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A REVIEW OF EMPIRICAL EQUATIONS FOR MISSILE IMPACT EFFECTS ON CONCRETE

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8) ABSTRACT

The literature on empirical equations for predicting penetration, perforation and scabbing is examined, and the relevant equations are given. These formulas are written in a nondimensional form in order to simplify comparison. It is seen that the various equations are quite different, and possible reasons for this are discussed. Finally, we try to make some recommendations as to which equations should be applied.

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CONTENTS

A REVIEW OF EMPIRICAL EQUATIONS FOR MISSILE IMPACT EFFECTS ON CONCRETE

1 INTRODUCTION

In this document, we will be looking at empirical formulas that exist for predicting the penetration into hard materials, especially concrete and rock.

Two observations can be made already at an early stage of this study. The good news is that a large number of experiments with penetration into rock and concrete have been performed, and that empirical formulas for the penetration depth have been constructed on the basis of these tests.

The bad news is that none of the formulas give the same results.

We shall look at the background of the various formulas in order to shed some light on why they are all different. Unfortunately, not all of the original data has been made available to us, but it should still be possible to draw some conclusions.

It turns out that most of the military formulas were constructed on the basis of extensive penetration tests performed in the US during World War II. After the war, there was not much military interest in the field of concrete penetration, and consequently very few advances were made until the seventies.

In the seventies, there was an increased interest from the nuclear industry in obtaining more accurate formulas for penetration into concrete. The old military formulas were inadequate since they were almost exclusively based on experiments with higher velocities and smaller projectiles than what was of interest to the civilian nuclear industry. Missile velocities of interest to the nuclear industry are generally in the range 30–180 *m/s*. As a result, several studies of low velocity penetration were carried out.

Before the seventies, we shall see that no care was taken in assuring that the empirical formulas were dimensionally correct. Consequently, all of the older equations are dimensionally wrong, i.e. the left hand side of the equation has different units than the right hand side. Fortunately, this situation has improved a lot in recent years, so that all modern formulas are nondimensional.

2 SOME EXISTING PENETRATION FORMULAS

We start by looking at penetration into very thick targets, i.e. targets in which the final penetration depth is much less than the target thickness.

Although none of the existing empirical formulas for this situation are identical, at least it turns out that quite a few of them can be written in the following form:

$$\frac{x}{D} = \alpha \frac{Mv^{\gamma_3}}{D^{\gamma_2}\sigma_c^{\gamma_1}} + \beta \tag{2.1}$$

The symbols have the following meaning:

x	Penetration depth
D	Diameter of the projectile
M	Mass of the projectile
v	Impact velocity of the projectile
σ_c	Compressive strength of target material
α, β, γ_i	Various nondimensional constants

There are other empirical equations that can not be written in the form of (2.1), and these will be examined in chapter 4. We start, however, by making some remarks on the dimensionality of equation (2.1).

Notice that the left hand side of (2.1) is nondimensional. The dimension of the right hand side is, however, dependent on the value of the γ_i–constants. It is clear that we can not choose γ_i freely if we want this side to be nondimensional as well. In fact, it can easily be shown that for equation (2.1) to be nondimensional, the only possible values for the γ_i–constants are the following:

$$\gamma_1 = 1 \quad , \quad \gamma_2 = 3 \quad , \quad \gamma_3 = 2 \tag{2.2}$$

If the variables above are the only ones included in the description of the problem, dimensional analysis gives us the following expression for x/D

$$\frac{x}{D} = f\left(\frac{Mv^2}{D^3\sigma_c}\right) \tag{2.3}$$

It is clear that equation (2.1) with values of γ_i as in (2.2) is a special case of (2.3). We shall soon see that in many of the empirical formulas, the γ_i –constants have other values than those given in (2.2). As a consequence, these equations are dimensionally wrong, which means that the left hand side of the equation does not have the same dimension as the right hand side. If such a formula is written on the form of (2.1), the left hand side will be nondimensional, but the right hand side will not.

We shall discuss the significance of the formulas not being nondimensional in chapter 7, where it will be shown that an equation need not be useless, even though it has the wrong dimensionality. When doing a real calculation, we just forget about the dimensional problems, insert the numerical value of each parameter, and pretend that everything ends up fine with the correct dimension. We have to be very careful when doing so, as the final result will then depend on which unit–system we have used, i.e. whether variables have been measured in meters, yards, inches, or whatever.

Most of the equations were originally published in units such as inches, pounds and similar. For convenience, all formulas have been converted to SI–units.

Formula	α	β	γ_1	γ_2	γ_3
Dim. analysis			1	3	2
Beth	$3.6 \cdot 10^{-4}N$	0.5N	0.5	2.78	1.5
ACE	$3.5 \cdot 10^{-4}N$	0.5	0.5	2.785	1.5
NDRC mod. ($x/D > 2$)	$3.8 \cdot 10^{-5}N$	1.0	0.5	2.9	1.8
Bernard (concrete)	$0.254 \cdot \rho^{-1/2}$	0	0.5	3.0	1.0

ACE = Army Corps of Engineers, NDRC = National Defence Research Commitee.

The parameter N is a socalled "nosefactor", which describes the shape of the projectile. Most of the equations calculate N in different ways, but for simplicity we have chosen to be a bit sloppy and use the same symbol for all of them. This parameter will typically be somewhere in the range [0.7,1.2]. For more details about this factor, one should consult appendix A.

The variable ρ is the density of the concrete. This quantity could have been included in the general formula (2.1) by saying that $x/D \propto \rho^{\gamma_4}$, but this seemed unnecessary since the density only appears in the Bernard formula, and γ_4 would consequently have been zero in all the other equations.

3 COMPARISON OF THE FORMULAS

There are some similarities between the equations since they all have the same general form of (2.1), but the γ_i–constants vary from equation to equation. As has already been noted, there also exist other formulas (that will be examined later) which do not fit the general form of (2.1), so clearly one has a lot of equations to choose from.

The question is how important these differences really are? To find out, we look at the γ_i–constants one by one:

3.1 Diameter dependence

The parameter that there appears to be least disagreement on is obviously γ_2 which describes how the final penetration depth depends on the diameter of the projectile. According to the formulas it should be somewhere in the range [2.785 , 3.0]. The small difference between the highest and lowest estimate is a good sign, and reinforces our belief in the validity of these equations.

3.2 Material dependence

The signs are also quite good when it comes to γ_1. This is a very important parameter because it gives the relationship between penetration depth and the material properties of the target, which is our primary interest. Interestingly, all the empirical formulas say $\gamma_1 = 0.5$. The only problem is the dimensional analysis which predicts $\gamma_1 = 1$.

Without the dimensional analysis, one might have been tempted to conclude that $\gamma_1 = 0.5$, but this would have been premature. We shall soon see that there are other formulas which give a different dependence on the target properties.

3.3 Velocity dependence

For γ_3, which describes the dependence on impact velocity, there is also a substantial disagreement between the equations. They seem to indicate that γ_3 lies somewhere in the range [1.0 , 2.0]. It is a good sign that γ_3 has a value in this range because this agrees with results that can be obtained from a simple quasianalytical approach. However, it is still not satisfactory to have such an inaccurate estimate of γ_3 , even though it apparently is in the right range.

An interesting point is that all formulas predict a nice "round" γ_3 of either 1.0, 1.5 or 2.0, with the notable exception of the NDRC–modified equation. Nobody seems to have found a value of γ_3 somewhere inbetween, say $\gamma_3 = 1.68$ or similar.

3.4 Evaluation

Unfortunately, we have been unable to get our hands on the "raw data" behind the Beth, NDRC and ACE–equation. Consequently we have had no way of properly evaluating these formulas. However, it is known that all of these equations are based on data from tests that were done during World War II (21). In total, more than 900 projectiles with diameters of 1.1 *cm*, 1.3 *cm*, 3.7 *cm*, 7.5 *cm* and 15.5 *cm* were fired against lighly reinforced concrete plates with thicknesses in the range [22,195] cm. The concrete strength varied from $\sigma_c = 27\,MPa$ to $\sigma_c = 44\,MPa$. New types of concrete have been developed after these tests took place, and it is not certain that new High Performance Concrete (HPC), with much larger compressive strength, can be correctly described by any of the formulas.

The ACE formula (6) is said to be valid in the velocity range [200,1000] m/s, so it is assumed that the the experiments were carried out in this range. The same goes for the NDRC formula (25). As mentioned above, the concrete was lightly reinforced, but since the amount of reinforcement did not seem to have a significant impact on penetration depth, no reinforcement parameter has been included in the equations.

