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EXAMINATION OF YAWED IMPACT USING A COMBINED NUMERICAL AND ANALYTICAL APPROACH

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Director of Research

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8) ABSTRACT

Performing sensitivity studies using 3D-hydrocodes have normally been an extremely time consuming process and still remains so even though computer speed continues to increase. To overcome this problem, a combined analytical and numerical approach based on cavity expansion theory (CET) was implemented as a user subroutine in Autodyn-3D. This enables us to model only the projectile explicitly, whereas the response of the target is incorporated through analytical theory. The runtime of the problem is then reduced to less than 1% of a corresponding full 3D simulation, and makes it possible to easily perform sensitivity studies for cases of non-normal impact. This can be very useful before full 3D-simulations are performed, or in its own right.

As an example of use, a parameter study of yawed impact has been performed for two different projectiles at different velocities. This was completed in a few hours whereas a corresponding full Autodyn-3D study would have taken many weeks.

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CONTENTS

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1 INTRODUCTION

Many situations in penetration mechanics exhibit symmetries that allow them to be effectively described as two-dimensional. A typical example is an axis symmetrical projectile impacting a cylindrical target. This symmetry can be taken advantage of in numerical simulations to reduce the runtime significantly. For example the widely used hydrocode Autodyn comes in two versions, 2D and 3D. In the 2D version, the user can choose between axial and planar symmetry to effectively model a symmetrical 3D problem in two dimensions.

Unfortunately, in cases of non-normal impact, neither planar nor axial symmetry is present. The situations can still be modelled numerically in Autodyn-3D, or a similar hydrocode, but in many cases with so large runtimes that parameter studies are difficult or impossible to perform. Examples of such situations are oblique impact and impact with yaw.

In this report we describe a combined numerical and analytical approach to non-normal impact. Autodyn-3D is used in combination with analytical theory to remove some of the timeconsuming numerical processes. As an example of use, the method is then applied to parameter studies of yawed impact for two different projectiles.

2 COMBINED ANALYTICAL AND NUMERICAL APPROACH

In hydrocodes, a physical situation is discretized both in time and space. The physical system is divided into elements, and on running a simulation the code usually performs calculations for all elements at each time step. Having a large number of elements should lead to more accurate results, but also increases the runtime of the simulation since more computations are necessary.

For a penetration problem in 3D, a reasonable description of the target may require an excessive number of elements, resulting in runtimes up to several days or weeks. Despite advances in computer speed, parameter studies are clearly difficult to perform in such cases.

In this article we present a combined analytical and numerical (hybrid) method, where only the projectile is modelled explicitly in the hydrocode. Instead of interacting with a target mesh, the projectile elements interact with a “virtual target” through boundary conditions on the projectile surface. The stress on a particular projectile surface element is calculated from analytical theory at each timestep and applied through the boundary condition. If the analytical calculations provide a good approximation of the exact numerical solution, the stresses on the projectile surface will be the same as if it was really interacting with a target subgrid and the projectile will consequently behave in the same way.

It is emphasized that this method gives no information about damage to the target, crater size etc. The method is therefore only relevant if our primary concern is what happens to the projectile, i.e. calculation of penetration depth, residual velocity etc.

As far as we are aware, this approach was first used by Warren and Tabarra [1], who combined analytical theory with the Lagrangian finite element code PRONTO 3D developed at Sandia. In our work, we have instead used the commercially available hydrocode Autodyn-3D [2]. This hydrocode is very well suited for implementation of such a method since the user has access to the physical variables during each time step, and these can be manipulated as desired using self-programmed user subroutines. At the same time we retain all the useful features of the commercial code.

