Space-based AIS - theoretical considerations and system parameter optimization

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8) ABSTRACT

The recently introduced Universal Automatic Identification System (AIS) is a ship-to-ship and ship-to-shore reporting system based on broadcasting of messages in the maritime VHF band. The AIS messages could also be received from space, and this report studies the behaviour of such a space-based AIS system with respect to the relevant parameters. The necessary theory is developed, and a method for optimizing the system with respect to ship detection probability, observation time, and number of ships that the system can handle is presented. Results show that the system can be optimized to handle 10 000 ships within the field of view with a ship detection probability of 99% if the ship reporting interval is set to 5.5 min, and the observation time is at least 36 min. In principle, the system could be optimized to handle any number of ships if the appropriate ship reporting interval is chosen, and the observation time is sufficiently long.

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Space-based AIS – theoretical considerations and system parameter optimization

1 INTRODUCTION

Internationally, there is a growing need to develop a global maritime surveillance capability. This stems from increased levels of hazardous cargo transports, smuggling of goods and humans, and growth in global terrorism. New cooperative systems for ship reporting are now being implemented to meet emerging requirements for detection, identification, and tracking.

One such system is the Universal Shipborne Automatic Identification System (AIS). AIS is a ship-to-ship and ship-to-shore reporting system based on broadcasting of messages in the maritime VHF band. The AIS messages could also be received from space, and the space-based AIS concept has been studied in several previous papers and reports (1)-(5). However, a space-based AIS sensor would see a much larger number of ships within its field of view than the system was originally designed for. As a result interference problems could occur, and some ships may not be detected. The ship detection probability for space-based AIS has been studied in two previous reports (2),(3), and an equation for the ship detection probability has also been derived (3).

This report studies in detail the behaviour of the space-based AIS system with respect to the relevant parameters. The necessary theory is developed, and a method for optimizing the system is presented. For given requirement to the ship detection probability and the number of ships that the system must be able to handle, the system can be optimized for shortest possible observation time by choosing the optimum ship reporting interval. The equation for the optimum ship reporting interval is derived together with the equation for the corresponding minimum required observation time. Results show that the system can be optimised to handle 10 000 ships within the field of view with a ship detection probability of 99% if the ship reporting interval is set to 5.5 min and the observation time is at least 36 min. In principle, the system could be optimised to handle any number of ships if the appropriate ship reporting interval is chosen, and the observation time is sufficiently long.

Chapter 2 presents the AIS system, while Chapter 3 gives a brief overview of the space-based AIS concept. Chapter 4 presents the theory, Chapter 5 presents results from the calculations, and Chapter 6 discusses the results. Chapter 7 gives a summary, while Appendix A gives details regarding the overlap factor and how to calculate its value.

2 THE AIS SYSTEM

AIS is a new element under the United Nation's SOLAS convention (6). It is a ship-to-ship and ship-to-shore reporting system that operates on two channels in the maritime VHF band. Ships send reports every 2-10 seconds with detailed information about vessel identity, position, heading, nature of cargo, etc. The range is typically 20 nautical miles for ship-to-ship communication and somewhat larger for ship-to-shore communications in coastal waters. Figure 2.1 illustrates the concept.

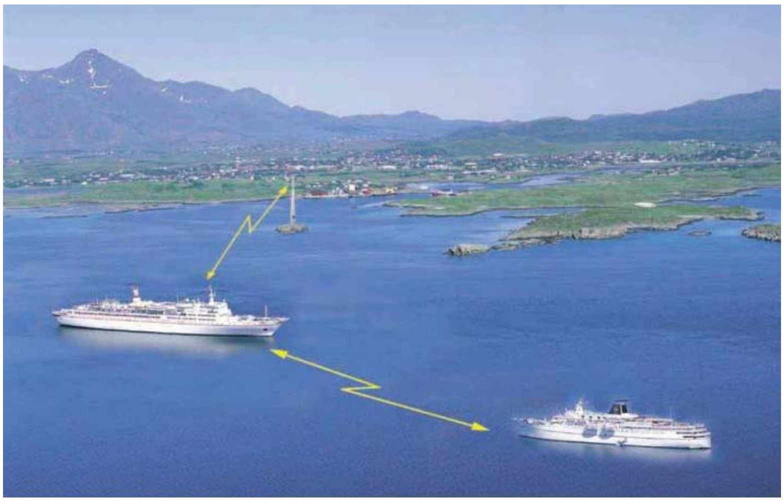

Figure 2.1 The AIS ship-to ship and ship-to-shore concept (Courtesy of Kongsberg Seatex).

The AIS reporting system is based on the broadcasting of digital messages that are entered into a 1 minute long message frame of 2250 message slots (7). The message entry is synchronized to the universal time coordinated (UTC), and the length of each message is limited to 256 bits. The two VHF maritime mobile channels 87B (AIS1) and 88B (AIS2) are allocated to AIS. Messages are broadcasted alternately on the two channels giving the system a total capacity of 4500 message slots per minute. The reporting between ships within communication range (~20 nm) is organized by a Self-Organizing Time Division Multiple Access (SOTDMA)-algorithm to avoid coinciding transmissions, see Figure 2.2.

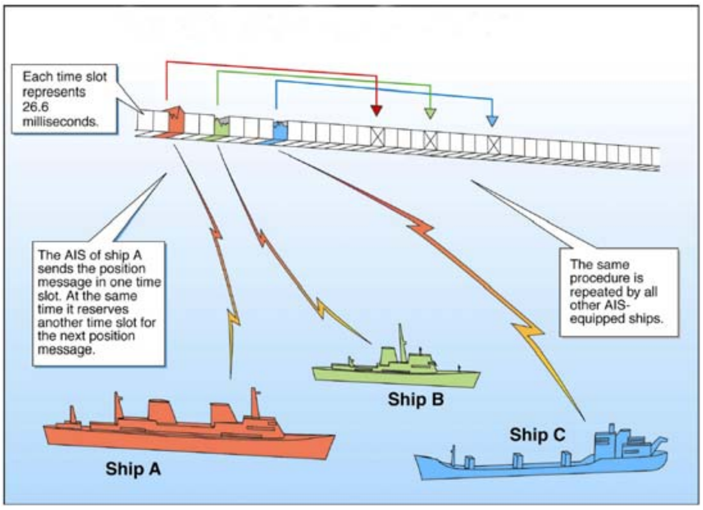

Figure 2.2 The AIS SOTDMA principle.

3 SPACE-BASED AIS

AIS signals can be detected from space by a standard AIS receiver for altitudes up to at least 1000 km (1). However, an AIS sensor in space would cover a much larger area on the ground than the AIS system was originally designed for. The reporting between ships within communication range (~20 nm) is organized by the SOTDMA-algorithm to avoid coinciding transmissions, but from space the AIS sensor will see more than one such organized area as illustrated in Figure 3.1. With many ships within the field of view interference problems will occur, and the AIS messages from some of the ships may not be detected. The ship detection probability for space-based AIS has been studied in two previous reports (2),(3).

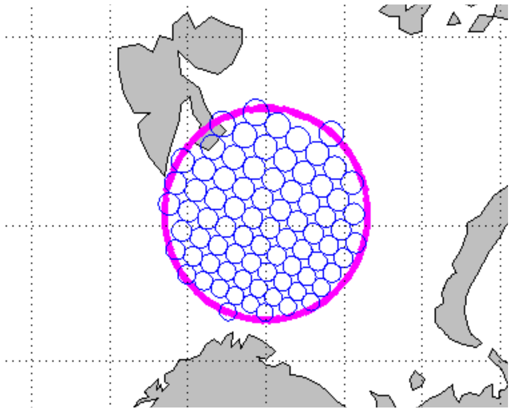

Figure 3.1 AIS sensor's field of view (big red circle) with several organized areas (small blue circles).

4 THEORY

We will in this report study the behaviour of the space-based AIS system with respect to the relevant parameters. The theory and equations necessary for this will be developed in this chapter.

In a previous report (3) the ship detection probability equation for the system was derived. This ship detection probability equation will be used as a basis for the present study. Important parameters when studying the system behaviour are:

P	- Ship detection probability [-]
N_{tot}	- Number of ships within the field of view [-]
T_{obs}	- Observation time [s]
ΔT	- Ship reporting interval [s]
n	- Number of reports transmitted during the observation time [-]
n_{ch}	- Number of channels used for the transmissions [-]
α	- Number of slots per second per channel [s^{-1}]
s	- Overlap factor [-]

For the existing AIS system the number of slots per second per channel is $\alpha = 37.5\ s^{-1}$ (2250 slots per minute/60 seconds) and the number of channels is $n_{ch} = 2$.

