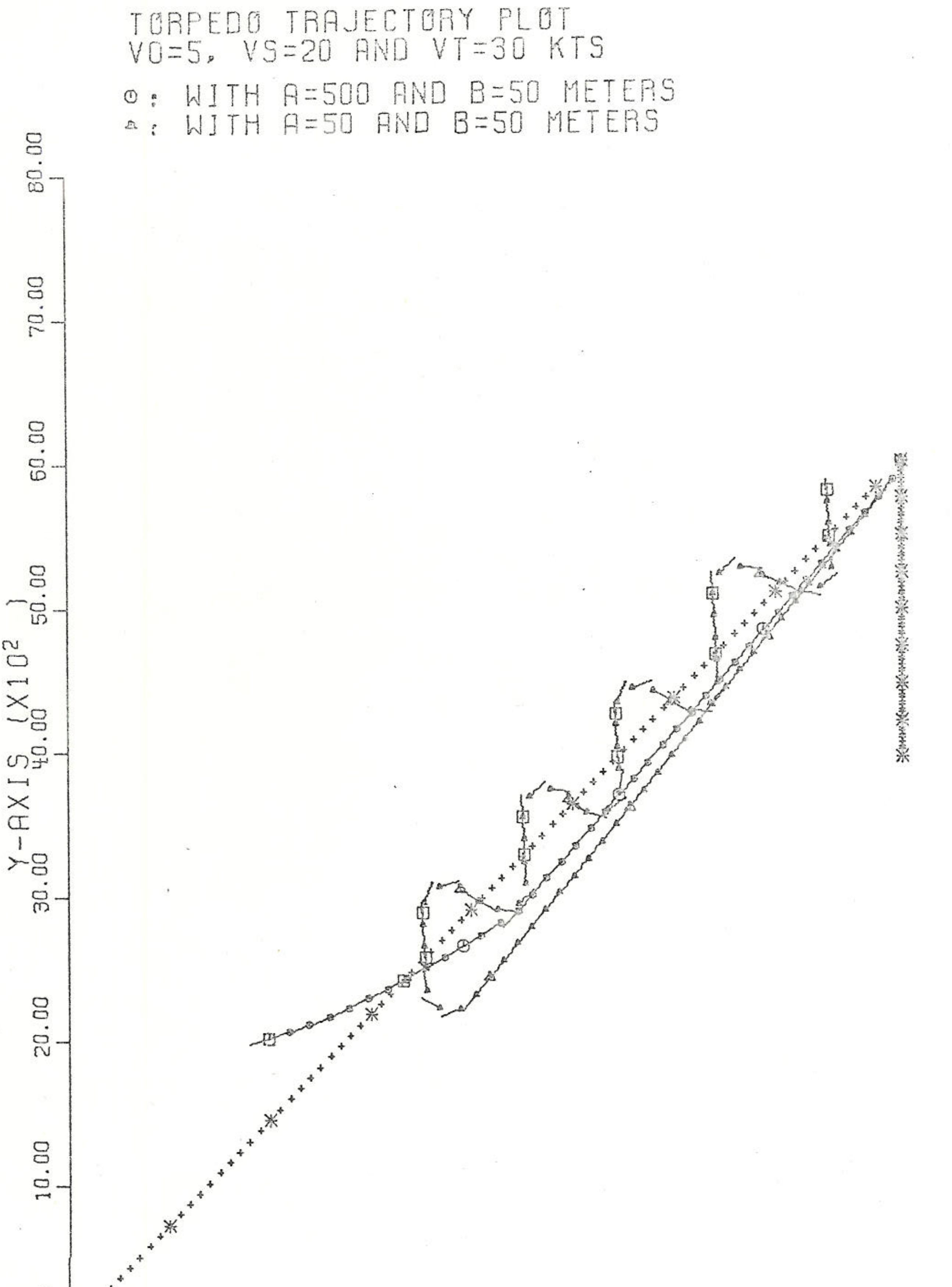


Figure 6.3 Trajectory plot

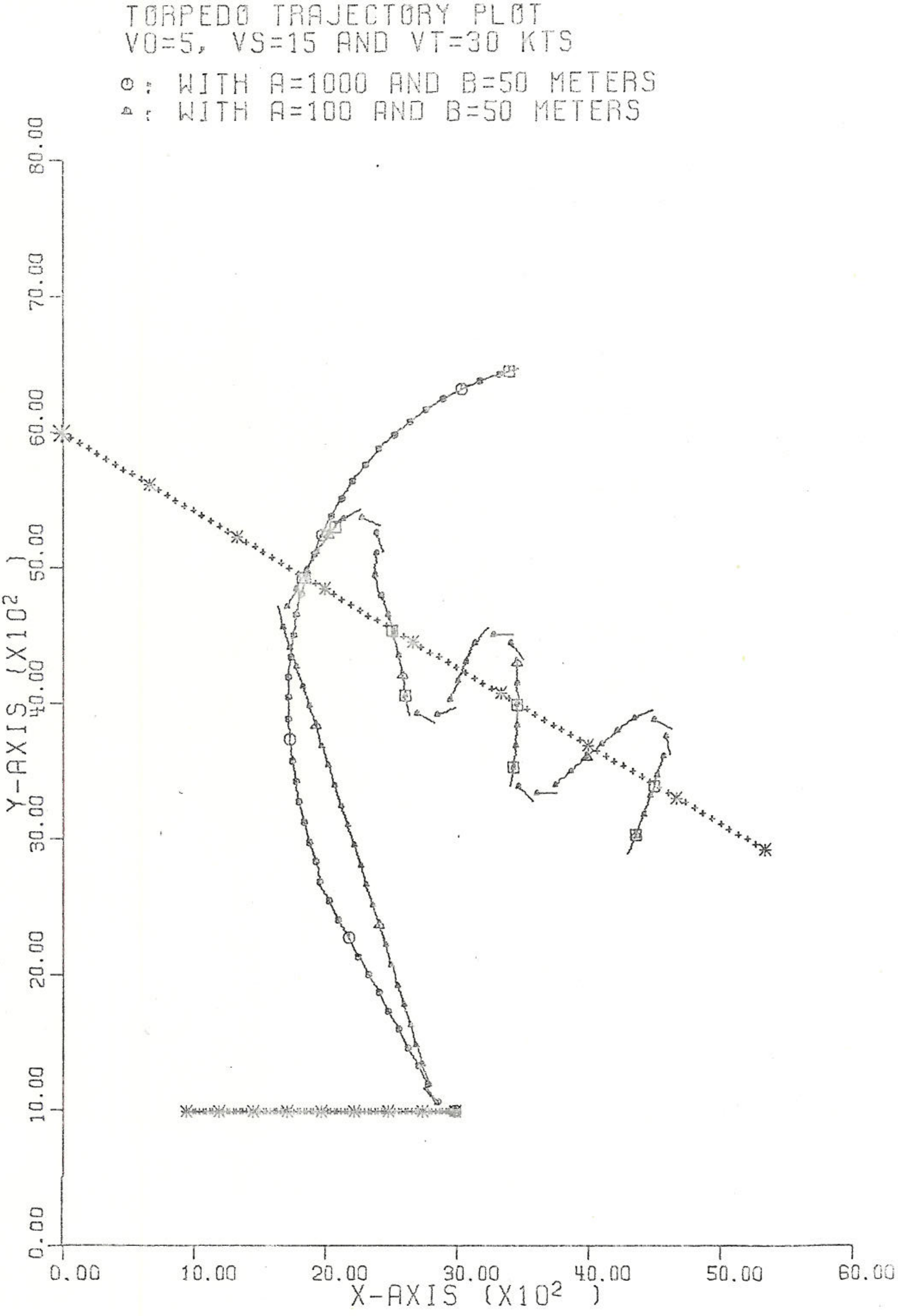