2.1 Overview

In brief the method works as follows:

The projectile is modelled using a Lagrangian processor. The target is not modelled explicitly. but instead a boundary condition is assigned to each of the projectile surface cells. This boundary condition is not a standard boundary condition, but is linked to our own user subroutine. Given the projectile velocity and the assumed material properties of the virtual target, for each step in the simulation, the user subroutine calculates the stress on that cell (from analytical theory) and applies it to the projectile surface. This is illustrated in Figure 2.1.
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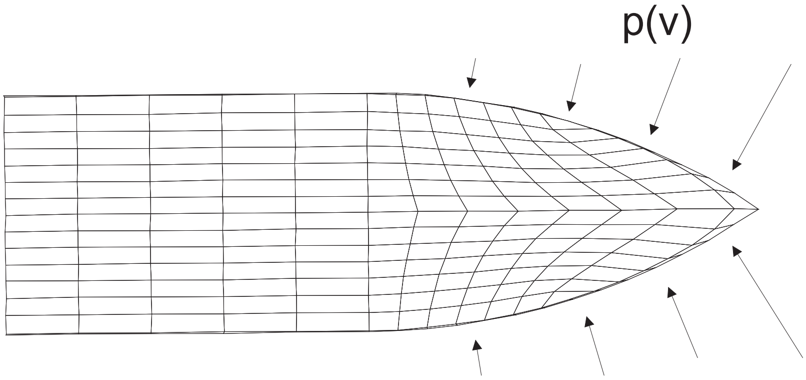

Figure 2.1: The projectile is modelled using a finite element mesh, whereas the target is modelled as a stress boundary condition applied to the projectile surface elements.

2.2 Subtasks performed during each cycle

During each cycle, for each cell on the projectile surface, the following subtasks are performed:

1. The outward pointing normal vector of each cell is found. (Since the projectile may bend, this has to be done every cycle.)
2. The node velocity in the direction of the normal vector for each of the four corner nodes is calculated. The average value is defined as the "cell velocity".
3. The distance to any free target surfaces is found.
4. The fraction of the cell currently inside the target is found.
5. Using the information obtained above combined with analytical theory, the final radial stress on the cell is calculated and applied.

Points 1 and 2 are "programming technical" and are not discussed further here.

Point 3 is not as trivial as one might first think. In fact, it is not even obvious how to define the distance to a free boundary, as illustrated in Figure 2.2. Both the perpendicular distance and the shortest distance to the surface can in principle be used. Currently we use the former approach, but this can be changed if there is evidence that the other approach is better. The calculation itself is carried out using a binary search algorithm.

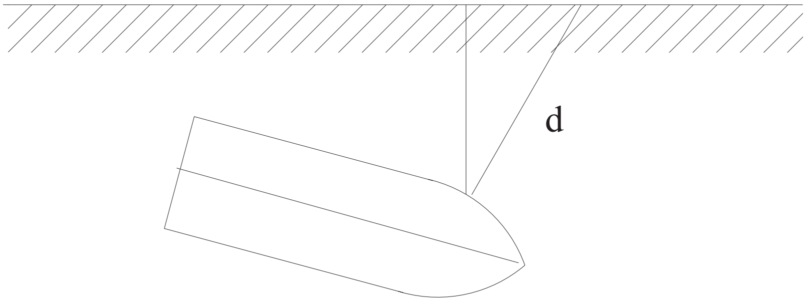

Figure 2.2 Two possible ways of defining the distance to a free boundary.

Point 4 makes sure that the pressure is turned on gradually as the cell enters the target. This is achieved by multiplying the stress with the number of nodes inside the target divided by four. This leads to a stepwise rise in the pressure, which is probably a close enough approximation for all purposes.

In Point 5 we use the obtained information to estimate the stress on the cell through an analytical calculation. In principle, any analytical penetration theory can be implemented. In our implementation we have used the approach based on cavity expansion theory (CET). This theory is briefly outlined in the next chapter. For more details the reader is referred to Teland [4].

3 CAVITY EXPANSION THEORY

Cavity expansion theory (CET) is often used to model penetration of rigid projectiles. In this theory the force on a penetrating projectile is estimated from the stress required to expand a cavity in the target material at a given velocity.