The overlap factor s describes the overlap that occurs when AIS messages that are sent in adjacent time slots from ships in different parts of the observation area partly overlap due to differences in the signal path lengths between each of the ships and the AIS sensor. Its value depends on the AIS sensor's altitude and field of view, see Appendix A. Figure 4.1 shows the overlap factor for different AIS sensor altitudes when the sensor has a field of view to the horizon[1].

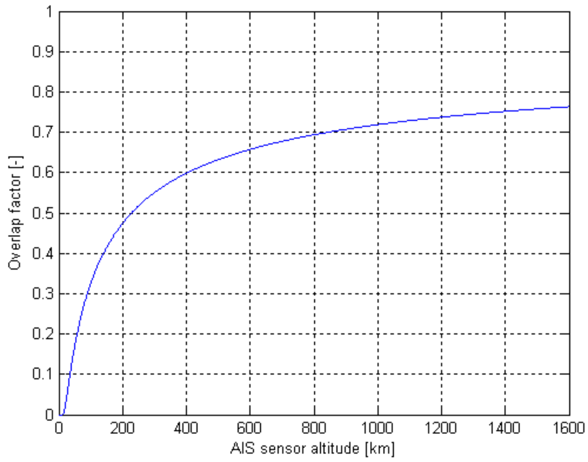

Figure 4.1 The overlap factor as a function of AIS sensor altitude. The sensor has a field of view to the horizon.

The number of reports and the ship reporting interval are related by

$$n = \frac{T_{obs}}{\Delta T} \tag{4.1}$$

and the system can be described by either of these two parameters. However, the system behaviour is different when the number of reports is kept constant compared to when the ship reporting interval is kept constant. We therefore give equations for both cases. All equations derived in this chapter are valid for $n_{ch} \cdot \Delta T \geq 2$ s.

Sections 4.1-4.3 give the equations for the ship detection probability, the observation time, and the number of ships. Sections 4.4-4.5 study the properties of the intersection point for the ship detection probability curves, and Sections 4.6-4.9 show how the system can be optimized. An approximate method for optimizing the system has already been presented in an earlier report (4) and paper (5), but here we develop the full theory for the optimization process.

4.1 Ship detection probability

The ship detection probability P for the system is given by (see Equation (3.33) in (3))

$$P_{\Delta T} = P_s = 1 - \left[1 - \exp\left(-\frac{(1+s)\cdot N_{tot}}{\alpha \cdot n_{ch} \cdot \Delta T}\right)\right]^{\frac{T_{obs}}{\Delta T}} \tag{4.2}$$

[1] The values for the overlap factor given here deviate slightly from the values found in (3). This is due to the fact that the present calculations (Appendix A) are more precise than the calculations performed in (3).

when the ship reporting interval ΔT is used as a parameter, and by

$$P_n = 1 - \left[1 - \exp\left(-\frac{n \cdot (1+s) \cdot N_{tot}}{\alpha \cdot n_{ch} \cdot T_{obs}}\right)\right]^n \tag{4.3}$$

when the number of reports n is used as a parameter. Here N_{tot} is the total number of ships within the field of view, T_{obs} is the observation time, n_{ch} is the number of channels used for the transmissions, α is the number of slots per second per channel, and s is the overlap factor.

4.2 Observation time

The observation time T_{obs} required to detect N_{tot} ships with a ship detection probability P can be found from Equations (4.2) and (4.3) and is given by

$$T_{obs}^{\Delta T} = \Delta T \cdot \frac{\ln\left(1-P\right)}{\ln\left[1 - \exp\left(-\frac{(1+s) \cdot N_{tot}}{\alpha \cdot n_{ch} \cdot \Delta T}\right)\right]} \tag{4.4}$$

when the ship reporting interval ΔT is used as a parameter, and by

$$T_{obs}^{n} = -\left(1+s\right) \cdot \frac{n \cdot N_{tot}}{\alpha \cdot n_{ch} \cdot \ln\left(1-\left(1-P\right)^{\frac{1}{n}}\right)} \tag{4.5}$$

when the number of reports n is used as a parameter. Here N_{tot} is the total number of ships within the field of view, P is the ship detection probability, n_{ch} is the number of channels used for the transmissions, α is the number of slots per second per channel, and s is the overlap factor.

4.3 Number of ships

The number of ships N_{tot} that the system can handle can be found from Equations (4.2) and (4.3) and is given by

$$N_{tot}^{\Delta T} = -\frac{1}{\left(1+s\right)} \cdot \left(\alpha \cdot n_{ch} \cdot \Delta T\right) \cdot \ln\left[1-\left(1-P\right)^{\frac{\Delta T}{T_{obs}}}\right] \tag{4.6}$$

when the ship reporting interval ΔT is used as a parameter, and by

$$N_{tot}^{n} = -\frac{1}{\left(1+s\right)} \cdot \frac{1}{n} \cdot \left(\alpha \cdot n_{ch} \cdot T_{obs}\right) \cdot \ln\left[1-\left(1-P\right)^{\frac{1}{n}}\right] \tag{4.7}$$

when the number of reports n is used as a parameter. Here P is the ship detection probability, T_{obs} is the observation time, n_{ch} is the number of channels used for the transmissions, α is the number of slots per second per channel, and s is the overlap factor.

4.4 Intersection point for the ship detection probability curves - different ship reporting intervals

The intersection point (${}_cN_{tot}^{\Delta T}$, ${}_cP_{\Delta T}$) for the ship detection probability curves for two different ship reporting intervals ΔT_a and ΔT_b can be found by setting

$$P_{\Delta T}(\Delta T_a) = P_{\Delta T}(\Delta T_b) \tag{4.8}$$

where $P_{\Delta T}$ is given by Equation (4.2), and solving for the number of ships ${}_cN_{tot}^{\Delta T}$. This gives

$${}_cN_{tot}^{\Delta T} = -\frac{1}{(1+s)} \cdot \alpha \cdot n_{ch} \cdot \Delta T_a \cdot \ln\left(1-\left[1-\exp\left(-\frac{(1+s)\cdot {}_cN_{tot}^{\Delta T}}{\alpha \cdot n_{ch} \cdot \Delta T_b}\right)\right]^{\frac{\Delta T_a}{\Delta T_b}}\right) \tag{4.9}$$

Note that Equation (4.9) is independent of the observation time T_{obs} . Equation (4.9) contains ${}_cN_{tot}^{\Delta T}$ on both sides of the equality sign and must be solved numerically.

The corresponding ship detection probability ${}_cP_{\Delta T}$ is found by substituting $N_{tot} = {}_cN_{tot}^{\Delta T}$ and $\Delta T = \Delta T_a$ (or ΔT_b) into Equation (4.2).

4.5 Intersection point for the ship detection probability curves - different numbers of reports

The intersection point (${}_cN_{tot}^{n}$, ${}_cP_n$) for the ship detection probability curves for two different numbers of reports n_a and n_b can be found by setting

$$N_{tot}^{n}(n_a) = N_{tot}^{n}(n_b) \tag{4.10}$$

where N_{tot}^{n} is given by Equation (4.7), and solving for the ship detection probability ${}_cP_n$. This gives

$${}_cP_n = 1-\left(1-\left[1-\left(1-{}_cP_n\right)^{\frac{1}{n_b}}\right]^{\frac{n_a}{n_b}}\right)^{n_a} \tag{4.11}$$

Note that Equation (4.11) is independent of the observation time T_{obs} , the number of channels n_{ch} , and the overlap factor s . Equation (4.11) contains ${}_cP_n$ on both sides of the equality sign and must be solved numerically.

The corresponding number of ships ${}_cN_{tot}^n$ is found by substituting $P = {}_cP_n$ and $n = n_a$ (or n_b) into Equation (4.7).

4.6 Optimum number of reports

In order to determine the optimum number of reports n_0 for the system, we must take the first derivative with respect to n of either Equation (4.3) (optimizing for the highest possible ship detection probability), Equation (4.5) (optimizing for the shortest possible observation time) or Equation (4.7) (optimizing for the largest possible number of ships). By setting the first derivative equal to zero we find the expression for the optimum number of reports n_0, which will be the same in each of the three cases mentioned above:

$$n_0 = \frac{(1-P)^{\frac{1}{n_0}} \ln(1-P)}{\left[1-(1-P)^{\frac{1}{n_0}}\right] \cdot \ln\left[1-(1-P)^{\frac{1}{n_0}}\right]} \tag{4.12}$$

Equation (4.12) can be written on the form

$$n_0 = \frac{1}{\ln C} \cdot \ln(1-P) \tag{4.13}$$

where C is a constant that is determined from

$$C = \frac{(1-C) \cdot \ln(1-C)}{\ln C} \tag{4.14}$$

and has the value $C \equiv \frac{1}{2}$. The equation for the optimum number of reports n_0 can then be written

$$n_0 = \frac{1}{\ln 2} \cdot \ln(1-P)^{-1} \tag{4.15}$$

Note that the optimum number of reports n_0 depends only on the ship detection probability P.