Figure 6.4 Trajectory plot

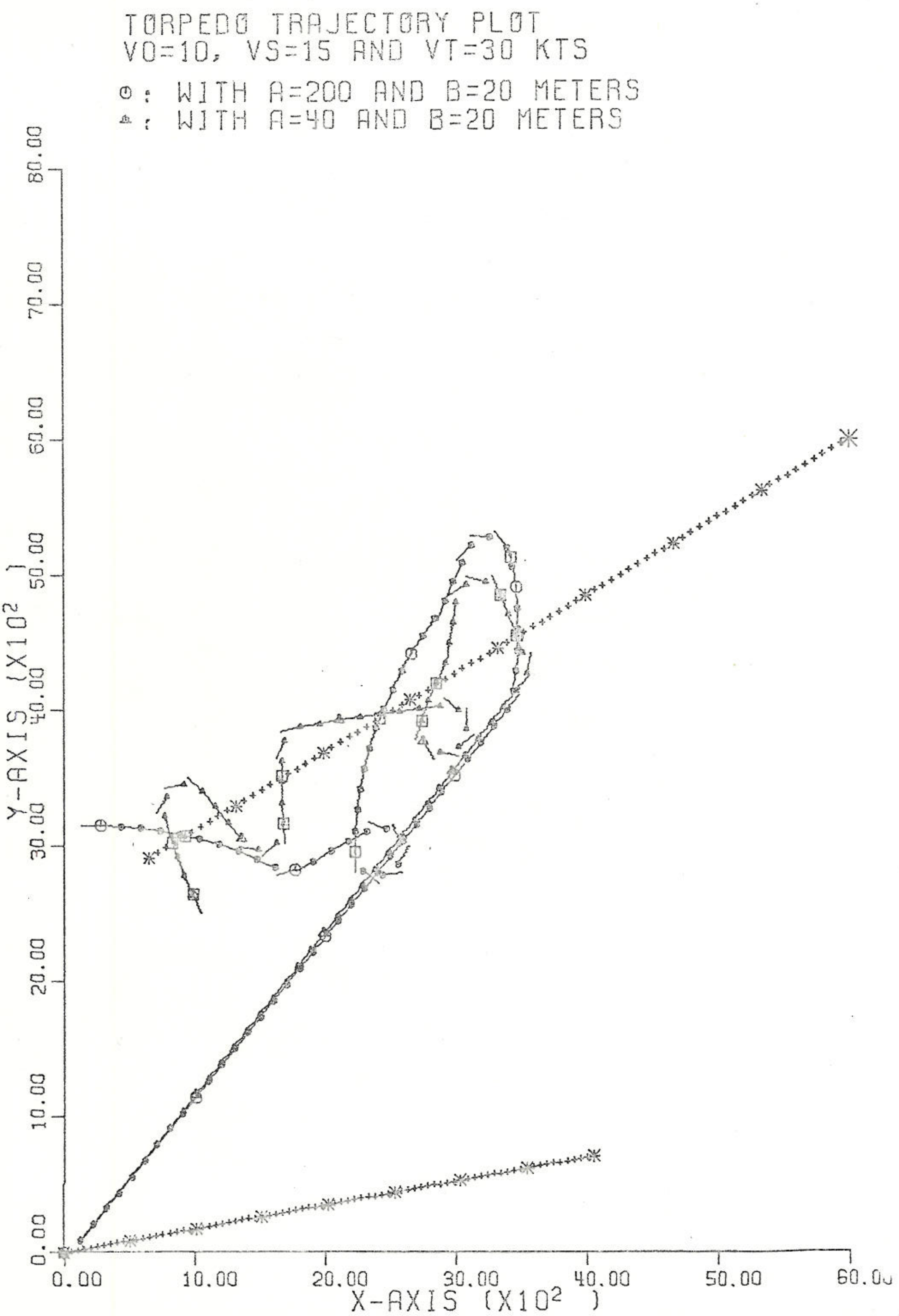


Figure 6.5 Trajectory plot

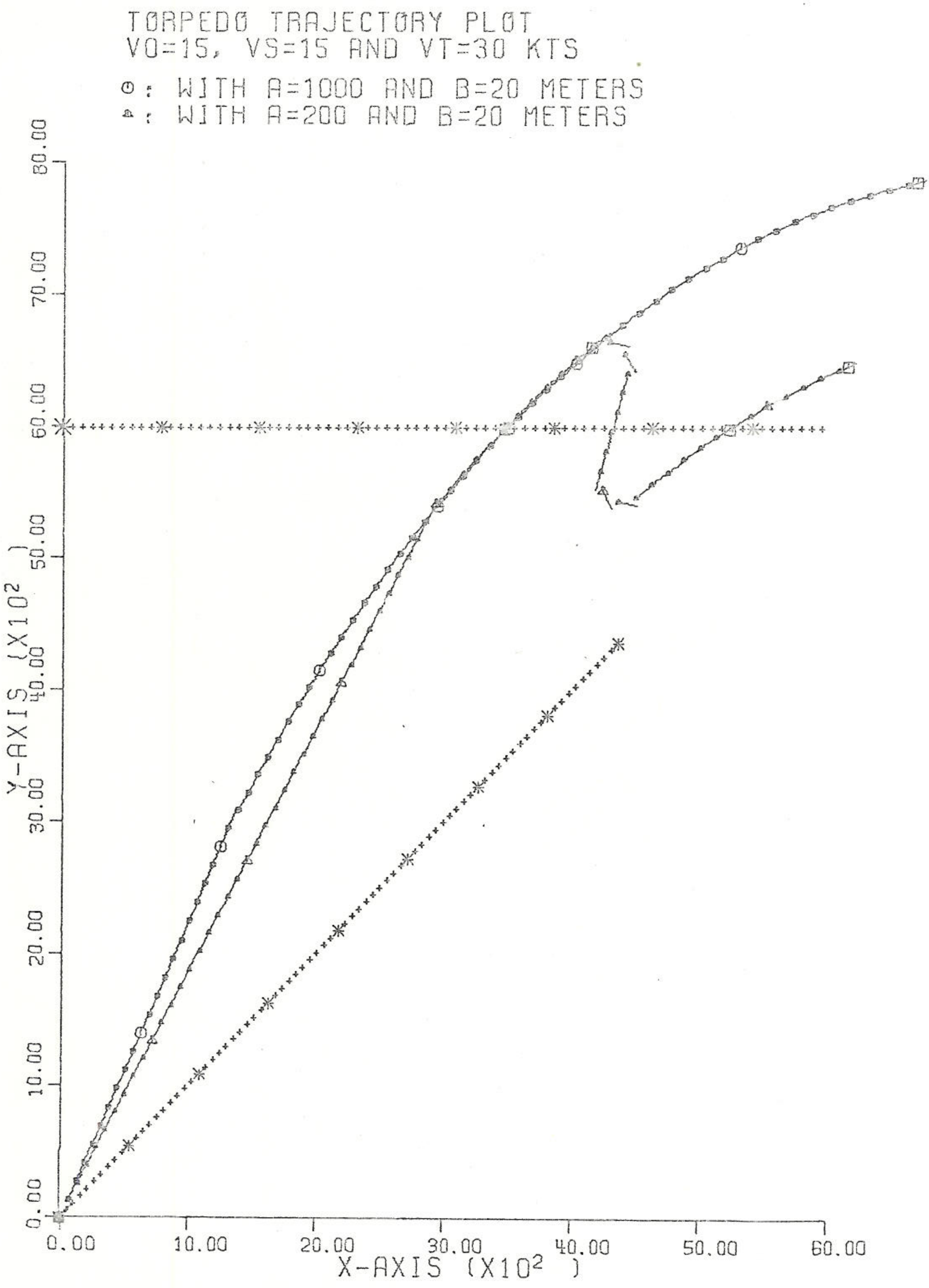