The first step is therefore to find a relationship between the radial stress σ_r and expansion velocity *u* of a cavity. This will depend on the material model, but for simple models an exact solution is possible. In an infinite medium, the radial stress can often be written on the following form:

$$\sigma_r\left(u\right) = A + Bu + Cu^2 \tag{3.1}$$

The constants *A, B* and *C* will depend on the applied material model. For more complex material models, the CET equations can not be solved analytically and a numerical solution is necessary. However, it turns out that Equation (3.1) is usually still a very good approximation. The constants *A, B* and *C* can then by found by curve fitting to the numerical solution.

The next step is to use Equation (3.1) to estimate the stress σ_n^p on a projectile penetrating the same material. The following relationship seems to be generally accepted:

$$\sigma_n^p(v) = \sigma_r\left(\vec{v}\cdot\vec{n}\right) = A + B\left(\vec{v}\cdot\vec{n}\right) + C\left(\vec{v}\cdot\vec{n}\right)^2 \tag{3.2}$$

where $\vec{n}$ is the normal vector of the projectile surface. Analytically, a total force can now be found by integrating the stresses over the projectile surface, whereas in our combined approach, the stress on each boundary cell is calculated and applied individually.

It must be noted that although CET in itself can be an exact theory, the application to penetration is just an estimate. There is a lot of empirical evidence that the approach works well for impact on relatively soft material, though. However, possible problems with applying CET to penetration are further discussed in [5].

4 BOUNDARY EFFECTS

The method as described above does not account for boundary effects. These can be included by multiplying the resistive pressure on the projectile by a so-called decay function *α(v,d)*:

$$\sigma_n^{red}\left(v,d\right) = \alpha\left(v,d\right)\sigma_n^p\left(v\right) \qquad 0 < \alpha \le 1 \tag{4.1}$$

The force is then calculated accordingly using the new expression for the resistive pressure.

The decay function *α(v,d)* could be found from CET as the ratio between the radial stress on an expanding cavity in a finite and infinite medium. Unfortunately, the CET equations for a finite medium can only be solved explicitly for very simple material models. In our implementation we have used a dynamic expression derived in [6], assuming the material to be perfectly plastic and incompressible.

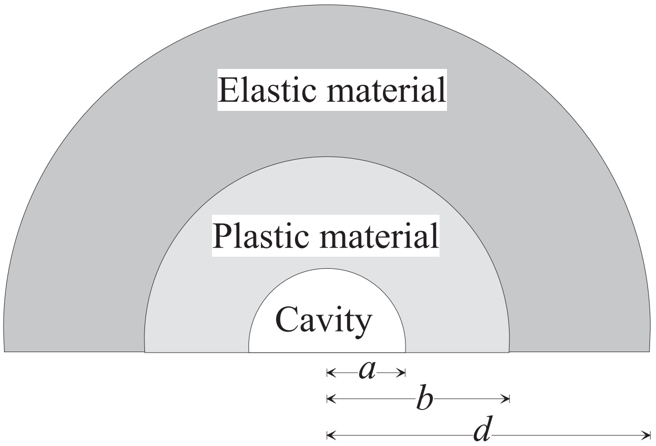

Figure 4.1 The physical problem in cavity expansion is to find the pressure at the cavity surface (r=a) as the cavity expands, while boundary conditions apply at the elastic-plastic boundary (r=b) and the elastic boundary (r=d).

We then obtain:

$$\alpha\left(d,v\right)=\frac{\dfrac{2Y}{3}\left(\ln\left(\left(\dfrac{b}{a}\right)^{3}\right)+1-\left(\dfrac{b}{a}\right)^{3}\left(\dfrac{a}{d}\right)^{3}\right)+\dfrac{\rho v^{2}}{2}\left(3+\left(\dfrac{a}{d}\right)^{4}-\dfrac{4a}{d}\right)}{\dfrac{2Y}{3}\left(1+\ln\left(\left(\dfrac{b}{a}\right)^{3}\right)\right)+\dfrac{3\rho v^{2}}{2}},\ d\geq b$$