4.7 Optimum ship reporting interval

The optimum ship reporting interval ΔT_0 is related to the optimum number of reports n_0 by

$$\Delta T_0 = \frac{T_{obs}^{\min}}{n_0} \tag{4.16}$$

where $T_{obs}^{\min}$ is the minimum required observation time. Substituting Equation (4.5), with $n = n_0$ where n_0 is given by Equation (4.13), for $T_{obs}^{\min}$ in Equation (4.16) gives

$$\Delta T_0 = -\frac{1}{\ln(1-C)} \cdot \frac{(1+s)}{\alpha \cdot n_{ch}} \cdot N_{tot} = \frac{1}{\ln 2} \cdot \frac{(1+s)}{\alpha \cdot n_{ch}} \cdot N_{tot} \tag{4.17}$$

Note that the optimum ship reporting interval ΔT_0 depends only on the number of ships N_{tot} (in addition to the number of channels n_{ch} and the overlap factor s which have fixed values for a given system), and that it *increases linearly* with *increasing* number of ships.

4.8 Optimum message density

The optimum message density q_0, i.e., the optimum number of messages per slot per channel, is given by

$$q_0 = \frac{N_{tot} / \Delta T_0}{\alpha \cdot n_{ch}} \tag{4.18}$$

Substituting Equation (4.17) for ΔT_0 in Equation (4.18) gives

$$q_0 = \frac{-\ln(1-C)}{(1+s)} = \frac{\ln 2}{(1+s)} = 0.693 \cdot \frac{1}{(1+s)} \tag{4.19}$$

Note that the optimum message density q_0 is independent of the ship detection probability P, the number of ships N_{tot}, and the observation time T_{obs}, depending only on the overlap factor s. For given s, i.e., for given AIS sensor altitude and field of view, the optimum message density is equal to a constant.

4.9 Optimization of the system

The system can be optimized so that, for given requirements to ship detection probability and number of ships that the system must be able to handle, the necessary observation time is as short as possible.

The minimum required observation time $T_{obs}^{\min}$ is found directly from Equation (4.16) by substituting Equation (4.15) for n_0 and Equation (4.17) for ΔT_0. This gives

$$T_{obs}^{\min} = \frac{1}{\ln 2} \cdot \Delta T_0 \cdot \ln(1-P)^{-1} = \frac{1}{(\ln 2)^2} \cdot \frac{(1+s)}{\alpha \cdot n_{ch}} \cdot N_{tot} \ln(1-P)^{-1} \tag{4.20}$$

The minimum required observation time depends on both the ship detection probability P and the number of ships N_{tot}, and *increases linearly* with the number of ships (or equivalently, *increases linearly* with the optimum ship reporting interval).

The following algorithm can be used to optimize the system:

1) Determine the number of ships N_{tot} that the system must be able to handle, and the required ship detection probability P.
2) Calculate the optimum ship reporting interval ΔT_0 from Equation (4.17).
3) Calculate the minimum required observation time $T_{obs}^{\min}$ from Equation (4.20).

We will illustrate the use of the algorithm with an example. Assume that we want to optimize the system to handle up to 10 000 ships simultaneously within the field of view ($N_{tot} = 10000$) with a ship detection probability of 99% ($P = 0.99$). We first calculate the optimum ship reporting interval from Equation (4.17) and then the observation time from Equation (4.20). We find that the optimum ship reporting interval is $\Delta T_0 = 5.5\,\text{min}$, and the minimum required observation time is $T_{obs}^{\min} = 36\,\text{min}$. For the calculations we have assumed that two channels are used ($n_{ch} = 2$) and that the overlap factor is $s = 0.7$ (corresponds to an AIS sensor field of view to horizon from 1000 km altitude). We conclude that for the system to be able to handle up to 10 000 ships with a ship detection probability of 99%, the ship reporting interval should be 5.5 min and the observation time at least 36 min.

5 RESULTS

We wanted to study the space-based AIS system's behaviour with respect to relevant parameters.

The following values were used as a basis for the calculations:

Number of ships, N_{tot}:	1450
Ship detection probability, P:	90%
Observation time, T_{obs}:	10 min
Ship reporting interval, ΔT:	10 s
Number of reports, n:	60
Number of channels, n_{ch}:	2
Number of slots per second per channel, α :	37.5 s^{-1}
Overlap factor, s:	0.7

An overlap factor of $s = 0.7$ corresponds to an AIS sensor at about 850 km altitude with field of view to horizon, see Figure 4.1.

One parameter's behaviour with respect to another parameter was studied by varying the second parameter and keeping all other parameters in the system constant. The values listed above constitute one consistent solution set for the system.

Sections 5.1-5.3 present figures that show the behaviour of the ship detection probability, the observation time, and the number of ships with respect to the other parameters in the system. Sections 5.4-5.5 present results for the intersection point for the ship detection probability curves, and Sections 5.6-5.9 present results from the optimization of the system.

5.1 Ship detection probability

5.1.1 Ship detection probability vs number of ships

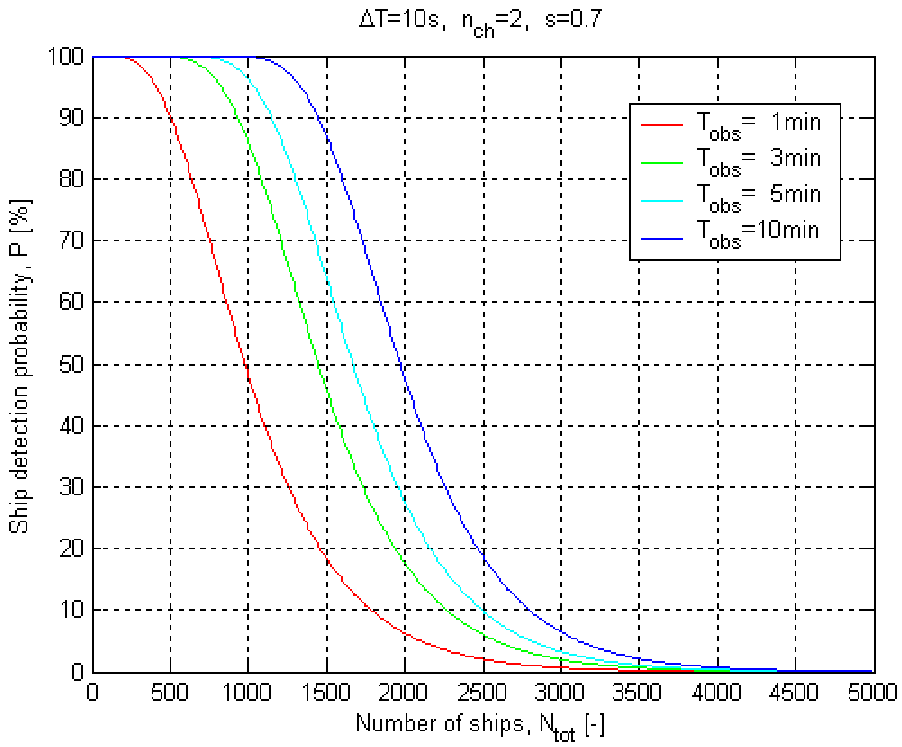

Figure 5.1 Ship detection probability as a function of number of ships for different observation times when the ship reporting interval is 10 s.