Figure 6.6 Trajectory plot

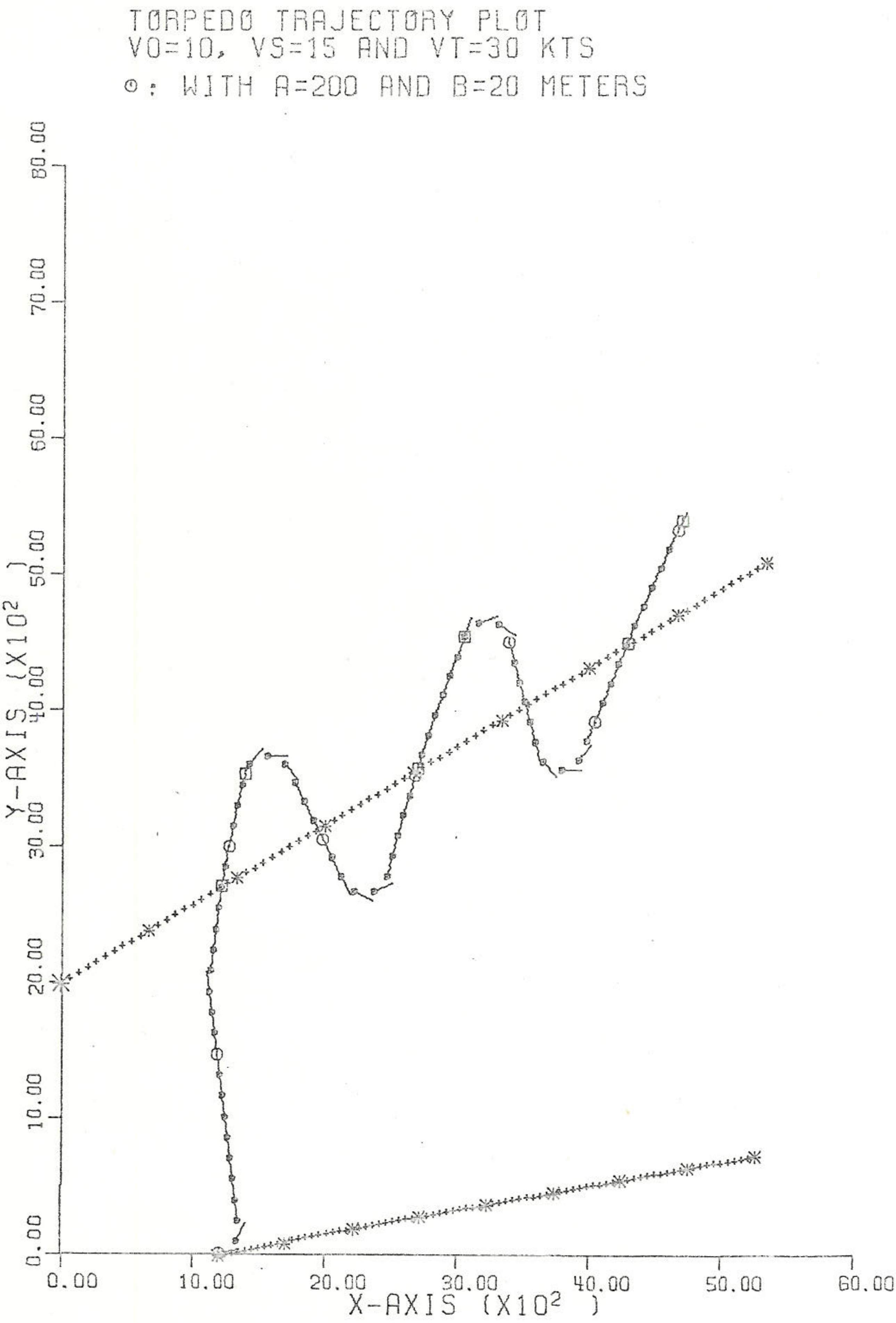


Figure 6.7 Trajectory plot