While the Beth and ACE equations are purely empirical, it is known that the NDRC–formula is based on a physical model of the impact process (21). In other words, it is a kind of semi–analytical formula. In this model, it is assumed that the force F on the projectile during penetration is given by:

$$F = \left(\frac{v}{D}\right)^{0.2} \frac{x}{2D} \ , \ x < 2D \tag{3.1}$$

$$F = \left(\frac{v}{D}\right)^{0.2} \quad , \quad x > 2D \tag{3.2}$$

However, it has been shown (both analytically and experimentally) that this model does not give a correct description of the penetration process (10),(13),(18).

In any case, this assumption leads to the following equations:

$$\frac{x}{D} = \sqrt{4KNMD^{-2.8}v^{1.8}} \quad , \quad \frac{x}{D} < 2 \tag{3.3}$$

$$\frac{x}{D} = KNMD^{-2.8}v^{1.8} + 1.0 \quad , \quad \frac{x}{D} > 2 \tag{3.4}$$

It is clear that the velocity dependence is determined theoretically, while the empirical data is used to estimate K, which contains the material dependence. However, if the underlying assumption is incorrect, there might be something suspect with the whole approach.

In 1946, the work on the NDRC–equation was abandoned without the factor K being completely defined. However, in 1966, work was continued by Kennedy (21), who suggested that $K \propto \sigma^{-0.5}$. He arrived at this result by using a curve fitting procedure on Beth's experimental data for larger missile diameters. His final result is therefore known as the modified NDRC equation.

In 1980, Degen (10) reviewed the assumptions behind the modified NDRC–formula, and concluded that they were invalid. He introduced a more realistic force assumption, but surprisingly this lead to exactly the same equation. So even though the NDRC force assumption was wrong, this did not seem to have any severe consequences.

Although concrete data were used in the development of the Bernard equation (4), this one is not really supposed to be valid as a concrete penetration formula. According to Bernard himself, the ACE formula fits the experimental data better than his own equation, which he instead recommends be used for rock penetration. Since the equation is constructed on the basis of only 11 tests, and the target parameters for those tests are not entirely certain, the accuracy of the prediction is said to be about $\pm$ 20%.

The Bernard equation was derived from experiments inside the following range:

$$\frac{x}{D} > 3 \tag{3.5}$$

$$M \in [5.9, 1066.0]\ kg \tag{3.6}$$

$$D \in [7.62, 25.88]\ cm \tag{3.7}$$

$$v \in [300, 800]\ m/s \tag{3.8}$$

Extrapolation outside this range is not recommended by Bernard.

4 OTHER MAJOR EMPIRICAL FORMULAS

As already mentioned there are several other empirical formulas which can not be put into the form of (2.1), and in this section we will describe the most important ones. Once again, for convenience, they are all written in SI–units.

4.1 Modified Petry formula

Probably the oldest available formula is the Petry equation (17), which was first developed in 1910. It is given by:

$$\frac{x}{D} = 0.06K\frac{M}{D^3}\log\left(1 + \frac{v^2}{20000}\right) \tag{4.1}$$

The coeffisient K was first assumed to have the following values depending on the amount of reinforcement:

$K = 0.00799$ for unreinforced concrete

$K = 0.00426$ for "normally reinforced concrete"

$K = 0.00284$ for "heavily reinforced concrete"

This equation is called the modified Petry I. We see that the concrete strength is not included in the equation, which is obviously a much too crude approximation. This was realised by Amirikian (2), who later found a connection between K and σ_c. Unfortunately, he did not write down an analytical expression, but instead found a graphic relation. This formula is called the modified Petry II. For details, one should consult his original report (2).

4.2 Bergman formula

Bergman's formula (3) from 1949 is the one most commonly used by the Norwegian Defence. It is actually based on the same data as the Beth, ACE and NDRC formula, but a different statistical analysis has been carried out, yielding a new equation.

Bergman's trick was to split the formula into two equations, depending on the impact velocity, instead of trying to write one equation in a closed form. For low velocities, x is just proportional to the velocity:

$$\frac{x}{D} = 3.5\frac{v}{v_1} \quad , \quad \frac{x}{D} < 3.5 \tag{4.2}$$

where v_1 is the impact velocity that gives $\frac{x}{D} = 3.5$. This velocity must be found from the following formula, which is valid for larger velocities:

$$\frac{x}{D} = 4.86 G v^{f(z)} \ , \ G = 7.57 \cdot 10^{-5} \frac{NM}{D^{2.8} \sigma^{0.5}} \ , \tag{4.3}$$

$$f(z) = 1.44 + \frac{0.18}{z} + 0.005z \ , \ z = G v^{1.73}$$

If the velocity dependence is disregarded, it can be shown that everything reduces to Beth's equation.

Bergman's equation is said to be valid in the following range:

$$v < 1200 \, m/s \tag{4.4}$$

$$M \in [0.02, 1000] \, kg \tag{4.5}$$

$$\sigma_c \in [30, 50] \, MPa \tag{4.6}$$

$$D \in [0.1, 0.4] \, m \tag{4.7}$$

4.3 Kar formula

In 1978, using regression analysis, Kar (20) revised the NDRC–formula to account for the type of missile material. He ended up with a very simple result, namely:

$$x = x_{NDRC} \left(\frac{E}{E_m} \right)^{1.25} \ , \ \frac{x}{D} < 2 \tag{4.8}$$

$$x - D = (x_{NDRC} - D) \left(\frac{E}{E_m} \right)^{1.25} \ , \ \frac{x}{D} > 2 \tag{4.9}$$

where E and E_m is the modulus of elasticity for the projectile material and mild steel, respectively. Kar is, however, cautious in recommending the use of this formula, because the new factor is only approximate, and should therefore only be used until sufficient test data is available to make a more accurate equation.

4.4 Haldar and Miller formula

In 1982, Haldar and Miller (16) reviewed the NDRC–equation. However, they were mostly interested in the impact of flying objects against nuclear power plants. Typical missiles have rather low impact velocities, which means that the penetration depth is usually lower than two diameters. Using low velocity data from Sliter (26), they constructed an empirical formula which fit the data better than the NDRC–equation for $x/D < 2$.

Their equation is split into three separate formulas:

$$\frac{x}{D} = -0.02725 + 0.22024I \quad , \quad 0.3 < I < 2.5 \tag{4.10}$$

$$\frac{x}{D} = -0.592 + 0.446I \quad , \quad 2.5 < I < 3 \tag{4.11}$$

$$\frac{x}{D} = 0.53886 + 0.06892I \quad , \quad 3 < I < 21 \tag{4.12}$$

where I is called the "damage potential" and is defined by:

$$I = 0.372N\frac{Mv^2}{D^3\sigma_c} \tag{4.13}$$

The nosefactor N is the same as used in the NDRC–formula.

An obvious advantage of these equations is that they are dimensionally correct. Haldar and Miller also compared their formula with data from other sources than Sliter, and found it to be more precise than the NDRC–equation. From a military point of view, there is a problem with this equation, as it is only valid when the penetration depth is less than twice the diameter of the projectile. This is usually not sufficient for military applications.

4.5 Adeli and Amin formula

In 1984, using data from Sliter (26), two new empirical equations were constructed by Adeli and Amin (1). They examined the data using a least squares technique, and found that the best fit was given by either a quadratic or a cubic polynomial:

$$\frac{x}{D} = 0.0416 + 0.1698I - 0.0045I^2 \tag{4.14}$$

$$\frac{x}{D} = 0.0123 + 0.196I - 0.008I^2 + 0.0001I^3 \tag{4.15}$$

where I is defined by:

$$I = N\frac{Mv^2}{D^3\sigma_c} \tag{4.16}$$

The formulas are valid in the following range:

$$v \in [27, 311]\ m/s \tag{4.17}$$

$$M \in [0.1, 343]\, kg \tag{4.18}$$

$$I \in [0.3, 21] \tag{4.19}$$

$$D < 12 \ , \ \frac{x}{D} < 2 \tag{4.20}$$

4.6 Hughes formula

The Hughes formula (18) from 1984 is also dimensionally correct. It is given by:

$$\frac{x}{D} = \frac{0.19N}{1 + 12.3 \ln\left(1 + 0.03 \frac{Mv^2}{\sigma_t D^3}\right)} \frac{Mv^2}{\sigma_t D^3} \tag{4.21}$$

Another interesting feature of this equation is that the σ_t parameter denotes the concrete *tensile* strength, whereas the relevant parameter in all the other equations has been the *compressive* strength. For concrete, the compressive strength is roughly ten times the tensile strength.