$$\alpha\left(d,a,v\right)=\frac{2Y\ln\left(\dfrac{d}{a}\right)+\dfrac{\rho v^{2}}{2}\left(3+\left(\dfrac{a}{d}\right)^{4}-\dfrac{4a}{d}\right)}{\dfrac{2Y}{3}\left(1+\ln\left(\left(\dfrac{b}{a}\right)^{3}\right)\right)+\dfrac{3\rho v^{2}}{2}},\ d<b \qquad (4.3)$$

$$\frac{b}{a}=\left(\frac{2G}{Y}\right)^{1/3}$$

The various geometrical parameters are defined in Figure 4.1. The cavity radius *a* is identified with the projectile radius in the application to penetration. We see then that the decay function *α(v,d)* is completely determined once the distance to the target boundary *d* is known.

Several alternative decay functions exist depending on the approximation used. However, comparison of the various expressions has shown that in practise the differences are usually small.

5 EXAMPLE OF USE: YAWED IMPACT SENSITIVITY STUDY

As a demonstration of the utility of the combined analytical and numerical approach, we will apply it to a parametric study for the case of yawed impact. We have not yet completed experiments for comparison, so at the moment these simulations should be regarded only as a demonstration of the potensial capability of the hybrid approach. The main point is that this study was completed in a few hours, whereas a similar parametric study using full 3D Autodyn simulations would have taken months.

Yawed impact means that the symmetry axis of the projectile has a different direction than the velocity vector $\vec{v}$. The yaw angle θ is defined as the angle between them. This is illustrated in Figure 5.1.

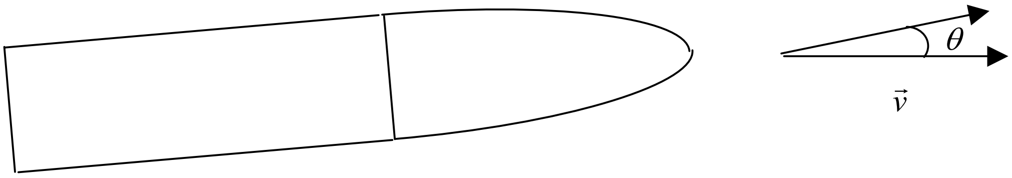

Figure 5.1: Definition of yaw angle.

Most of the earlier work on yawed impact has been concentrated on eroding long rods impacting at very high velocities. Our approach is not applicable to this case because the assumption of rigid projectile does not hold due to the occurrence of mushrooming and erosion of the projectile. Instead we will study impact at velocities that are sufficiently low for deformation of the projectile nose not to occur. However, we will allow bending of the main projectile body. Although in principle this means that the projectile is not rigid, this does not turn out to be a problem.

5.1 Projectile

Simulations were carried out for two different projectiles. Both had a diameter of 7.11 mm, but the first was quite long with a length/diameter ratio of 8.34 and a mass of 20.8 g. The exact dimensions of the were L=59.3 mm, l=11.8 mm, and $2a$ = 7.11 mm (see Figure 5.2). This is the same projectile geometry as was used in [6]. In [3] we tested our combined analytical and numerical approach on this projectile for oblique impact and obtained good results compared with experiments in [6].

The other projectile was a scaled down version with a length/diameter ratio of 4.17, having dimensions L=29.65 mm, l=5.9 mm, 2a = 7.11 mm and mass 10.4 g.

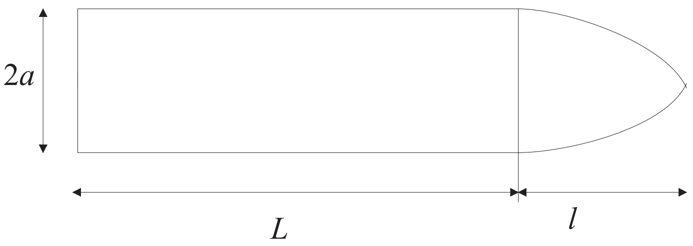

Figure 5.2: Dimensions of the projectile.