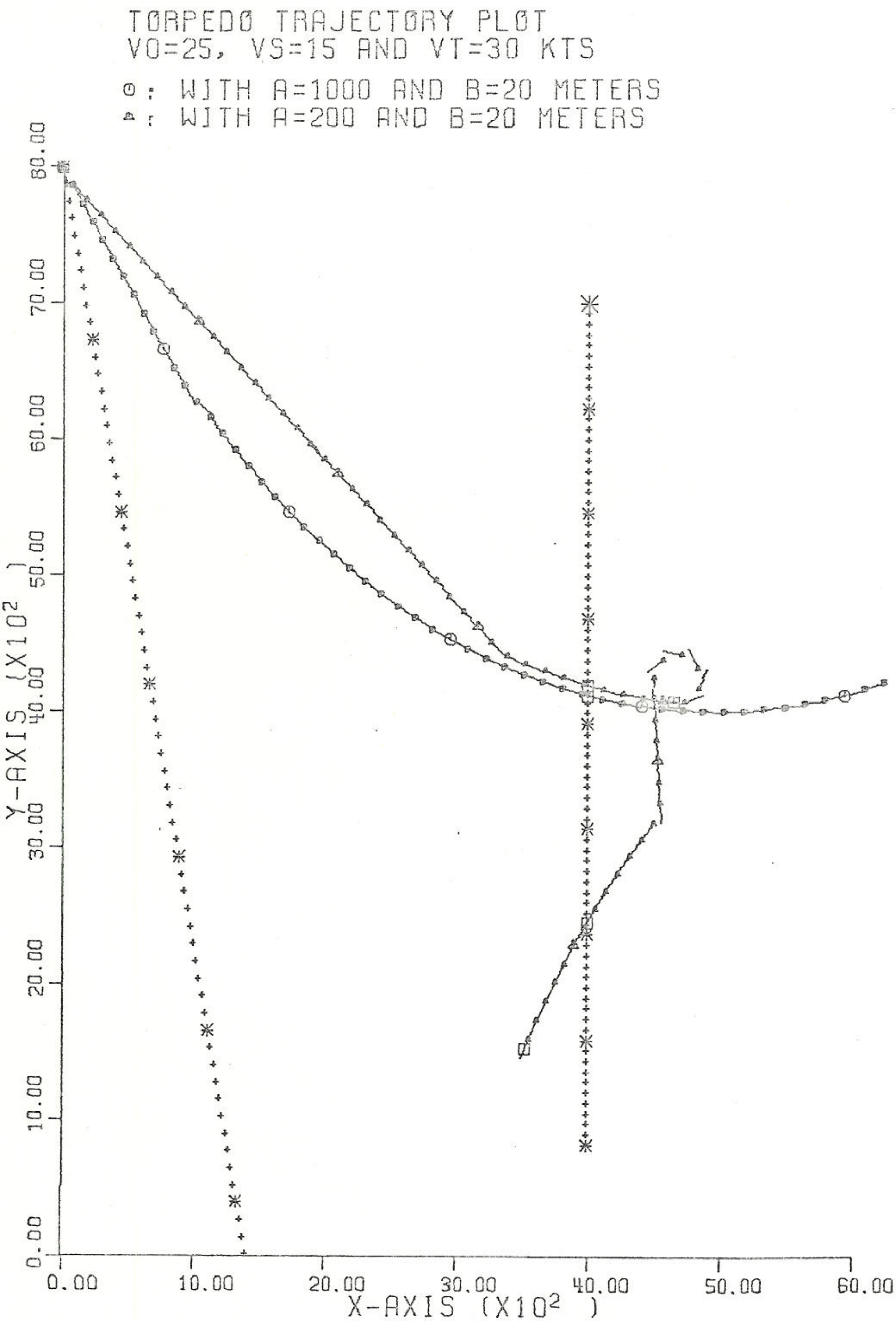


Figure 6.8 Trajectory plot

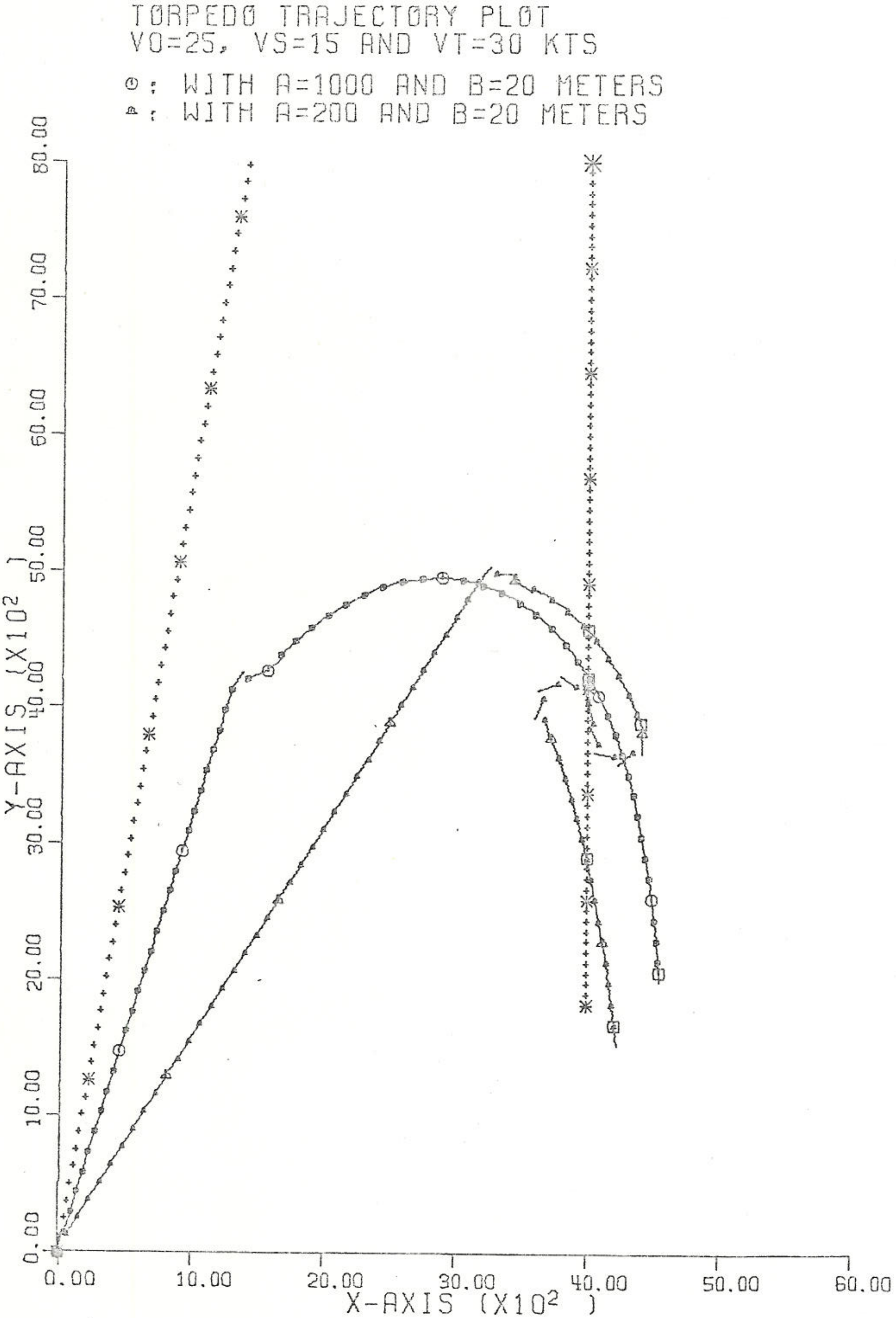


Figure 6.9 Trajectory plot