Just like the NDRC–equation, the Hughes formula is semi–analytical, i.e. based on both analytical methods and experimental data. Hughes, however, used an allegedly more realistic assumption for the force on the projectile during penetration. It was assumed that on impact the force rises instantaneously to the maximum value F_s, and then falls off parabolically according to:

$$F = F_s\left(1 - \frac{x}{x_p}\right)^2 \tag{4.22}$$

This gives a final penetration depth of:

$$\frac{x}{D} = \beta \frac{I}{S(I)} \ , \ I = \frac{Mv^2}{\sigma D^3} \tag{4.23}$$

Experimental data is supposed to determine the value of β and the functional form of $S(I)$. Hughes was also unable to obtain the ACE and NDRC data, so he instead used these formulas to generate "pseudo–data". He also mixed in some more recent data from Sliter (26) and Berriaud (5), giving the following result:

$$\beta = 0.19N \ , \ S(I) = 1 + 12.3 \ln(1 + 0.03I) \tag{4.24}$$

which of course gives exactly (4.21). Hughes pointed out that the formula is only valid as long as neither scabbing nor perforation occurs, and from experimental data he derived two mathematical criteria for this. These are given in chapter 5 and 6.

Hughes's formula is valid for $I < 3500$, which roughly corresponds to impact velocities up to 1050 m/s (given reasonable values for the other parameters). The projectiles are supposed to be hard cylindrical missiles impacting the target normally, while the concrete is lightly reinforced. For very low values of I, the theory is expected to predict too deep penetration since elastic and global effects are neglected in the force assumption. On comparison with other formulas, we shall indeed see that this is the case.

4.7 Young formula

Young's formula (30) is another very well known equation. Just as Bergman's formula, it has to be split into two separate equations depending on the impact velocity. For a cylindrical projectile, the equations are given by

$$\frac{x}{D} = 1.10 \cdot 10^{-3} \frac{KSN}{D} \left(\frac{M}{D^2}\right)^{0.7} \ln(1 + 2.16 \cdot 10^{-4} v^2) \ , \ v < 61 m/s \tag{4.25}$$

$$\frac{x}{D} = 2.14 \cdot 10^{-5} \frac{KSN}{D} \left(\frac{M}{D^2}\right)^{0.7} (v - 30.4) \ , \ v > 61 m/s \tag{4.26}$$

where S is a penetrability constant of the material. For semi–infinite concrete targets with a cure time of more than one year, this is given by

$$S = 13.93\left(\frac{11 - P}{\sigma^{0.3}}\right) \tag{4.27}$$

where P is the percentage of rebar. The constant S will typically be in the range [0.7,1.1].

For hard target materials, such as concrete, K is given by:

$$K = 0.43 M^{0.28} \quad M < 181\ kg \tag{4.28}$$

$$K = 1.0 \quad M > 181\ kg \tag{4.29}$$

There are several assumptions related to this equation:

$$x > 2L \tag{4.30}$$

$$v < 1350\, m/s \tag{4.31}$$

where L is the noselength. Also, the weight of the projectile should be "more than a few pounds", according to Young.

Young's formula is seen to be linear for larger velocities and approximately quadratic for small velocities. It is also clear that Young's formula essentially gives $\gamma_1 = 0.3$, which is a lower estimate than what is given by any of the other formulas.

4.8 British formula

There is also a quite common British formula (29):

$$\frac{x}{D} = 4.81 \cdot 10^{-8}(1 - \frac{\sigma}{1.66 \cdot 10^{8}})\frac{Mv^{1.5}}{D^{2.8}d^{0.2}} \tag{4.32}$$

where d is the aggregate size.

It is clear that this formula has a velocity and diameter dependence which is similar to the other formulas, but the same can not be said about the material dependence. The penetration depth apparently decreases linearly with the concrete hardness, which is different from the predictions of all the other formulas.

4.9 Tolch and Bushkovitch formula

Of other significant formulas, one should include the equation of Tolch and Bushkovitch (27) from 1947. They conducted experiments on penetration into many different kinds of rock, and summarised their results in the following formula:

$$\frac{x}{D} = 2.217 \cdot 10^{-7}k\frac{Mv}{D^{2.83}} \tag{4.33}$$

The constant k has a different value depending on whether we are dealing with soft or hard materials:

Hard rock: $k = 2.7$

Soft rock (Concrete): $k = 4.7$

These two values are averages of values for several materials, but given the large experimental uncertainties, it is probably appropriate only to give the numbers as averages. If we do not take the average value, Tolch and Bushkovitch found the following approximate relation between penetration depth and compressive strength:

$$x \propto \sigma^{-1/3} \tag{4.34}$$

This would correspond to a value of $\gamma_1 = 1/3$, which is a quite weak material dependence compared with most other formulas.

The Tolch and Bushkovitch equation is based on very little data, which might be a problem. Usually, only two or three experiments have been performed on each type of material, and this does not give enough information to enable any certain conclusions about the parameter dependence to be drawn. One could perhaps even question whether any conclusions at all can be made from so little data. In any case, we note that the formula would give $\gamma_3 = 1.0$, which is inside the same range as the value given by the other equations which have been studied.

In the original paper of Tolch and Bushkovitch (27), the constant corresponding to γ_2 is given as exactly 17/6. It is not a good idea to assign such an accurate value to a constant which has been derived from experimental data. Things are made even worse by the fact that their average experimental value for γ_2 is 2.89 and not $17/6 \approx 2.83$. The reason that the researchers have used 17/6 instead of the value suggested by their data, is that they wanted to make a "compromise" with Beth who, as we have seen, found $\gamma_2 = 2.78$. Apparently $\gamma_2 = 17/6$ seemed like the perfect choice.

None of the experiments were performed on concrete, so it is not clear whether the formula can be applied to such a material. The experimental impact velocities were in the range 300 – 1000 m/s.

4.10 Forrestal formula

Recently a new formula for penetration into concrete has been created by Forrestal et al. (12), (13). This formula is even partly analytical. It is assumed that the projectile acts as a rigid body, and that the force on it is given by (12):

$$F = \pi \frac{D}{8}\left(\sigma_c S(\sigma_c) + N\rho_t V_1^2\right)x, \quad 0 < x < 2D \;, \quad V_1^2 = \frac{2Mv^2 - \pi D^3 \sigma_c S(\sigma_c)}{2M + \pi D^3 N \rho_t} \tag{4.35}$$

$$F = \frac{\pi}{4} D^2 \left(\sigma_c S(\sigma_c) + N\rho_t v^2\right), \quad x > 2D \tag{4.36}$$

The dimensionless empirical constant S is a function of the compressive strength. These equations may look similar to the NDRC force assumptions, but they rest, however, on a much more solid basis, as they can be analytically derived from cavity–expansion theory (23) under the assumption that the projectile is a rigid body.

The experiments were all performed for approximately the same σ_c, so S is assumed constant. Now an expression for the penetration depth can be derived analytically, and it is found to be:

$$\frac{x}{D} = \frac{2M}{\pi D^3 \rho_t N} \ln\left(1 + \frac{N\rho_t V_1^2}{\sigma_c S(\sigma_c)}\right) + 2 \tag{4.37}$$

The functional dependence of S on σ_c is not found from theory, and must be determined experimentally. No analytical expression for S has been given by Forrestal et.al, but experimentally it is indicated that S decreases with increasing compressive strength.

Several tests have been performed to validate formula (4.37), and it appears to agree quite well. The experiments were done with the following range of parameters:

$$v \in [277, 945]\ m/s \tag{4.38}$$

$$\sigma_c \in [11.7, 15.0]\ MPa\ ,\quad [32.4, 40.1]\ MPa\ ,\quad [90.5, 108.3]\ MPa \tag{4.39}$$

$$M = 0.064\ kg\ ,\ 0.9\ kg\ ,\ 5.9\ kg \tag{4.40}$$

$$D = 1.27\ cm\ ,\ 2.69\ cm\ ,\ 7.62\ cm \tag{4.41}$$

Notice especially that the tests mostly involved rather small projectiles. In (4.39)– (4.41) above, $M = 0.064\ kg$ corresponds to $D = 1.27\ cm$ etc.