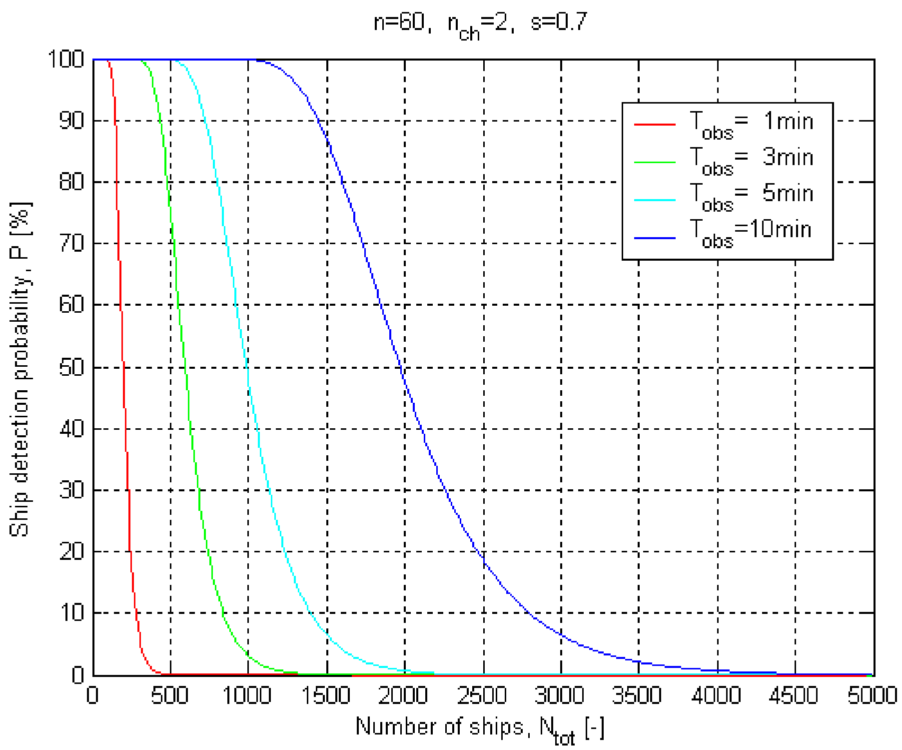

Figure 5.2 Ship detection probability as a function of number of ships for different observation times when the number of reports is 60.

5.1.2 Ship detection probability vs observation time

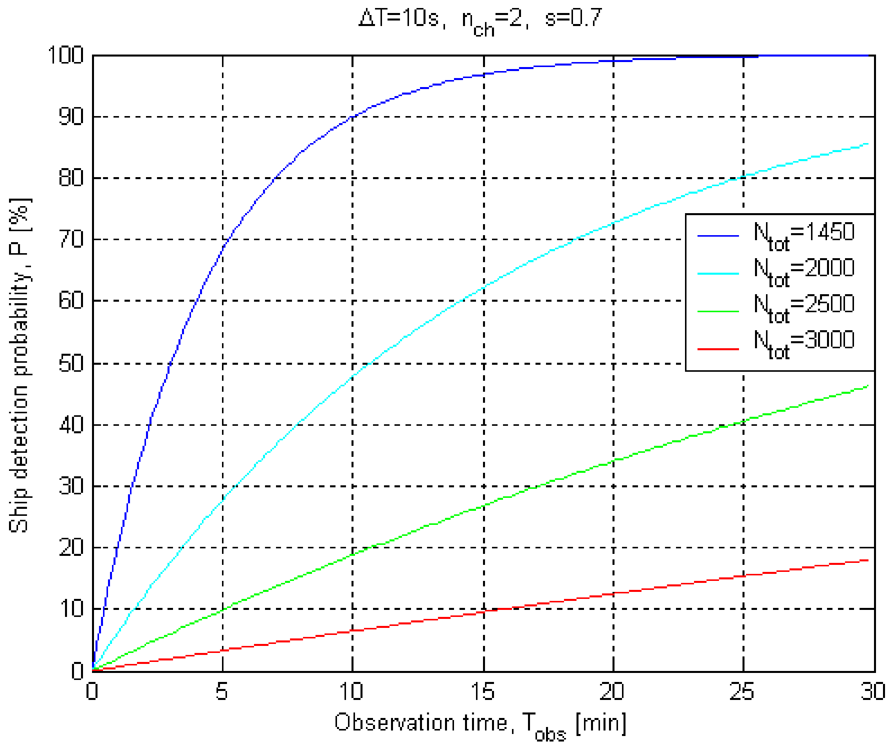

Figure 5.3 *Ship detection probability as a function of observation time for different numbers of ships when the ship reporting interval is 10 s.*

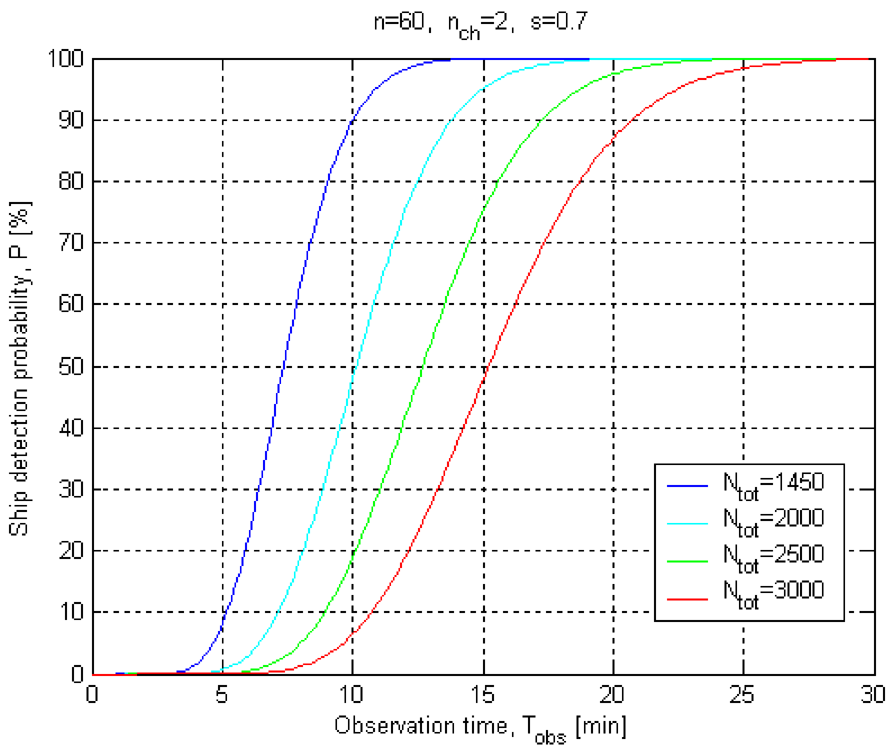

Figure 5.4 *Ship detection probability as a function of observation time for different numbers of ships when the number of reports is 60.*

5.1.3 Ship detection probability vs ship reporting interval

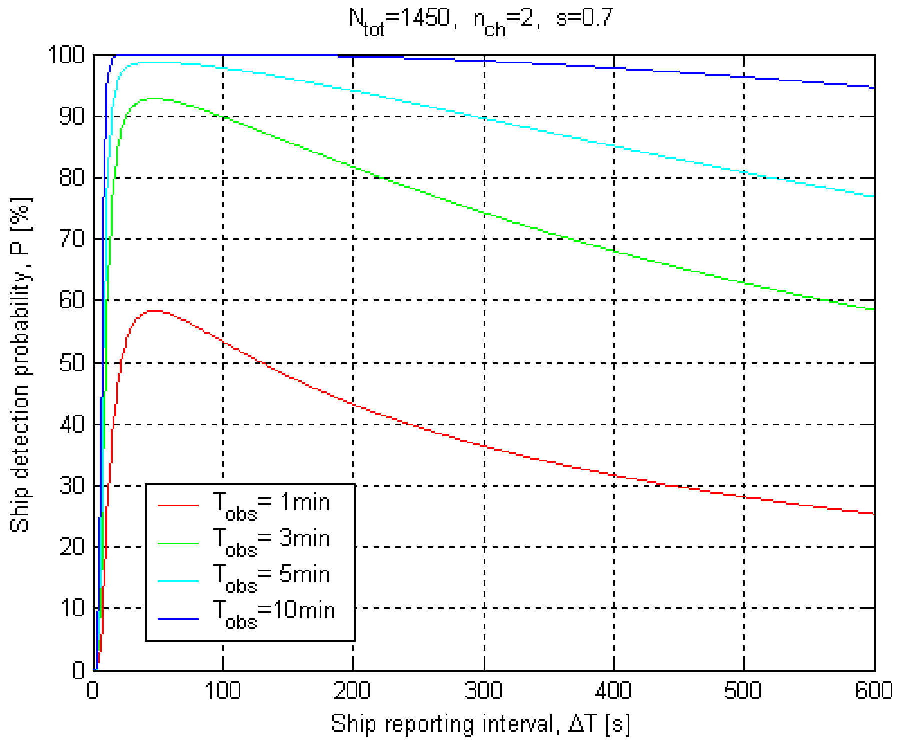

Figure 5.5 Ship detection probability as a function of ship reporting interval for different observation times when the number of ships is 1450.