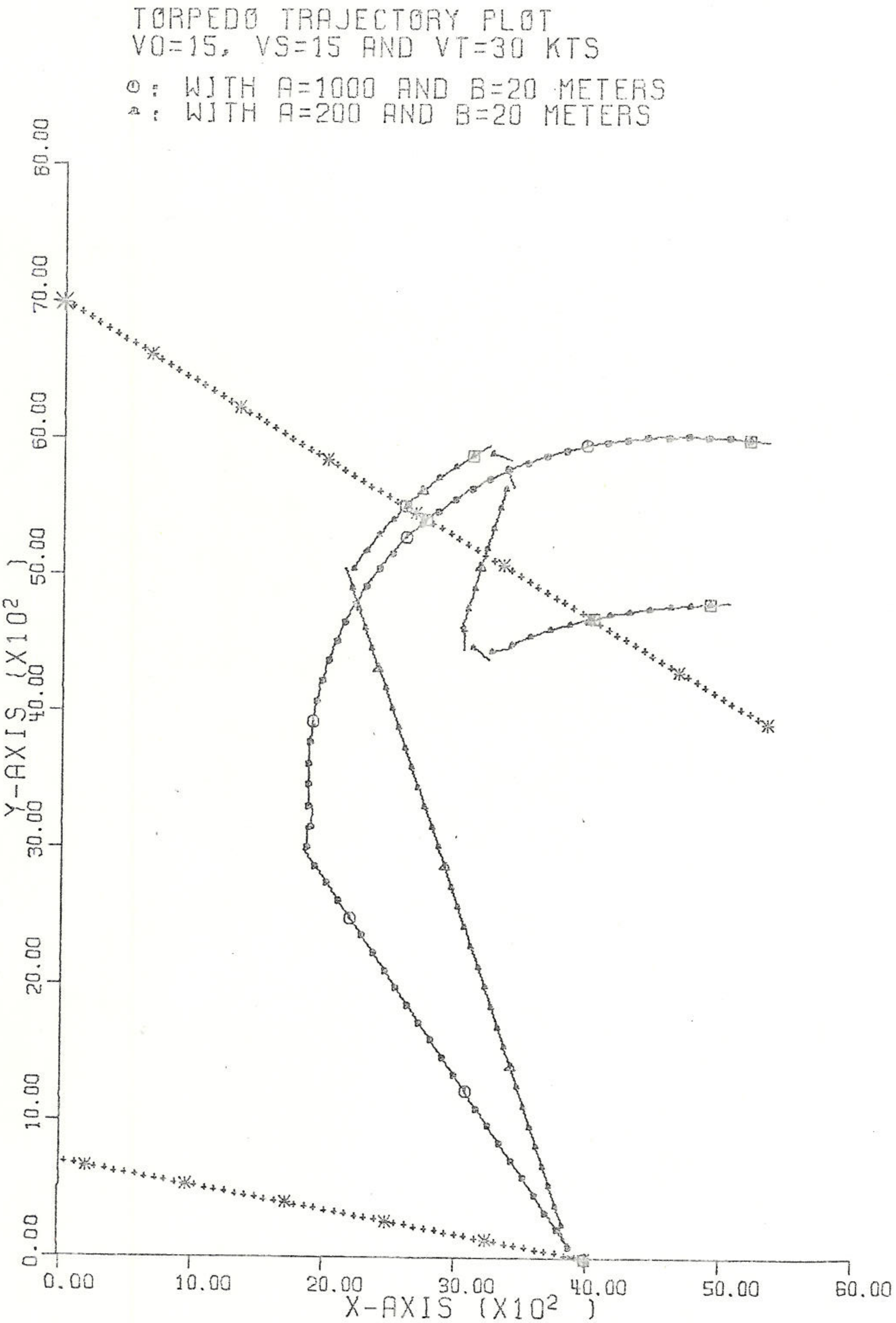


Figure 6.10 Trajectory plot

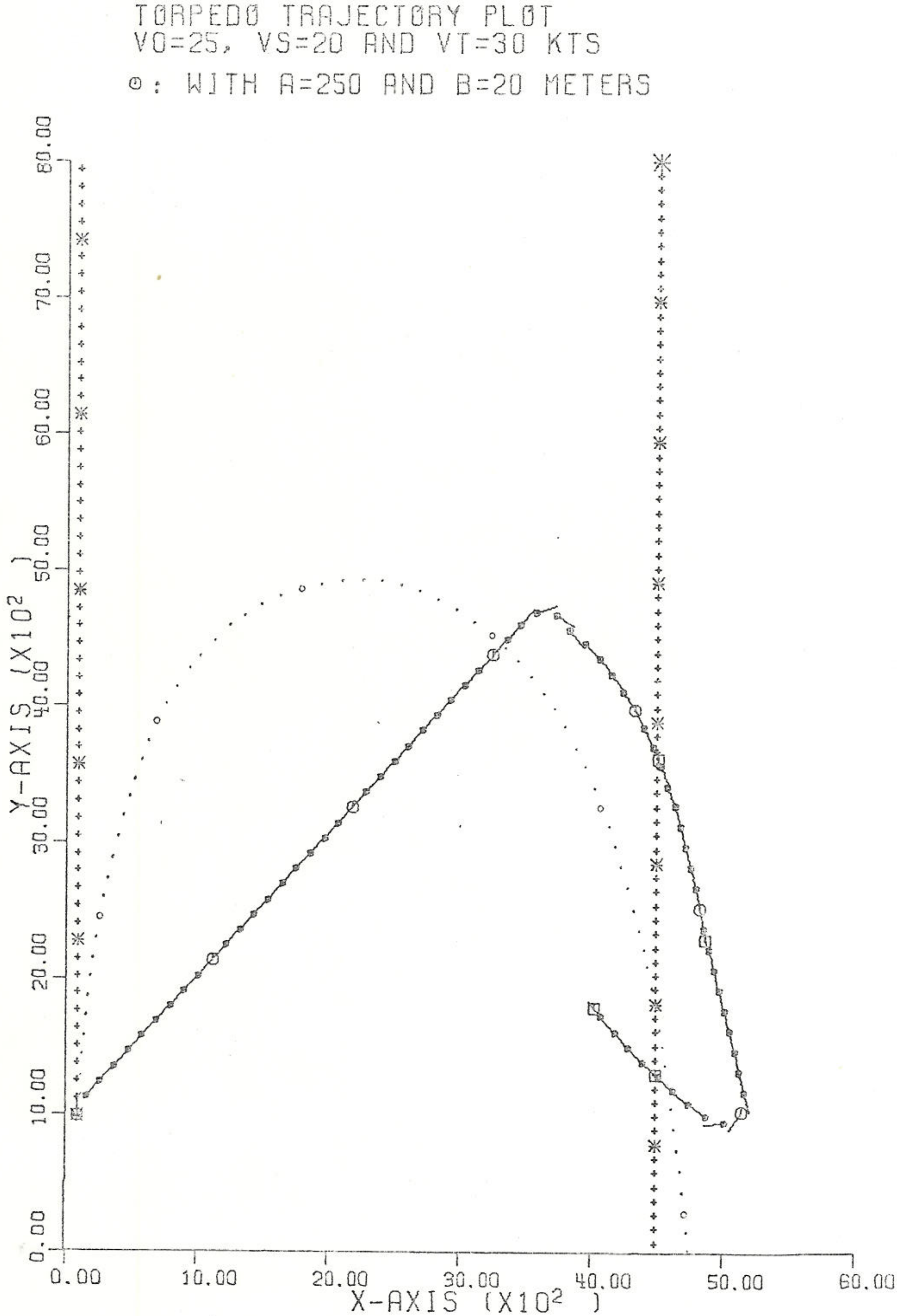