4.11 TBAA formula

The British Textbook of Air Armament (TBAA) (14) gives the following equation for penetration into concrete:

$$\frac{x}{D} = 2.6104 \frac{M}{D^3 \sigma_c^{0.5}} \left(\frac{D}{C}\right)^{0.1} \left(\frac{v}{533.4}\right)^{97.51 \sigma_c^{-0.25}} \tag{4.42}$$

where C is the maximum size of coarse aggregate in the concrete. The validity range of the formula is given as:

$$\sigma_c \in [5.5, 69.1]\ MPa \tag{4.43}$$

$$M \in [0.14, 9975]\ kg \tag{4.44}$$

$$D \in [0.013, 0.96]\ m \tag{4.45}$$

$$v < 1130\ m/s \tag{4.46}$$

A new feature of this equation is that the velocity exponent depends on the concrete compressive strength. An implication of this is that for concrete with larger values of σ_c, the penetration depth is not as velocity dependent as for less solid types of concrete. However, if we insert the allowed values of σ_c, it is seen that the exponent will be in the range $[1.07, 2.01]$, which is the same range as in all the other equations.

5 PERFORATION

So far we have examined the penetration depth in very thick targets. If we do not make the assumption about large target thickness, two other important phenomena may take place, namely perforation and scabbing.

Perforation is the entry of a missile into the target and its exit out of the back face.

Scabbing is the ejecting of concrete pieces from the back face, with at least a size equal to the thickness of the concrete layer between the surface and the reinforcement (18) , (26).

An interesting quantity is the required target thickness to prevent perforation/scabbing. This is called the perforation/scabbing thickness, and is usually expressed as a function of the penetration in an infinite medium.

In this chapter, we review the various equations that exist for predicting perforation, and then we look at scabbing in the next chapter. The perforation thickness is denoted by h.

5.1 NDRC formula

According to the NDRC–formula, the perforation thickness is given by:

$$\frac{h}{D} = 1.24\frac{x}{D} + 1.32 \quad , \quad 1.35 < \frac{x}{D} < 13.5 \tag{5.1}$$

$$\frac{h}{D} = 3.19\left(\frac{x}{D}\right) - 0.718\left(\frac{x}{D}\right)^2 \quad , \quad \frac{x}{D} < 1.35 \tag{5.2}$$

where $\frac{x}{D}$ is the penetration depth according to the NDRC penetration formula. These equations were derived by Chelapati and Kennedy (21),(8),(9),(22).

According to Chang (7), the NDRC perforation formula is too conservative, which means it predicts perforation in cases where the projectile does not go through.

5.2 CEA–EDF formula

The CEA–EDF (Commissariat à l'Energie Atomique – Electricité de France) formula was proposed in 1977 by Berriaud et.al. (5). This equation was based on data from new French experiments conducted by the CEA–EDF. The formula is given by:

$$h = 0.82\left(\frac{M}{D^3}\right)^{0.5} \frac{v^{0.75}}{\sigma_c^{0.375}\rho_c^{0.125}} \tag{5.3}$$

The equation is valid inside the following range:

$$v < 200\, m/s \tag{5.4}$$

$$\rho_c \in [150, 300]\, kg/m^3 \tag{5.5}$$

$$M \in [20, 300]\, kg \tag{5.6}$$

$$\frac{D}{d} \in [0.24, 2.9] \tag{5.7}$$

5.3 Degen formula

A few years later, in 1980, using the experimental data of several different sources (5), (11), (15), Degen (10) statistically derived another perforation equation:

$$\frac{h}{D} = 2.2\frac{x}{D} - 0.3\left(\frac{x}{D}\right)^2 \quad , \quad \frac{x}{D} < 1.52 \tag{5.8}$$

$$\frac{h}{D} = 0.69 + 1.29\frac{x}{D} \quad , \quad 1.52 < \frac{x}{D} < 13.41 \tag{5.9}$$

where x/D is given by the NDRC–penetration formula. The experiments covered the following ranges:

$$v \in [25, 310]\ m/s \tag{5.10}$$

$$M \in [15, 340]\ kg \tag{5.11}$$

$$\sigma_c \in [28, 43]\ MPa \tag{5.12}$$

$$D \in [10, 31]\ cm \tag{5.13}$$

$$H \in [15, 60]\ cm \tag{5.14}$$

where H is the wall thickness. The reinforcement was between 160 kg/m^3 and 350 kg/m^3, but variation of this parameter did not produce any significant effect.

5.4 Chang formula

Chang (7) derived a perforation formula in 1981, almost exclusively using classical mechanics. The reason why his equation is still classified as empirical is that he applied a Bayesian statistical approach on some test data in order to determine a constant. His final equation reads:

$$h = 2.79\sqrt{\frac{M}{D^3\sigma_c}}\,v^{0.75} \tag{5.15}$$

As always, we also state the validity range of the formula:

$$v \in [16.7, 311.8]\ m/s \tag{5.16}$$

$$M \in [0.1, 343.6]\ kg \tag{5.17}$$

$$\sigma_c \in [23.2, 46.4]\ MPa \tag{5.18}$$

$$D \in [2.0, 30.5]\ cm \tag{5.19}$$

5.5 Hughes formula

Hughes (18) has the following equations for perforation:

$$\frac{h}{D} = 3.6\frac{x}{D} \quad , \quad \frac{x}{D} < 0.7 \tag{5.20}$$

$$\frac{h}{D} = 1.58\frac{x}{D} + 1.4 \quad , \quad \frac{x}{D} > 0.7 \tag{5.21}$$

The validity range of these equations are the same as for his penetration formula, which is given earlier in the report. Here x/D is the penetration depth predicted by the Hughes penetration formula.

5.6 Adeli and Amin formula

Using a least–squares fit on test data from both Europe (5) and the US (26), Adeli and Amin (1) derived yet another perforation formula:

$$\frac{h}{D} = 0.906 + 0.3214I - 0.0106I^2 \quad , \quad I = N\frac{Mv^2}{D^3\sigma_c} \tag{5.22}$$

The validity range is the same as for their penetration formula.

5.7 Petry formula

In 1950, Amirikian (2) suggested that the perforation thickness was given by the following simple formula:

$$\frac{h}{D} = 2\frac{x}{D} \tag{5.23}$$

6 SCABBING

In this chapter, we list the scabbing equations which are available in the literature. Their validity range is always the same as the corresponding perforation formula, unless otherwise is stated. The scabbing thickness is denoted by s.

6.1 NDRC formula

Just as the corresponding perforation formula, this scabbing equation was derived by Kennedy and Chelapati (8), (9), (22) a long time after the tests were actually carried out:

$$\frac{s}{D} = 7.91\frac{x}{D} - 5.06\left(\frac{x}{D}\right)^2 \quad , \quad \frac{x}{D} < 0.65 \qquad (6.1)$$

$$\frac{s}{D} = 2.12 + 1.36\frac{x}{D} \quad , \quad 3 < \frac{s}{D} < 18 \qquad (6.2)$$

6.2 Chang formula

For scabbing, Chang (7) devised the following formula using his Bayesian statistics and classical mechanics approach:

$$s = 3.14\left(\frac{M}{D^3\sigma_c}\right)^{0.4} v^{0.67} \qquad (6.3)$$

6.3 Hughes formula

Hughes (18) gives the following expressions for scabbing thickness:

$$\frac{s}{D} = 5.0\frac{x}{D} \quad , \quad \frac{x}{D} < 0.7 \qquad (6.4)$$

$$\frac{s}{D} = 1.74\frac{x}{D} + 2.3 \quad , \quad \frac{x}{D} > 0.7 \qquad (6.5)$$

6.4 Bechtel Corporation formula

The Bechtel Corporation (24) has proposed the following empirical formula for calculating the scabbing thickness for cylindrical hard missiles:

$$s = 39.02\left(\frac{M}{D^3}\right)^{0.4}\left(\frac{v}{\sigma}\right)^{0.5} \qquad (6.6)$$

The equation was based on 12 tests with solid missiles and 9 tests with half pipe missiles inside the following experimental range:

$$v \in [37.1, 144.4]\, m/s \qquad (6.7)$$

$$M \in [3.6, 97.1]\, kg \qquad (6.8)$$

$$D \in [20.3, 21.8]\ cm \tag{6.9}$$

$$t \in [30.5, 61.0]\ cm \quad , \quad [7.6, 22.9]\ cm \tag{6.10}$$

$$\sigma_c \in [30.3, 39.7]\ MPa \tag{6.11}$$

The parameter t is the target thickness. Notice that this equation is based on experiments in a very limited diameter range.