The projectile material model was the 4340 steel model in the Autodyn material library.

5.2 Target

For target we assumed a 6061-T6511 aluminium cylinder. Warren and Forrestal [7] have performed lots of work to determine the constants *A, B* and *C* for such a material. Their result was $A = 5.0394Y$, $B = 0.983\sqrt{\rho Y}$ and $C = 0.9402\rho$, with the material parameters given in Table 5.1.

Table 5.1 Material parameters for the aluminium target

Yield stress *Y*	276 *MPa*
Youngs modulus *E*	69 *GPa*
Poisson ratio	0.33
Density	2.71 g/cm^3

The target size was made large enough to avoid boundary effects influencing on the results, with radius of 100 cm and length/width of 200 cm.

6 RESULTS

Simulations were run for different angles of yaw at two velocities, 500 m/s and 1000 m/s. A number of simulations were completed within a few hours, significantly less than what would have been possible with full Autodyn-3D simulations.

The results are given in Tables 6.1 and 6.2 and are shown graphically in Figures 6.1 and 6.2.

As expected, yaw seems to be a much larger problem at high impact velocities. We note for instance that for the 20.8 g projectile, a yaw of 2 degrees results in a decrease of 12% at 1000 m/s, but only around 1% at 500 m/s.

Further parameter studies, varying the projectile length/diameter etc, can now easily be performed using the combined analytical and numerical approach described here.

20.8 g	500 m/s		1000 m/s	
Yaw angle	Pen. depth (cm)	Rel. pen.	Pen. depth (cm)	Rel. pen
0	5.17	1.0	17.82	1.0
0.5	5.16	0.998	17.73	0.995
1.0	5.15	0.996	17.22	0.966
1.5	5.14	0.994	16.48	0.925
2.0	5.12	0.990	15.66	0.879
2.5	5.09	0.985	15.14	0.850
3.0	5.05	0.977	14.47	0.812
3.5	5.00	0.967	13.63	0.765
4.0	4.95	0.957	12.88	0.729
4.5	4.88	0.944	12.26	0.688
5.0	4.81	0.930	11.80	0.662
6.0	4.66	0.901	11.14	0.625
7.0	4.50	0.870	10.60	0.595

Table 6.1: Simulation results for 20.8 g projectile.

10.4 g	500 m/s		1000 m/s	
Yaw angle	Pen. depth (cm)	Rel. pen.	Pen. depth (cm)	Rel. pen
0	2.49	1.00	7.55	1.0
0.5	2.49	1.00	7.54	0.999
1.0	2.48	0.996	7.53	0.997
1.5	2.48	0.996	7.50	0.993
2.0	2.47	0.992	7.45	0.987
2.5	2.47	0.992	7.39	0.979
3.0	2.46	0.988	7.32	0.970
3.5	2.44	0.980	7.23	0.958
4.0	2.43	0.976	7.13	0.944
4.5	2.41	0.968	7.03	0.931
5.0	2.40	0.964	6.93	0.918
6.0	2.35	0.944	6.72	0.890
7.0	2.31	0.928	6.52	0.864

Table 6.2: Simulation results for the 10.4 g projectile.