Figure 6.11 Trajectory plot with superimposed line-of-sight trajectory

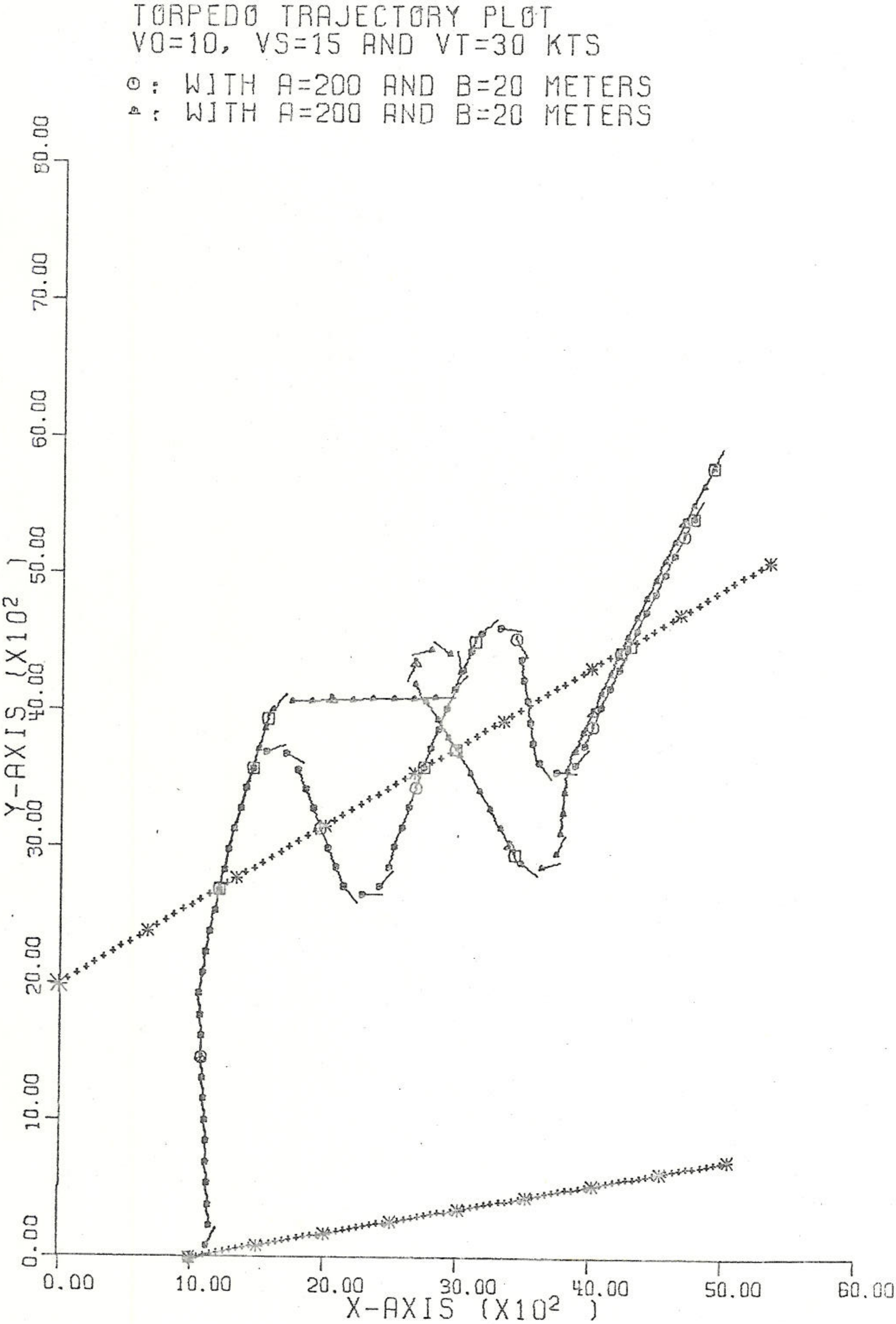