6.5 Stone and Webster formula

Another scabbing formula is the one by Stone and Webster (19). It is based on 7 tests with solid missiles and 21 tests with pipe missiles.

$$s = \left(\frac{Mv^2}{cD^3}\right)^{1/3} \tag{6.12}$$

where c is a coefficient that depends on t/D. The experimental range of the parameters was:

$$v \in [27, 157] m/s \tag{6.13}$$

$$M \in [1.9, 12.8] kg \tag{6.14}$$

$$D \in [4.1, 8.9] cm \tag{6.15}$$

$$t \in [11.4, 15.2] cm \tag{6.16}$$

$$\sigma_c \in [22.1, 30.3] MPa \tag{6.17}$$

Notice that this equation is based on tests with rather thin concrete plates.

6.6 Petry formula

This scabbing equation was suggested by Amirikian (2) in 1950:

$$\frac{s}{D} = 2.2\frac{x}{D} \tag{6.18}$$

6.7 Adeli and Amin formula

The last formula that will be mention is the Adeli/Amin (1) scabbing formula. Just like their perforation formula, it is totally empirical and based on curve fitting.

$$\frac{s}{D} = 1.8685 + 0.4035I - 0.0114I^2 \quad , \quad I = N\frac{Mv^2}{D^3\sigma_c} \tag{6.19}$$

7 COMPARISON BETWEEN THE EQUATIONS

In the previous chapters, we have listed a very large number of equations giving a relation between penetration depth, perforation thickness, scabbing thickness and various parameters describing the situation. It is, however, obvious that the formulas are all quite different, and consequently their predictions will not be identical.

This is not really surprising considering what a complex field missile impact/penetration is, especially when one takes into account all the different methods which have been used in order to derive the equations. Some researchers have used a purely empirical approach, while others have applied a semi–analytical method. Some have based their work solely on their own experimental data, while others have reinterpreted data from various other sources.

It is also important to note that the formulas are valid for different range of parameters. Whether this is a sufficient explanation for their differences, will be investigated in chapters 8 and 9.

In this chapter, we will try to compare the various formulas and see how different they really are. This is done most easily if we define nondimensional quantities and express the formulas in terms of them.

7.1 Nondimensional quantities

Let us now focus on the relation between penetration depth x and impact velocity v. The empirical formulas say that x could be proportional to v, it could be proportional to v^2, or have even another kind velocity dependence. How important is this difference?

A mathematical function $f(z) = z$, will in general be different from the function $g(z) = z^2$, but notice that if z is somewhere in the range $[0,1]$, there is not that much difference between the functions f and g. The same should apply to the relation between penetration depth and impact velocity. However, the situation is a little bit different since v is a dimensional quantity. To be able to compare the various empirical formulas, it is convenient to write them in a nondimensional form, i.e. define nondimensional quantities and express the equations in terms of them.

When trying this approach, we once again run into the problem of having equations which are dimensionally incorrect. As has been stressed, only the newer formulas have their dimensions correct, so these are the only formulas that in principle might be accurate inside any parameter range. This doesn't mean that any of them actually *are* exactly right. They need not even be the best of the equations, but they *could* be right. None of the older formulas *can* be 100% correct, but they *could* still be excellent approximations, possibly better than the new ones.

On this topic, we might also add that for an equation to be applicable in all ranges, it necessarily must have $\beta = 0$. Unless this is the case, the equation will predict penetration even when the impact velocity is zero, something which is obviously unphysical. It is seen that many of the formulas do not even clear this hurdle, but this is not surprising as they were usually not designed to work for such low velocities. The real worry is the difference in important parameters and the dimensionally incorrect formulas.

7.2 Comparison of nondimensional formulas for penetration

Now we can start comparing the various equations. We are mainly interested in the relation between penetration depth and impact velocity, so we define the nondimensional velocity Z, and the nondimensional penetration depth X in the following manner:

$$Z = \sqrt{\frac{M}{D^3\sigma_c}}v \quad , \quad X = \frac{x}{D} \tag{7.1}$$

Expressing some of the formulas in terms of these parameters gives us:

ACE $\quad X = a_1 Z^{1.5} + 0.5 \quad a_1 = 0.35 \cdot 10^{-3}\left(\frac{M\sigma_c}{D^3}\right)^{0.25} = 0.232\left(\frac{kg}{m^2}s\right)^{0.5}$

Hughes $\quad X = \frac{a_2}{1 + 12.3\ln(1 + 0.3Z^2)} \quad a_2 = 1.9$

NDRC mod. $\quad X = a_3 Z^{1.8} + 1.0 \quad a_3 = 3.8 \cdot 10^{-5}\left(\frac{M}{D}\right)^{0.1}\sigma_c^{0.4} = 0.078\left(\frac{kg}{m}\right)^{0.5}s^{-0.8}$

Bernard $\quad X = a_4 Z \quad a_4 = 0.254\sqrt{\frac{M}{D^3\rho}} = 0.772$

If the formulas really had been dimensionally correct, the "constants" a_1 and a_3 would have been real nondimensional constants, but instead they have turned out to have a dimension. In order to compare the equations, it is therefore necessary to give numerical values to the variables. Above, the following reasonable values for a projectile impacting a concrete wall have been inserted:

$$M = 500\,kg \quad , \quad D = 0.3\,m \quad , \quad \sigma_c = 30\,MPa \quad , \quad \rho = 2000\,kg/m^3$$

This gives us the following relations:

$$Z = 0.00248v \quad , \quad v = 40.25Z \tag{7.2}$$

In the Hughes equation, the material parameter is the concrete tensile strength σ_t instead of compressive strength σ_c, as in all the other equations. It has been assumed that $\sigma_t = 0.1\sigma_c = 3\ MPa$. In reality, the relationship between these two parameters is more complicated, but the assumption above has sufficient accuracy for our present purposes.

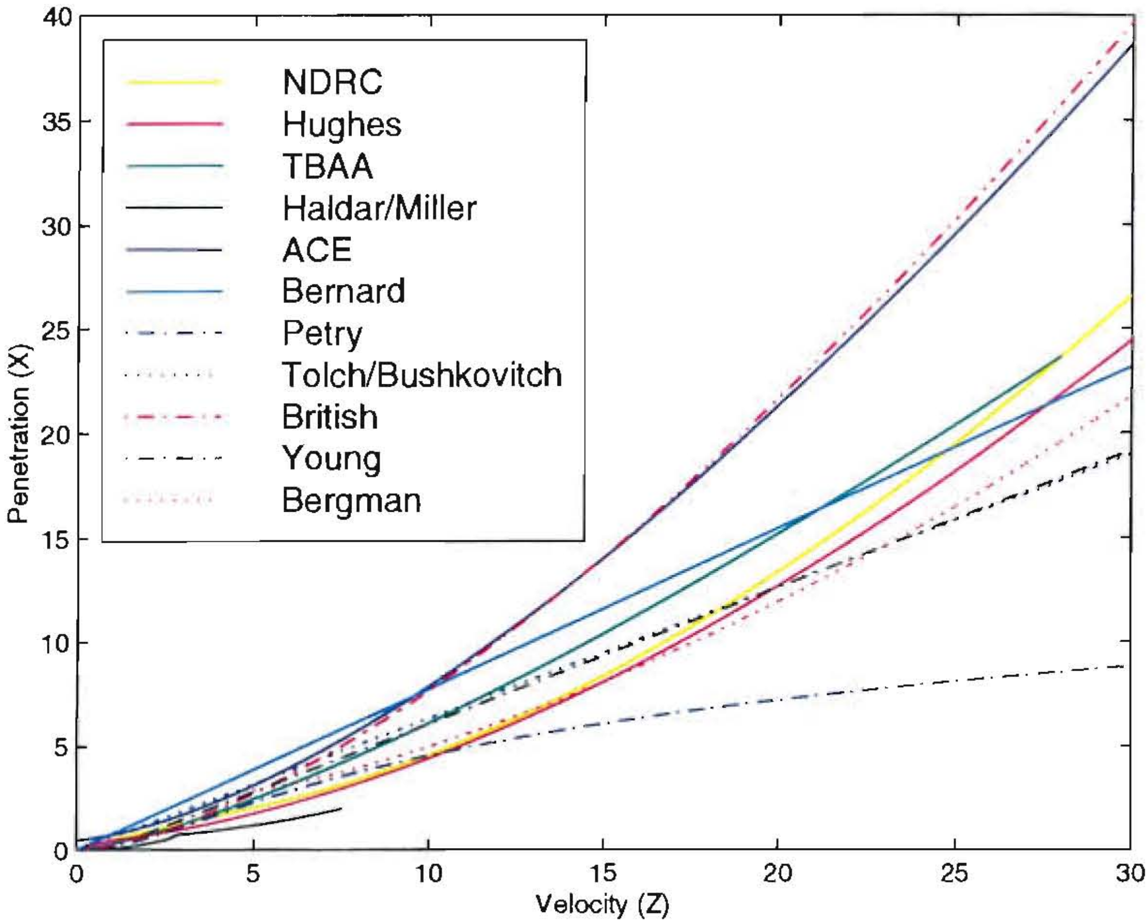

Figure 7.1: *Comparison of the non–dimensional penetration equations into infinite targets.*

In the equations which require a nosefactor N to be given, we have inserted the value corresponding to a flatnosed projectile. See appendix A for details.