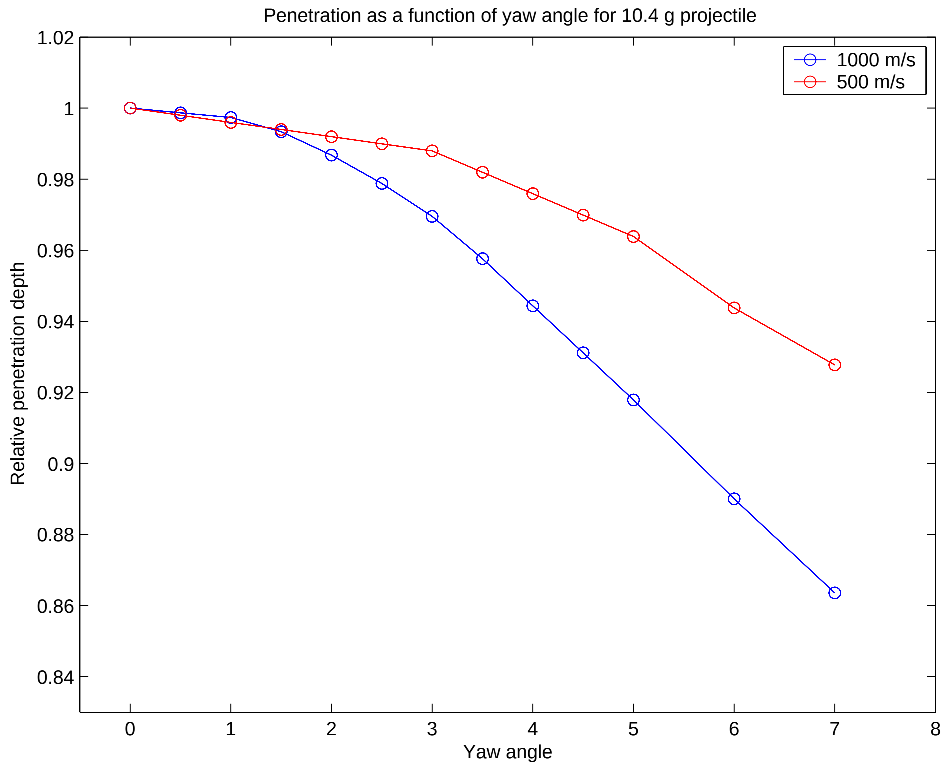

Figure 6.1: Penetration as a function of yaw angle for 10.4 g projectile.

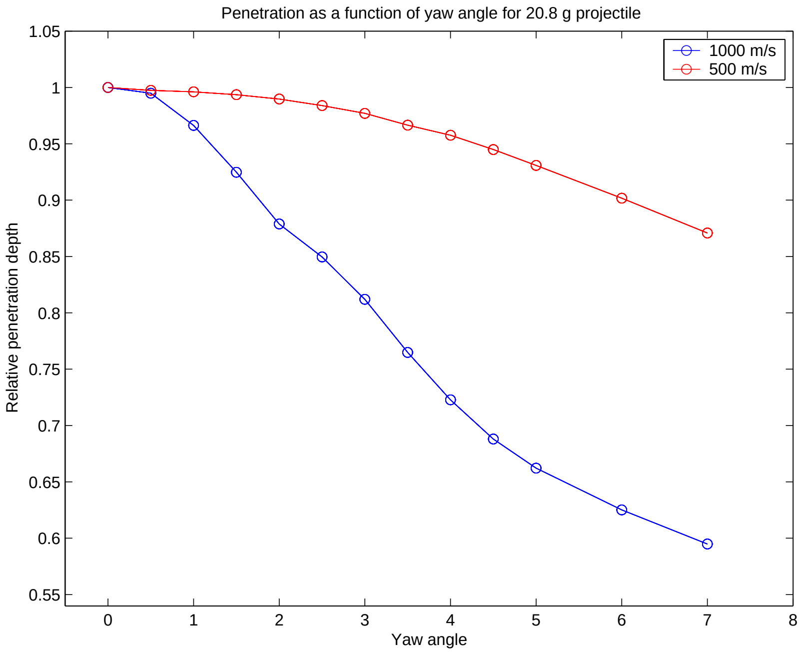

Figure 6.2: Penetration as a function of yaw angle for 20.8 g projectile.

7 CONCLUSION

A hybrid approach for performing simulations of non-normal impact has been implemented. In this article it was applied to the problem of yawed penetration, and it was seen that parameter studies of penetration depth as a function of yaw angle could easily be performed.

As expected, the penetration depth was much more sensitive to yaw at a high impact velocity than at low impact velocity.

It is believed that the approach described here can be a very powerful tool for calculations of penetration. The method is especially useful for performing sensitivity studies and to perform preliminary simulations before doing full 3D simulations.

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