Figure 6.12 Trajectory plot with delayed start of multipass

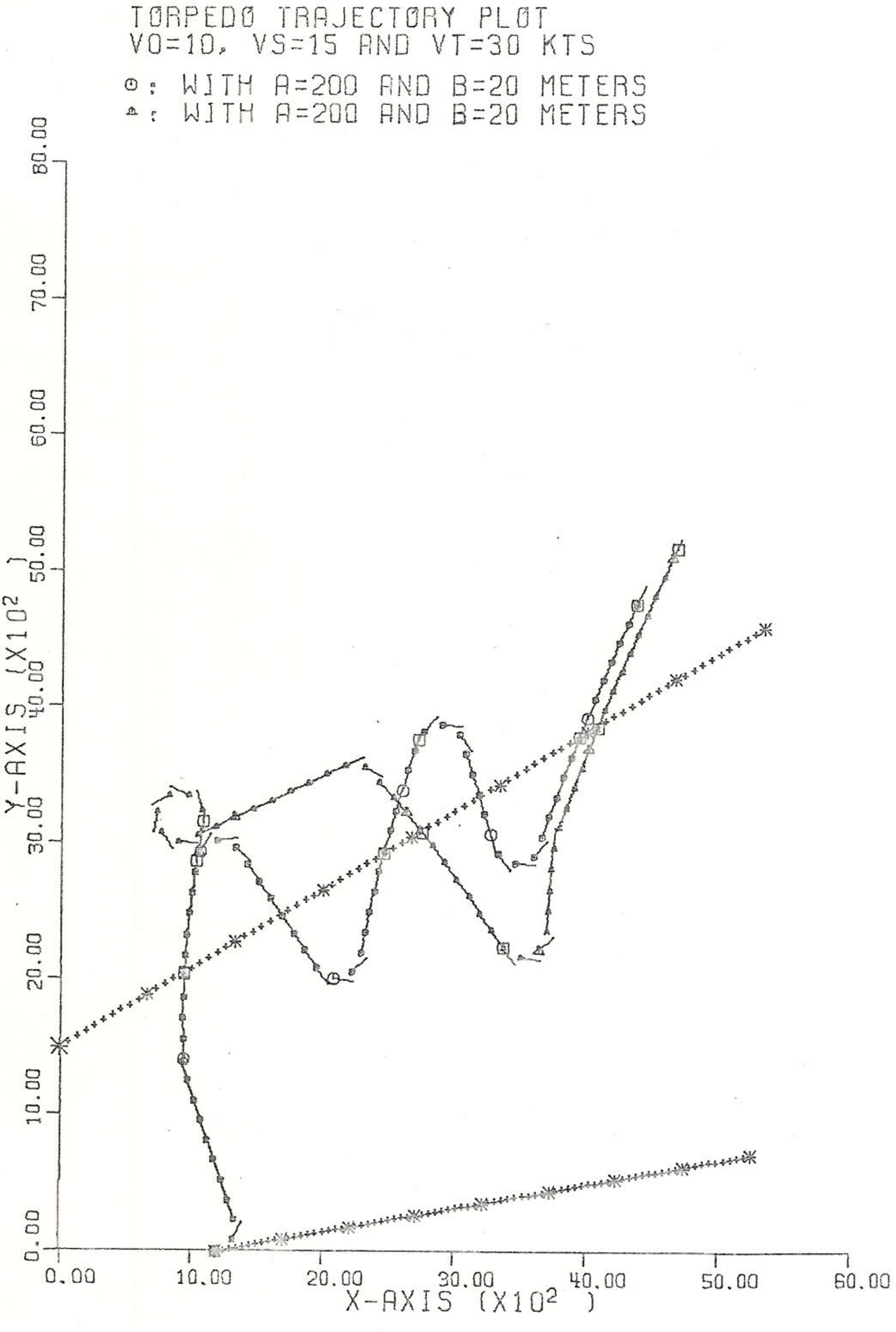


Figure 6.13 Trajectory plot with delayed