In figure 7.1, the various equations have been plotted. We notice that except for the ACE, Petry and British equations, the disagreement does at first view not seem catastrophic. If these formulas are excluded, the highest penetration estimate is, however, for $Z = 30$ (corresponding to approximately $1200\ m/s$) still about 40% larger than the lowest estimate This is not really a satisfactory situation.

One might get the impression from figure 7.1, that for low velocities the equations approximately agree with each other. To explore this idea further, all formulas have been plotted for low velocities $Z < 10$ (corresponding roughly to $v < 400\ m/s$) in figure 7.2. This makes it quite clear that the highest penetration estimate is, for $Z = 10$, almost 100% larger than the lowest estimate. So not only do the formulas not agree with each other at low velocities, but the relative disagreement is actually even worse than for high velocities.

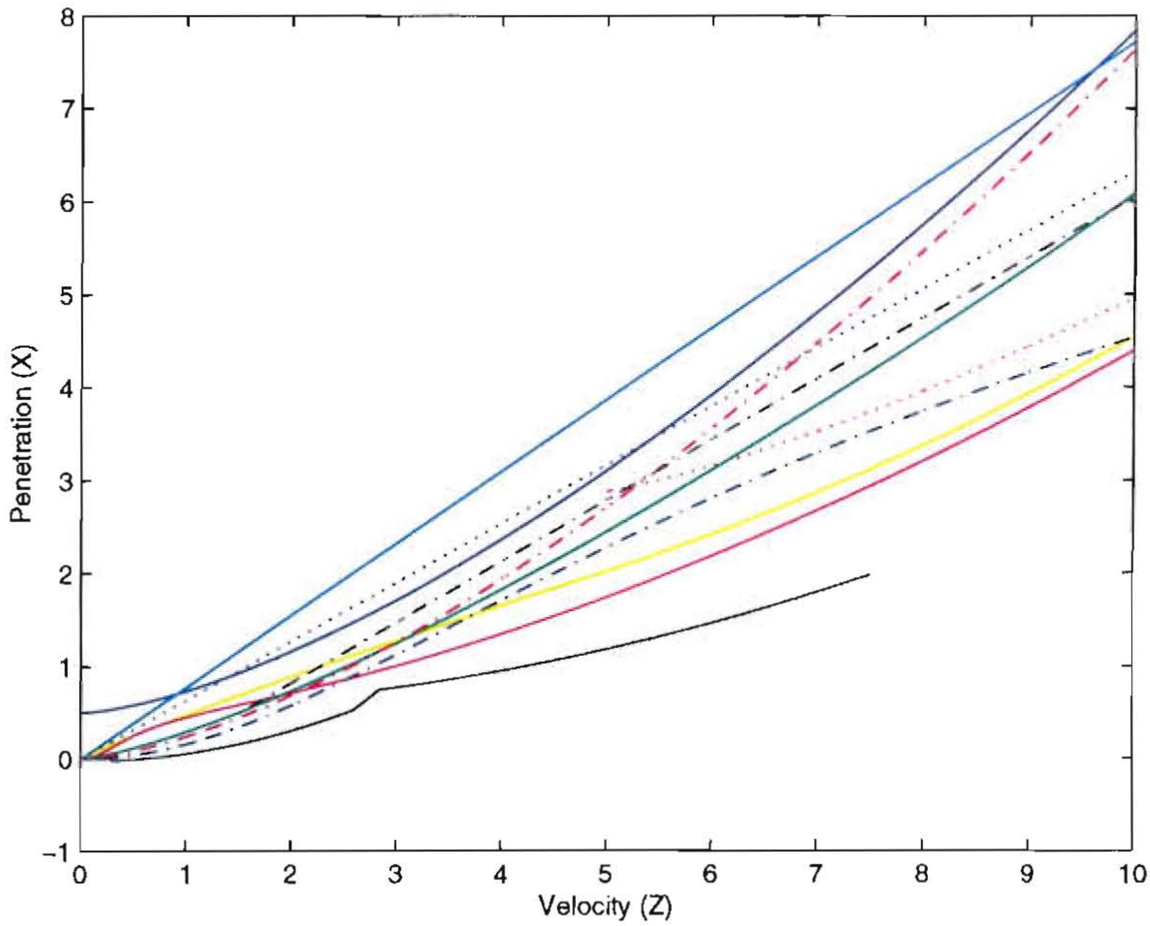

Figure 7.2: The nondimensional penetration equations into infinite targets for low velocities (same colour codes as in figure 7.1).

7.3 Comparison of nondimensional formulas for perforation

We are able to use exactly the same procedure in creating nondimensional formulas for perforation as we have done for penetration. The nondimensional velocity is the same as for penetration, while the nondimensional perforation thickness H is defined by:

$$H = \frac{h}{D} \tag{7.3}$$

Using the same parameter values as for penetration, we have plotted the perforation formulas in figure 7.3. All of the equations, except for Hughes and NDRC, are only valid for low velocities, so we have plotted them all for $Z < 10$. It is seen that for $Z = 10$, the spread between lowest and highest estimate is only about 30%, so there is better agreement here than for the penetration formulas.

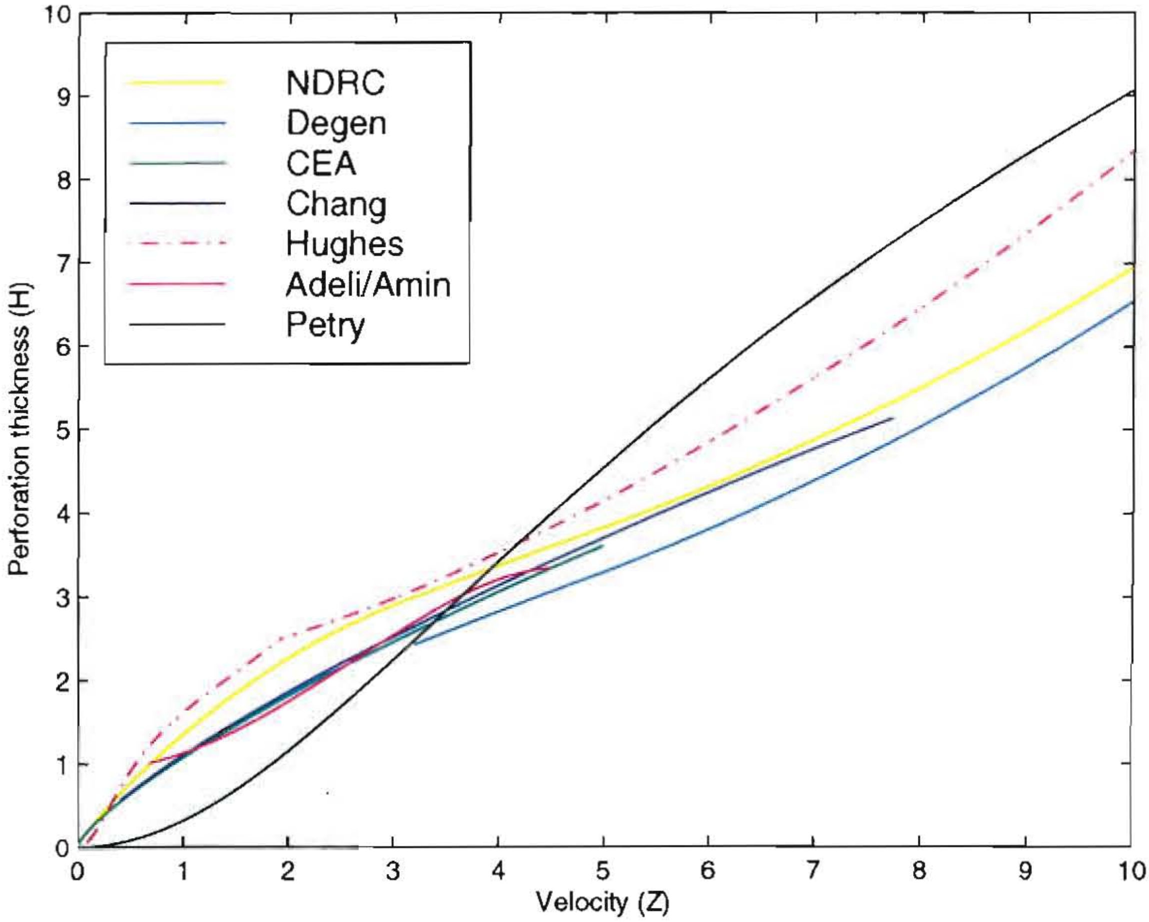

Figure 7. 3: Comparison of the nondimensional perforation equations.