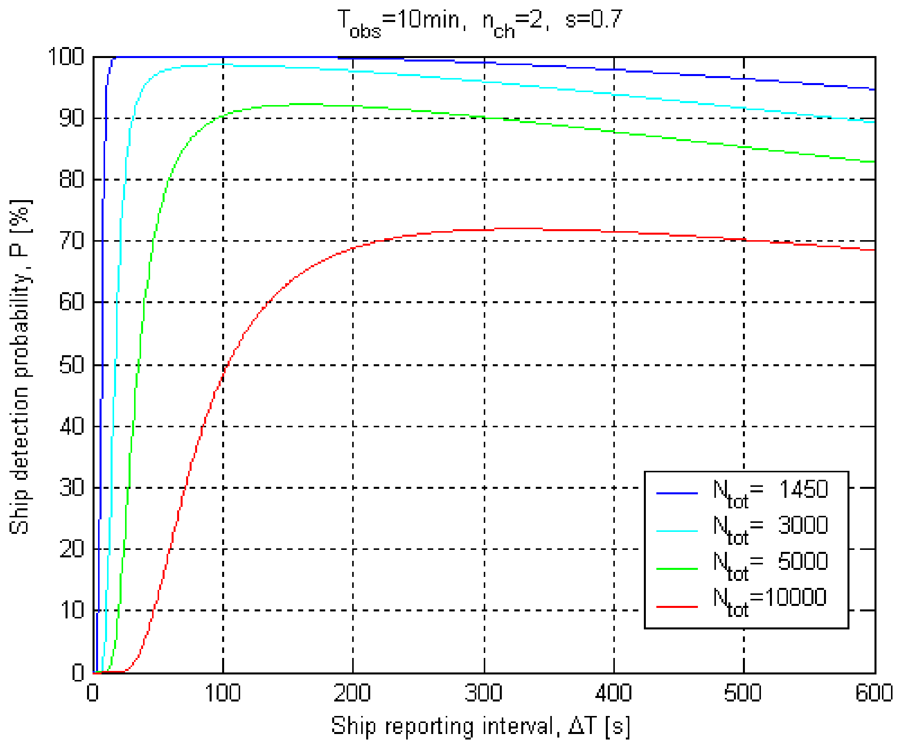

Figure 5.6 Ship detection probability as a function of ship reporting interval for different numbers of ships when the observation time is 10 min.

5.1.4 Ship detection probability vs number of reports

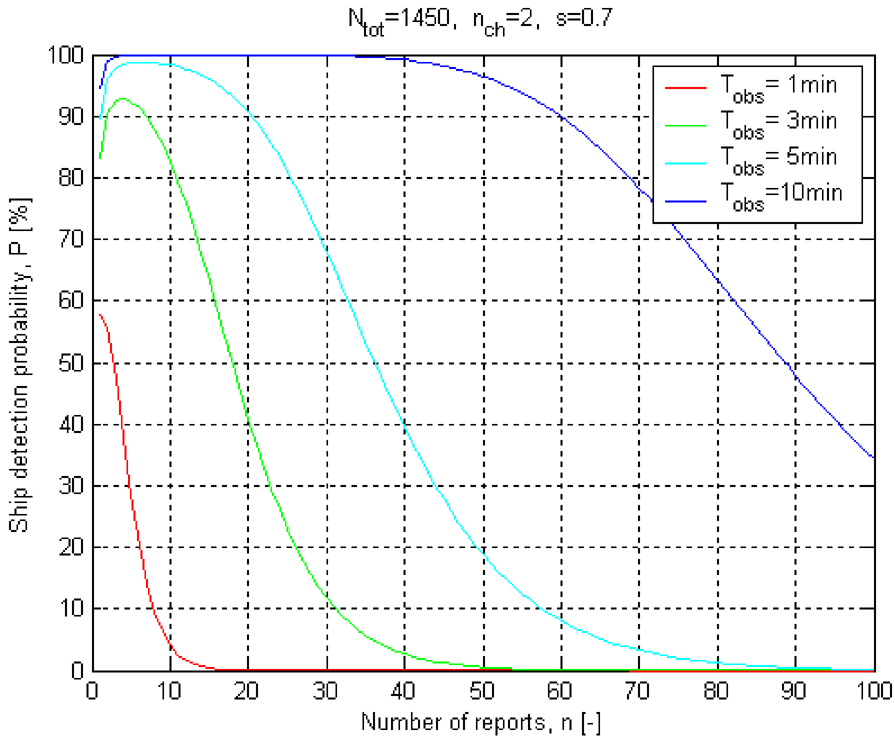

Figure 5.7 Ship detection probability as a function of number of reports for different observation times when the number of ships is 1450.

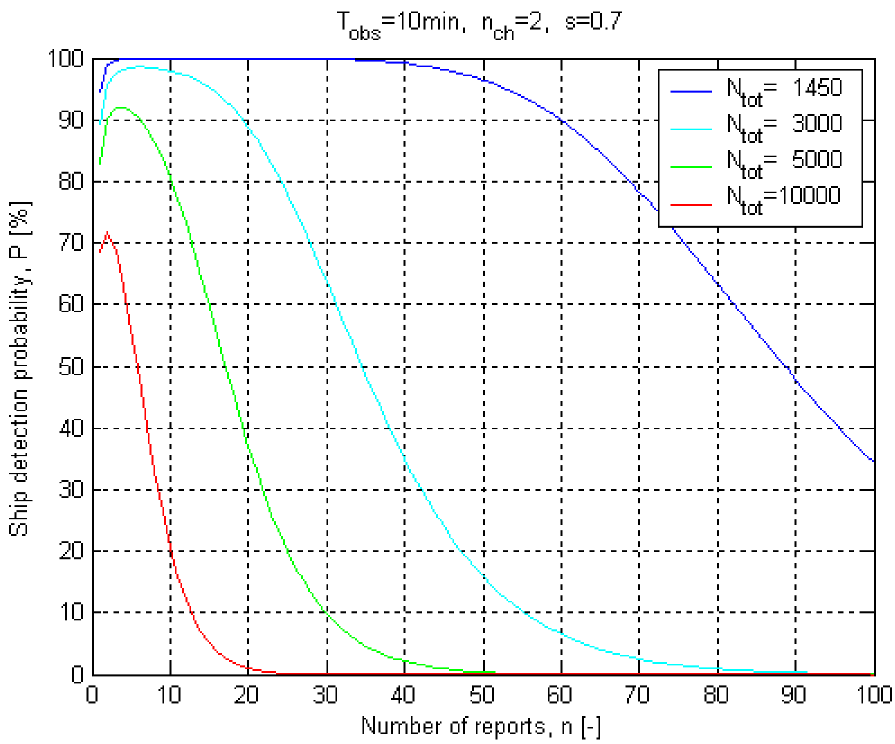

Figure 5.8 Ship detection probability as a function of number of reports for different numbers of ships when the observation time is 10 min.

5.1.5 Ship detection probability vs number of channels

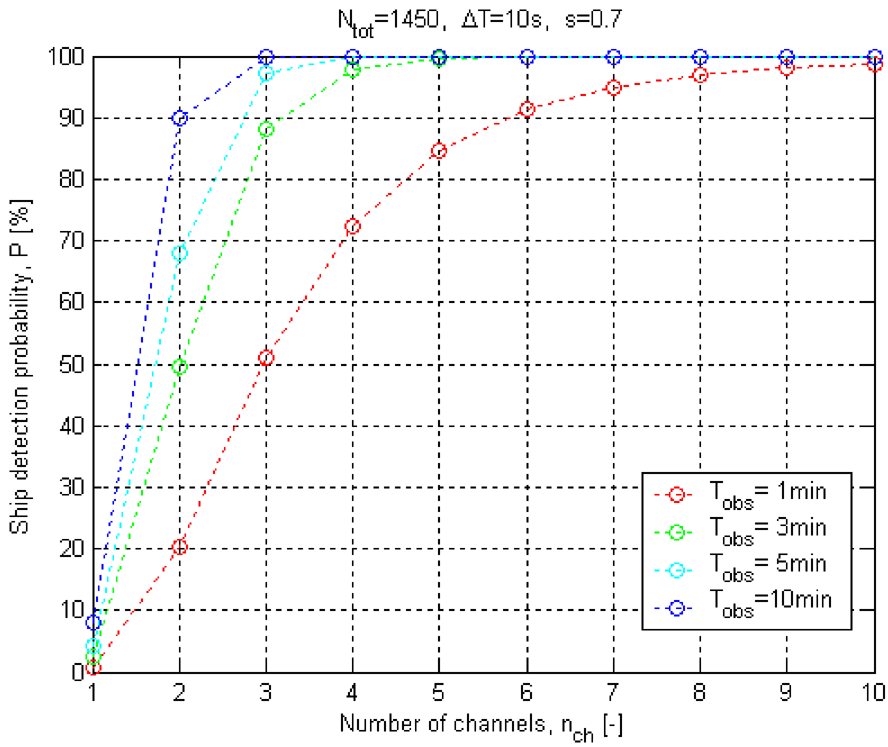

Figure 5.9 *Ship detection probability as a function of number of channels for different observation times when the number of ships is 1450 and the ship reporting interval is 10 s.*