start of multipass

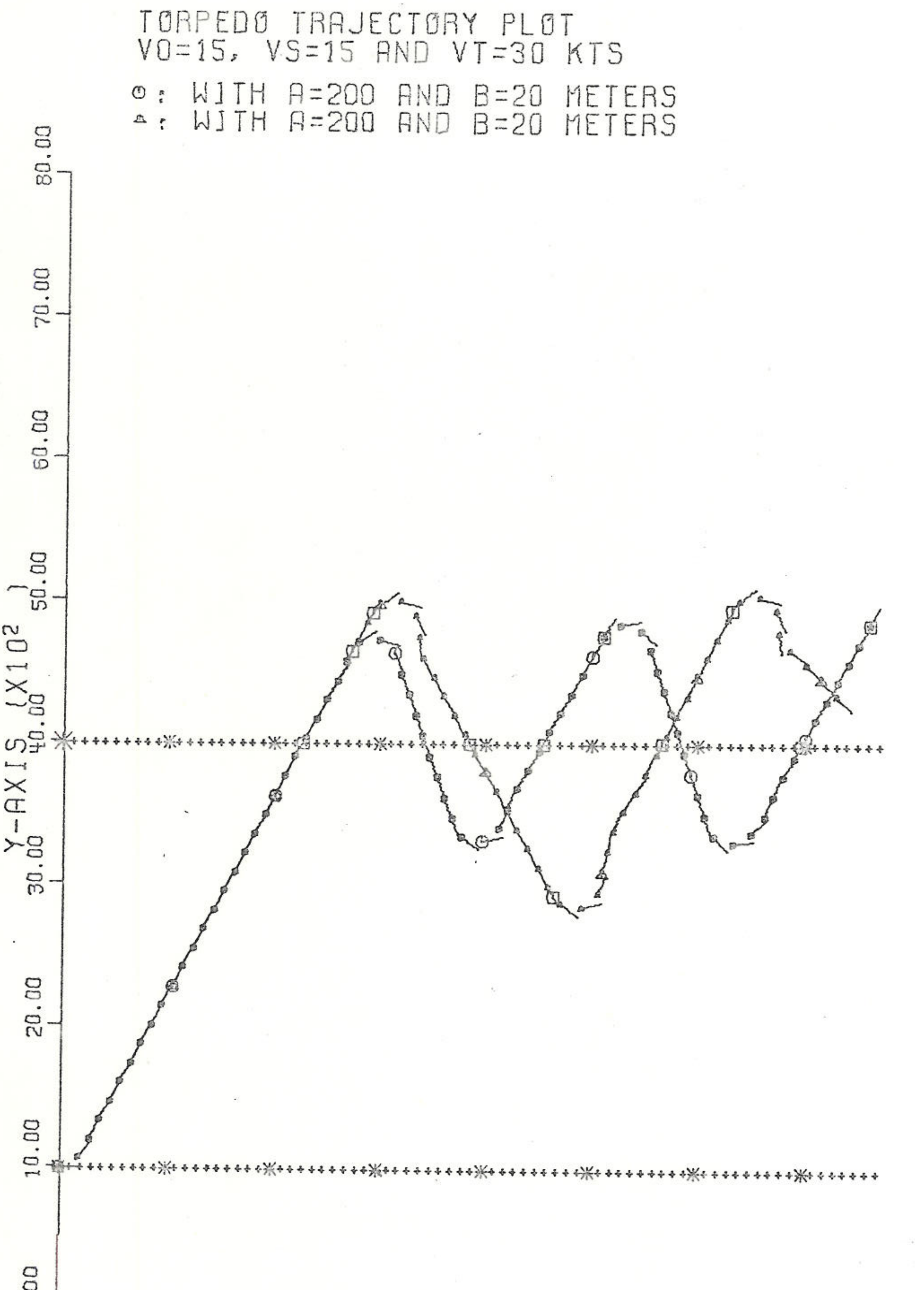


Figure 6.14 Trajectory plot with delayed start of multipass