7.4 Comparison of nondimensional formulas for scabbing

For scabbing, the situation is once again very similar. We define the nondimensional scabbing thickness S by:

$$S = \frac{s}{D} \tag{7.4}$$

In figure 7.4, we have plotted the various formulas. The spread between lowest and highest estimate seems to be of approximately same magnitude as the perforation spread, i.e. roughly 30%.

7.5 Observations

There is no doubt that there are large differences between the various equations. Especially for penetration, the situation is quite bad with predictions sometimes differing by more than 100%. For perforation and scabbing, the agreement is better, but still not quite satisfactory.

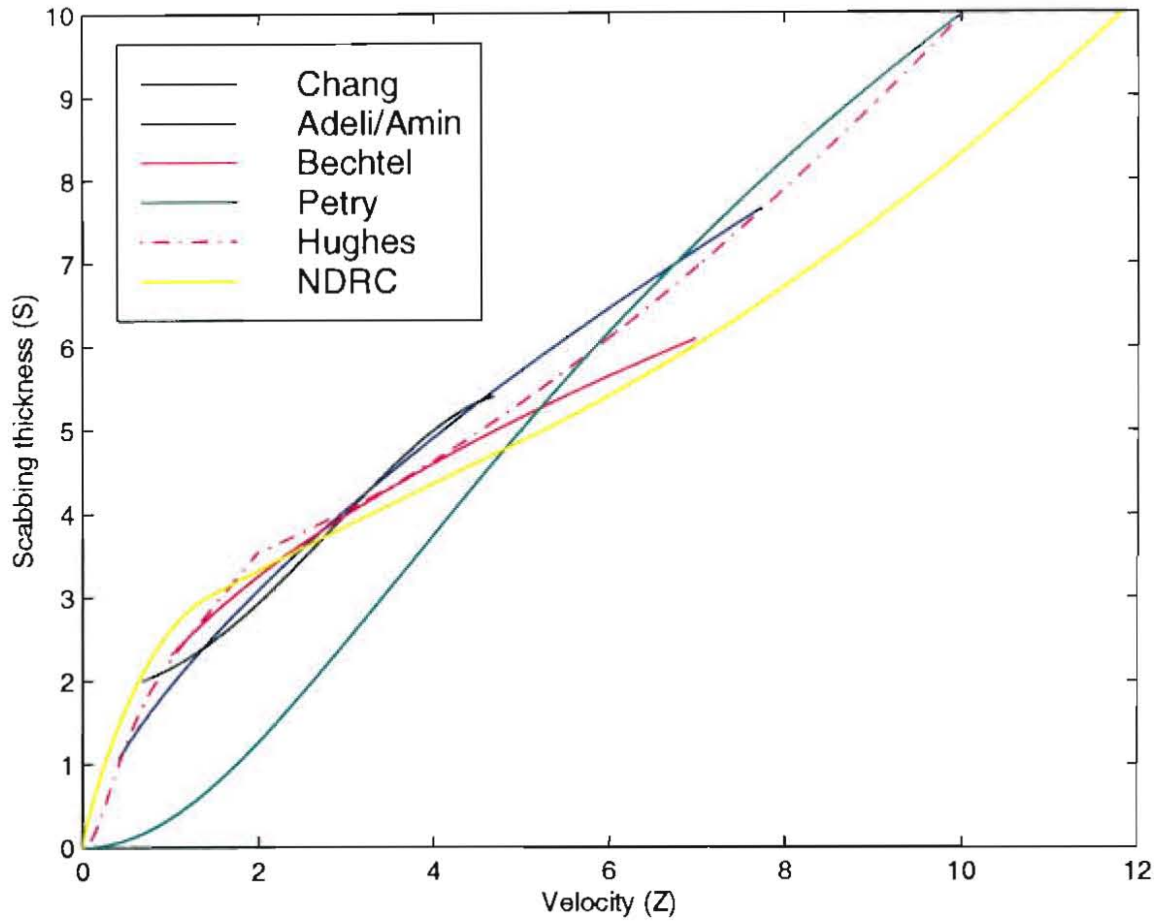

Figure 7. 4: Comparison of the nondimensional scabbing equations.

8 WHY ARE THE FORMULAS NOT ALL THE SAME?

In this chapter, we shall investigate the question of why there are such big differences between the equations. We start with some general remarks about possible explanations.

8.1 Dimensional problems

First we address the question of how it is possible derive dimensionally incorrect equations from experimental data. This is related to the mathematical problem of finding the correct formulas without solving the full set of differential equations describing the problem. By using dimensional analysis, it is often found that a final answer can only be written in a few different ways if it is to be dimensionally correct.

Also, it can easily be shown that including more variables in describing a problem, means more possible ways of writing the final result. When only a few variables is included in the description, there is often basically only one way of writing the final equation. As an example, it was mentioned earlier that if the penetration depth is to be described by equation (2.1), dimensional analysis forces the γ–constants to take on the respective values of 1, 3 and 2. If other variables had been allowed in the formula, say the target density, there would have been several other possible dimensionally correct formulas.

Here lies a clue to the reason for the troublesome dimensions. Experiments with penetration are time- and moneyconsuming, making it very difficult to vary all kinds of relevant parameters, as this would mean too many experiments. A consequence of this is that when such experiments are performed, only what is believed to be the most important of the parameters are varied.

By using the experimental data, one tries to find the functional relationship of these few parameters, but since there in reality are more parameters involved than has been studied, this is not an easy task. One might interpret an effect which is caused by a variable not being studied to be caused by a variable being studied. This could in the end lead to a dimensionally incorrect result.

For instance, it might seem reasonable to expect the penetration depth to depend on the target density, but this parameter is only included in a few of the equations. Including this density as a parameter could possibly fix some of the dimensional problems that otherwise occur. It is important to be aware that those formulas which do not depend on the density, probably have been found from experiments done with only one specific density. Unless the penetration actually is independent of the target density (which seems physically unreasonable), those formulas are in principle only valid for that specific density under which the tests have been performed.

8.2 Experimental uncertainties

It must also be remembered that there is a large degree of uncertainty associated with penetration experiments. As can be seen by studying the penetration problem analytically, a lot of effects take place during the penetration event and contribute to the final outcome.

In the end, the data one collects from the experiments, might not be so easy to interpret. As one of many examples, we mention Beth's formula and Bergman's formula, which are both based on the same data, but nonetheless are quite different, due to the fact that they used different statistical methods for analysing the data.

The semi-analytical formulas are also very sensitive to the underlying assumptions. Hughes used the NDRC-formula to generate pseudo-data, but since the force on the projectile is assumed to depend on penetration depth in a different way than in the NDRC-formula, the final result is quite different.

When the experimental uncertainties are large, some of the data obtained may be conflicting and misleading. A natural solution is to discard such data, but since one does not know in advance which data are correct and which are not (if we knew this, there would be no need for any experiments!), this is a very difficult task. Consequently, a researcher might be able to obtain almost whatever equation he wants, just by selecting the appropriate datapoints.

As already mentioned, penetration experiments are very expensive. Ideally, one would like to perform thousands of experiments to ensure the validity of the formulas, but in the real world this is impossible. Instead, one has to settle for performing as many experiments as possible, which generally is only a few. If we take Bernard's formula as an example, it is based on only 11 experiments. This formula depends on four parameters, and it should be quite clear that 11 experiments do not give much of an opportunity to study the functional form of them all. Another example is the Tolch/Bushkovitch equation, which has been obtained partially by fitting two datapoints with the best possible (straight) line.