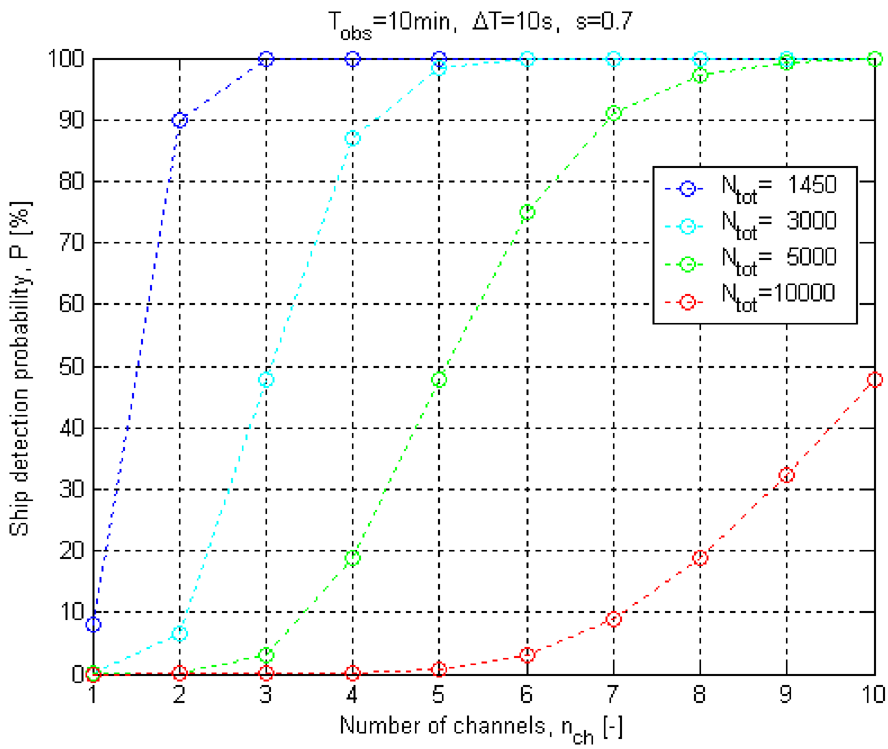

Figure 5.10 *Ship detection probability as a function of number of channels for different numbers of ships when the observation time is 10 min and the ship reporting interval is 10 s.*

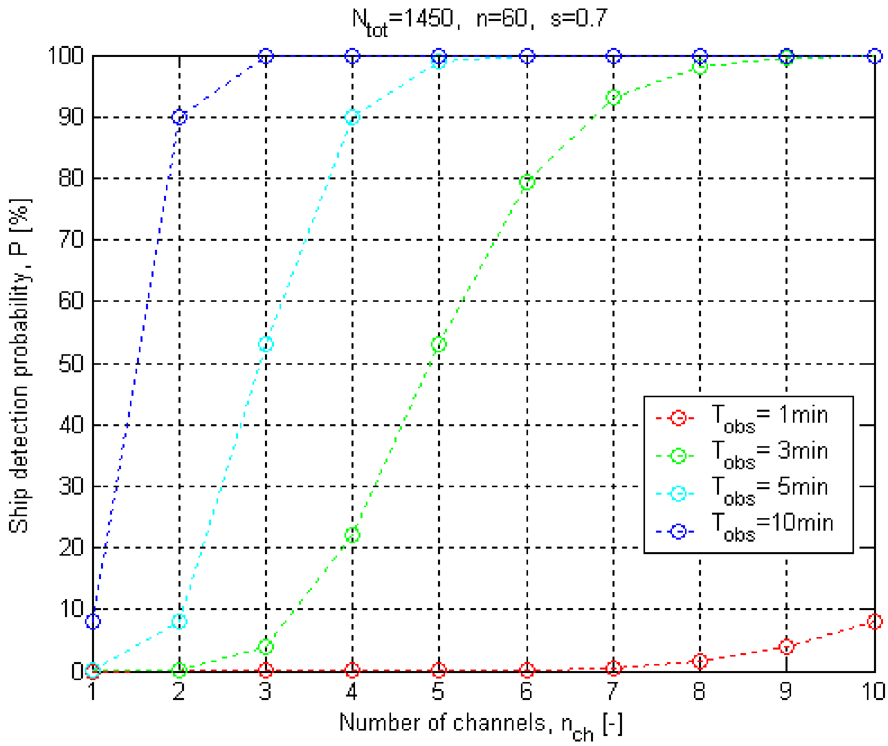

Figure 5.11 Ship detection probability as a function of number of channels for different observation times when the number of ships is 1450 and the number of reports is 60.

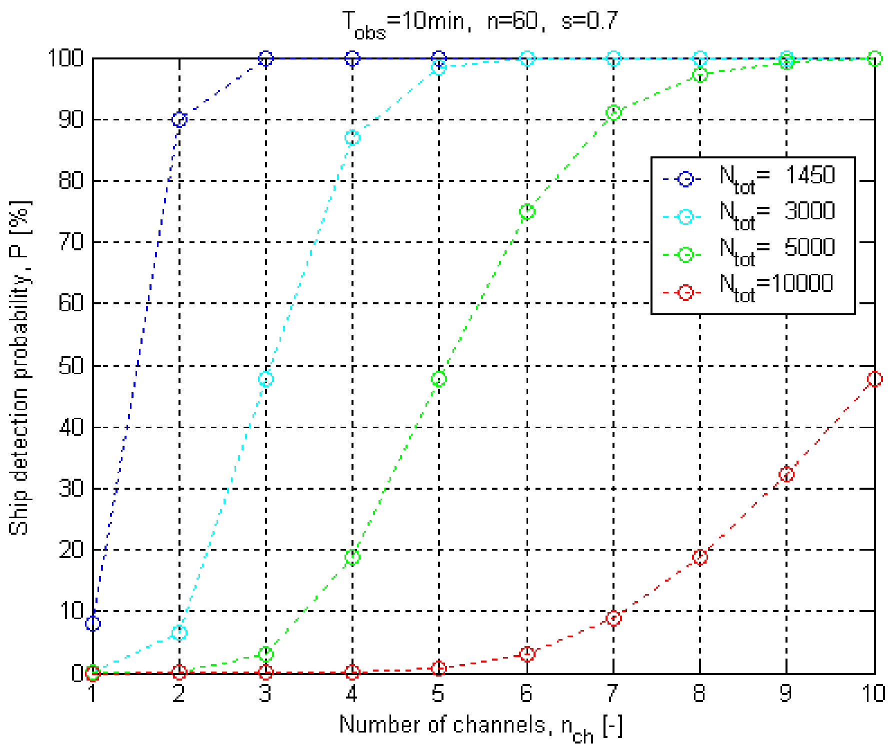

Figure 5.12 Ship detection probability as a function of number of channels for different numbers of ships when the observation time is 10 min and the number of reports is 60.

5.2 Observation time

5.2.1 Observation time vs ship detection probability

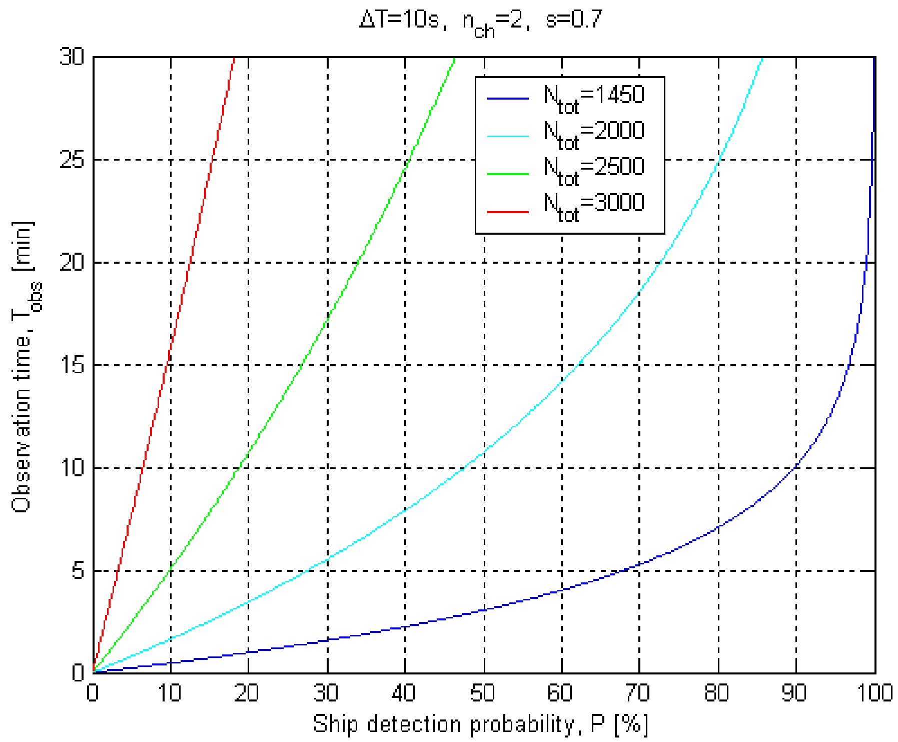

Figure 5.13 Observation time as a function of ship detection probability for different numbers of ships when the ship reporting interval is 10 s.