In order to obtain sufficient data, we have seen that some researchers have used experimental data from different sources. There is a drawback to this, as the researchers themselves are not able to totally control the circumstances under which the various experiments are performed. It is a basic principle of science that all experiments should be reproducable. In penetration mechanics it could seem that this is only true in principle, but not in reality since experiments are too expensive to be done over and over again. Consequently, the only remaining alternative is to take previous results for granted, something which increases the uncertainty in the resulting formulas.

8.3 Different ways of analysing the data

It has been noticed that some of the equations are split into separate formulas, mostly depending on the impact velocity. Other equations are the same for all velocities. That some of the researchers have needed to use two (or more) equations to describe the penetretation event might suggest that γ_3 in (2.1) is not really a constant, and that it in fact depends on the impact velocity. This is certainly what Bergman and Young seemed to believe.

If this assumption is correct, then it could help explaining why there is so much disagreement about γ_3. The various experiments have not been performed at the same velocities, and therefore give different results since γ_3 is velocity dependent. Inside a certain range, it might be approximately constant, though. Maybe the experiments that give $\gamma_3 = 1$ have been performed at completely different velocities than those which say $\gamma_3 = 2$? Both results could actually be correct, but they are just not valid in the same range.

We can test our hypothesis by comparing the data behind the Haldar/Miller ($\gamma_3 = 2$) and Bernard ($\gamma_3 = 1$) equations. On examination of the experimental data, it is seen that the former only uses data for quite low impact velocities, i.e. in the range 30–300 m/s, while the latter is based on data in the range 300–800 m/s. This observation seems to agree very well with what we stated above. The hypothesis is further supported by the fact that the Tolch/Bushkovitch formula is also based on data in approximately the same range as the Bernard equation. However, due to the little amount of data used to derive this formula, one probably should not read too much into this.

Another possibility is that the differences could be due to the various experiments being performed with different kinds of concrete. Since concrete is such a complex material, its material properties depend on several parameters. Perhaps some of the experiments have been on reinforced concrete while others have been on normal concrete, or different aggregate sizes have been used in the various experiments. This *could* lead to different results for the penetration depth.

9 EARLIER REVIEWS OF THE VARIOUS EQUATIONS

Before presenting our own conclusions, let us look at other reviews of the available equations that have been done.

9.1 Kennedy

In 1975 Kennedy (21) examined most of the (at that time) available equations. These were ACE, NDRC, BRL, Amman/Whitney and mod. Petry I+II.

For penetration, he found that in the range [150,300] *m/s*, the ACE and NDRC–equation were in reasonable agreement with experimental data, while all the other formulas seemed to underpredict the penetration depth. For lower velocities, not much data was available at that time, but results from the Calspan Corporation (28) showed the NDRC–equation to be the most accurate here as well. All the other formulas seemed to grossly underpredict penetration in this velocity range.

His conclusion was to recommend the NDRC–equation for penetration since it seemed to be "valid" over the whole velocity range.

As for perforation and scabbing, he also recommended the NDRC–equation, but only until something better became available. According to him, this formula had the advantage of not being purely empirical, which made him more confident in extrapolating its results outside the test range.

9.2 Sliter

In 1980, Sliter (26) performed another review of the empirical equations for low impact velocities. Of the older formulas, only the NDRC–equation was considered, though. This was partly because of Kennedy's conclusion that it was consistently better than the other old formulas.

Sliter compared the NDRC penetration equation with newer test data and found reasonably good agreement for $\frac{x}{D} \in [0.6, 2.0]$. Reasonably good means within $\pm$ 25%. accuracy. For smaller values of $\frac{x}{D}$, the agreement was not equally satisfactory, though.

For scabbing, he concluded that further investigation was needed before any of the empirical equations could be applied with confidence for large diameter missiles. In the meantime, one could use scabbing formulas developed over a limited range of impact parameters.

Examples are the Stone/Webster and Bechtel equations which, as we have seen, are not valid for a large range of parameters, but turns out to be very good where they are valid.

For perforation, the CEA–EDF formula and the NDRC–formula was compared, with the result being that the former gave much better agreement with data. However, he warns that perforation is more dependent on the amount of reinforcement than scabbing and penetration resistance, so consequently the formula should not be applied outside the range of reinforcement, for which it was derived.

9.3 Adeli and Amin

In 1985, Adeli and Amin (1) built on the work of Sliter and performed yet another review of the available equations for penetration, perforation and scabbing. Together with the NDRC, ACE and modified Petry equations, they also included the new equations from Haldar/Miller, Hughes and themselves. In their evaluation they used the recent data from Sliter.

For penetration, they drew the following conclusions after comparing the formulas with the experimental data:

ACE and the modified Petry I overpredicted the penetration depth with a big margin, while generally the Hughes, Haldar/Miller and their own formulas were the most accurate ones. More precisely, for $x/D > 0.6$, the NDRC, Haldar/Miller, Hughes and Adeli/Amin were the best ones, while for $x/D < 0.6$, the modified Petry II, Haldar/Miller and Adeli/Amin were more accurate.

Adeli/Amin then performed a statistical comparison to find the best overall fit. If all data points were included in the analysis, the quadratic Adeli/Amin were found to give the least coeffisient of variation. However, in an analysis using only data points corresponding to a velocity larger than 144 *m/s*, the NDRC–equation was found to be slightly better than this formula.

Their main conclusion on penetration was to recommend their own quadratic formula for velocities below 144 *m/s*. For velocities between 144 *m/s* and 310 *m/s* they recommended the NDRC–formula or the quadratic formula.

On scabbing they compared all relevant equations, finding the Chang, Bechtel, and their own formulas to give the best predictions. These are the formulas they recommend. The NDRC, ACE and Hughes formulas also agree quite well with the data, but they are more conservative.

For perforation, they also compared the available formulas and found that Adeli/Amin, Chang, Degen and CEA–EDF to be the best formulas. For velocities below 310 *m/s* these formulas were recommended.

Notice that Adeli/Amin only compared the formulas in the low velocity range and for non–deformable missiles.

10 SUMMARY

The review by Adeli/Amin is the most recent and consequently they have been able to build on Kennedy and Sliter, while using all the available new data. We therefore have some confidence in their conclusions. However, being mainly interested in threats against nuclear reactors, they only looked at the low impact velocities.

From a military point of view, the high velocity regime $v > 300\ m/s$ is of larger interest. For high velocities there are unfortunately very little data available, and what is available is very old and almost unobtainable. Nearly all high velocity penetration formulas are based on data from the American tests during World War II. It is difficult to say how much confidence can be put into these equations without having seen the original data. We are therefore hesitant in making any firm recommendations for this velocity range.

For low velocities, we endorse the conclusions by Adeli/Amin, although with some reservations. When examining the test data, it is seen that the data is very much scattered. This is obviously due to experimental uncertainties rather than the final penetration formula being very "irregular". In our opinion there is no point in using higher and higher polynomials to approximate the datapoints even closer. Instead, what is needed is more reliable data.

For high velocities, there is an even greater need for more (reliable) data. It must be said though, that the approach of Forrestal et.al. looks quite promising and might be something to build on in the future.

For perforation and scabbing, we also endorse the conclusions of Adeli/Amin for low velocities.

In the high velocity regime, there are only the NDRC and Hughes equations available. There is no point in recommending any of these, as we have no data to base our judgement on. We know, however, that the NDRC–equations are based on very old data, while the Hughes equations partially are based on "what the NDRC–data might have been".

To summarise our recommendations, we advice that until a better theoretical understanding of the penetration phenomenon is established, one should be very careful in applying any of the equations. It is especially important to understand that each formula should only be applied inside the range for which the original tests took place.

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APPENDIX

A NOSEFACTORS

As we have seen, many of the empirical formulas include a nosefactor N in the equation. Unfortunately, there are several different definitions of this factor, which may ultimately lead to some confusion. Here is an overview of the various definitions:

Let S denote the length of the nose (the part of the projectile which is curved), D the projectile diameter and R the curvature radius Then we have the following definitions:

NDRC: $N = 0.72 + 0.25S/D \ , \quad N \in [0.72, 1.17]$

Bergman: $N = 0.7 + 0.27S/D \ , \quad N \in [0.8, 1.2]$

Young: $N = 0.56 + 0.183S/D$ Ogival projectile
$N = 0.56 + 0.25S/D$ Conical projectile

Forrestal: $$N = \frac{8(R/D) - 1}{24(R/D)^2}$$

Hughes gives no analytical formula for his nosefactor, but instead gives its value for the following special cases:

Hughes:
$N = 1.00$ Flat nose
$N = 1.12$ Blunt nose
$N = 1.26$ Spherical nose
$N = 1.39$ Very sharp nose

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