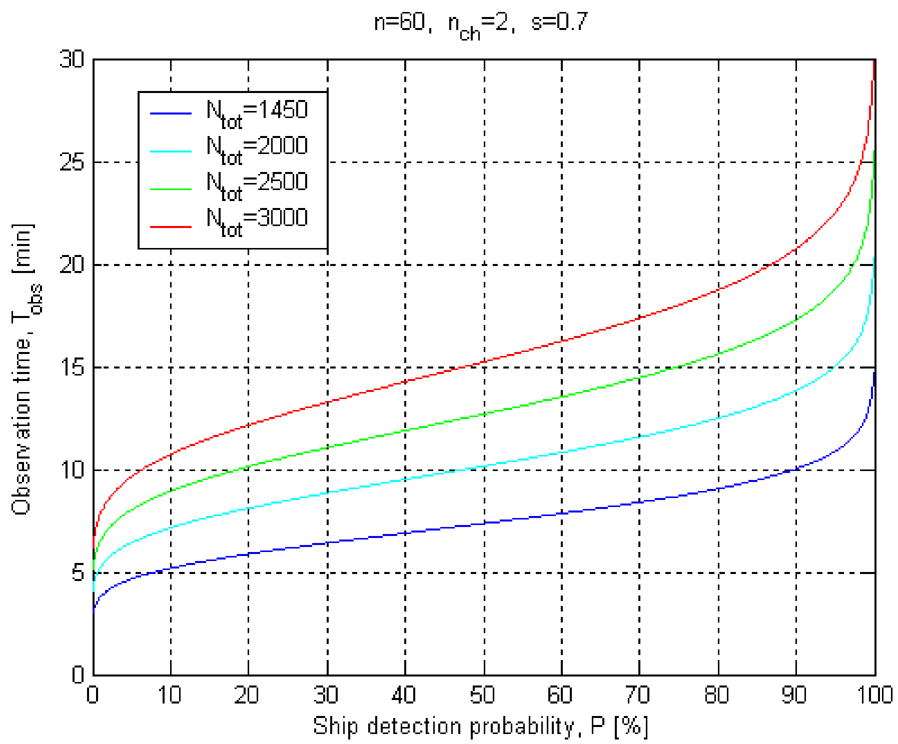

Figure 5.14 Observation time as a function of ship detection probability for different numbers of ships when the number of reports is 60.

5.2.2 Observation time vs number of ships

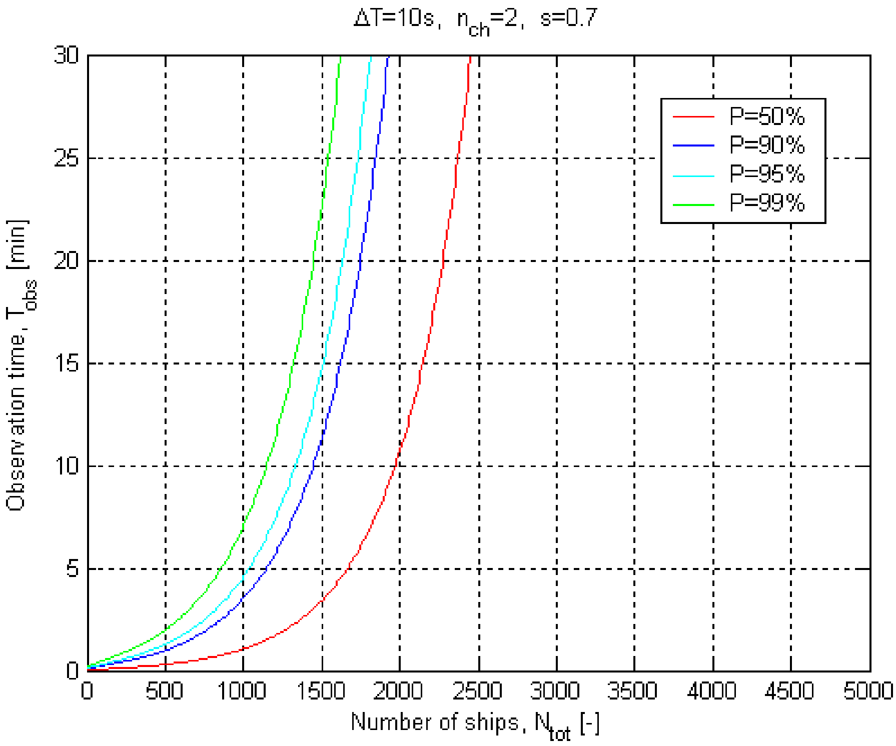

Figure 5.15 Observation time as a function of number of ships for different ship detection probabilities when the ship reporting interval is 10 s.

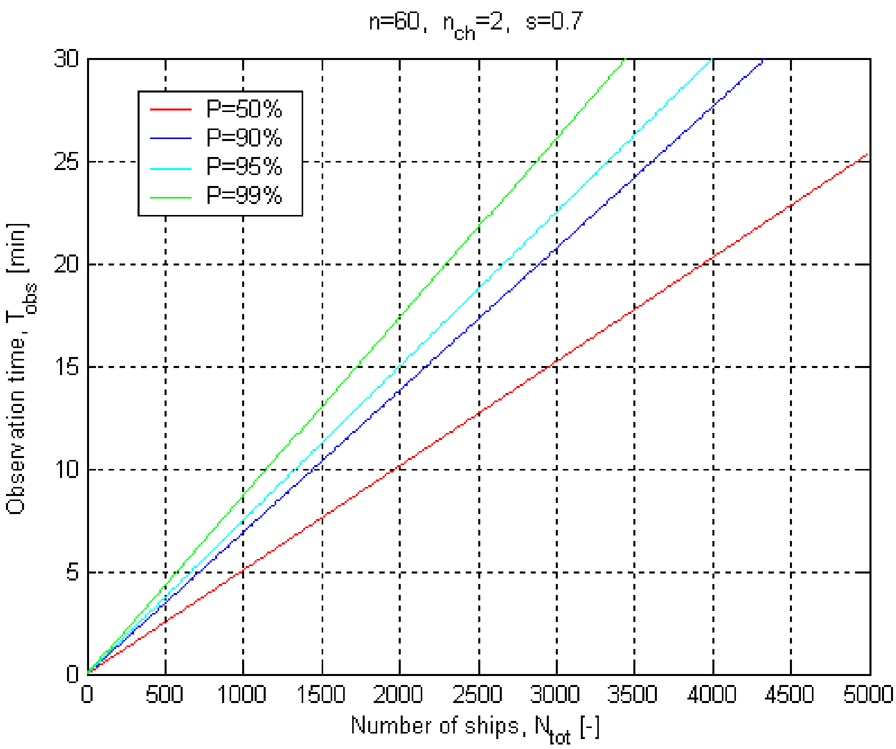

Figure 5.16 Observation time as a function of number of ships for different ship detection probabilities when the number of reports is 60.

5.2.3 Observation time vs ship reporting interval

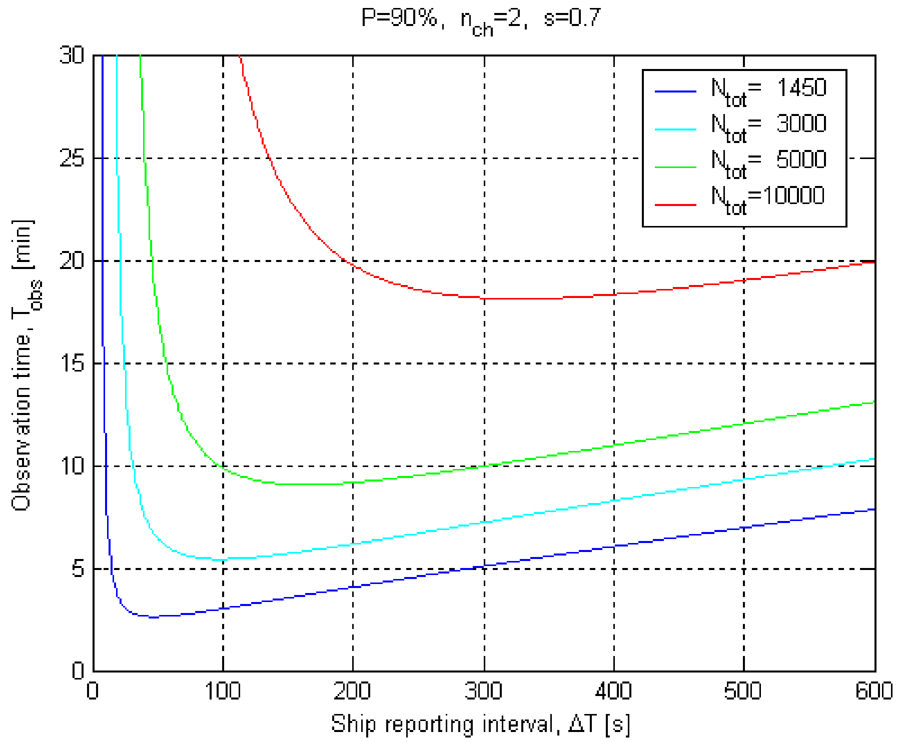

Figure 5.17 Observation time as a function of ship reporting interval for different numbers of ships when the ship detection probability is 90%.

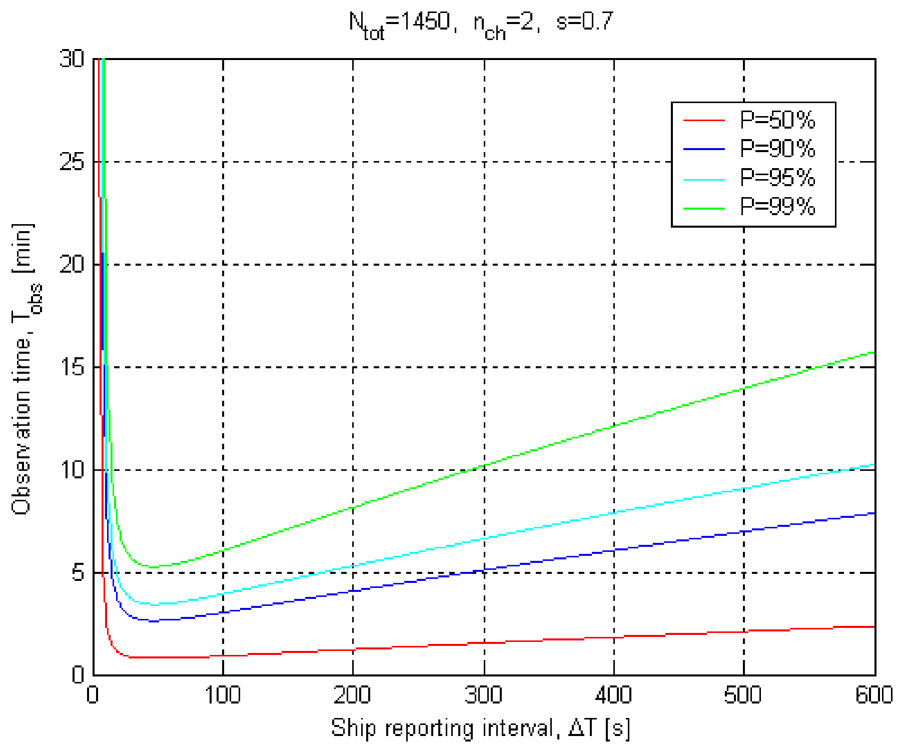

Figure 5.18 Observation time as a function of ship reporting interval for different ship detection probabilities when the number of ships is 1450.

5.2.4 Observation time vs number of reports

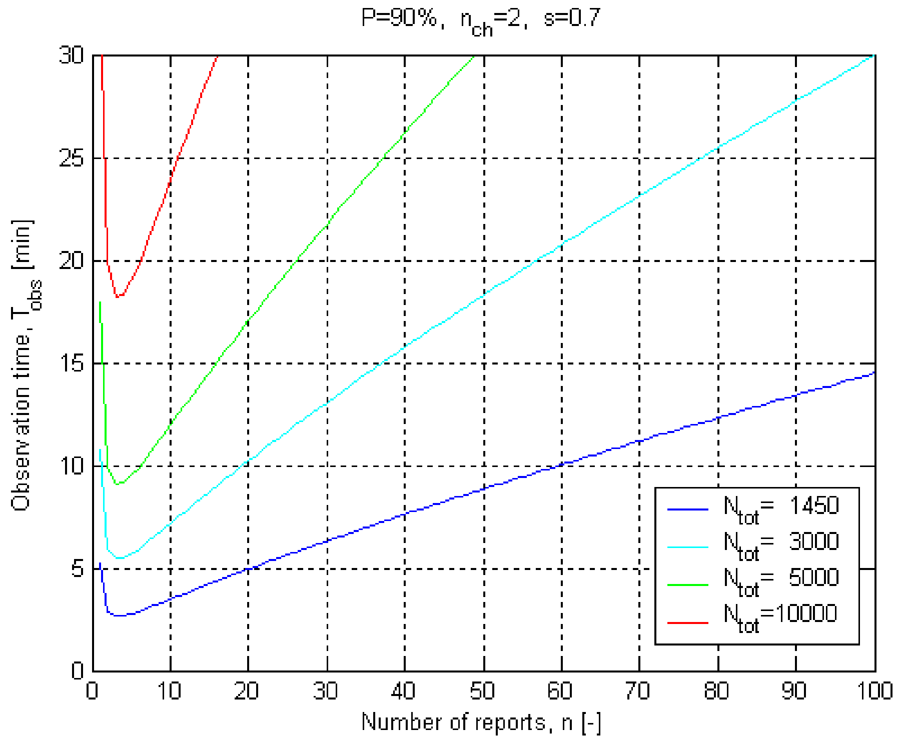

Figure 5.19 Observation time as a function of number of reports for different numbers of ships when the ship detection probability is 90%.

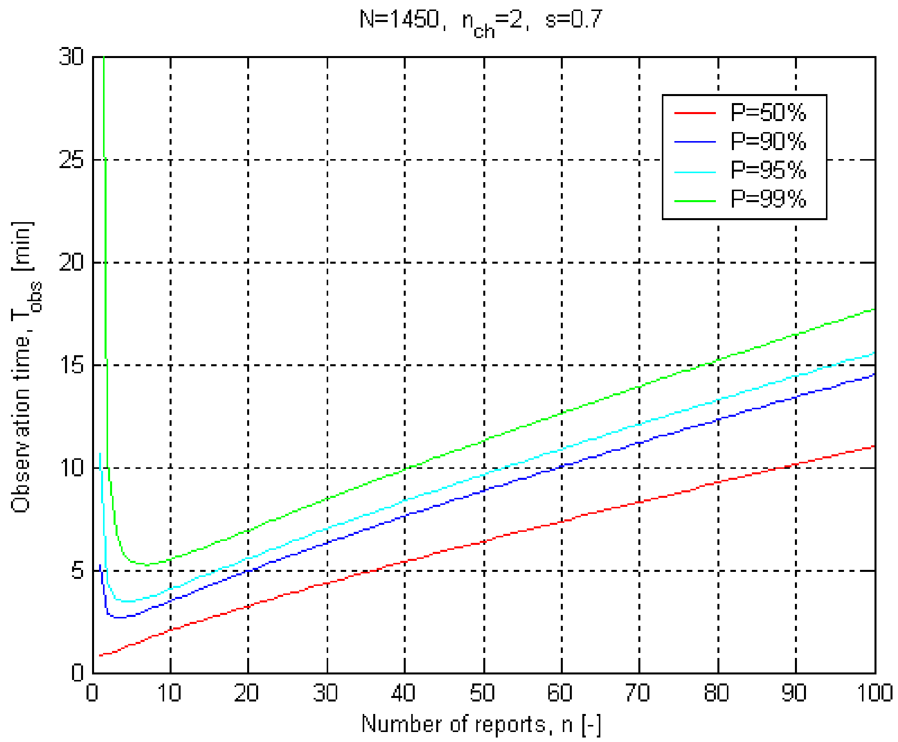

Figure 5.20 Observation time as a function of number of reports for different ship detection probabilities when the number of ships is 1450.

5.2.5 Observation time vs number of channels

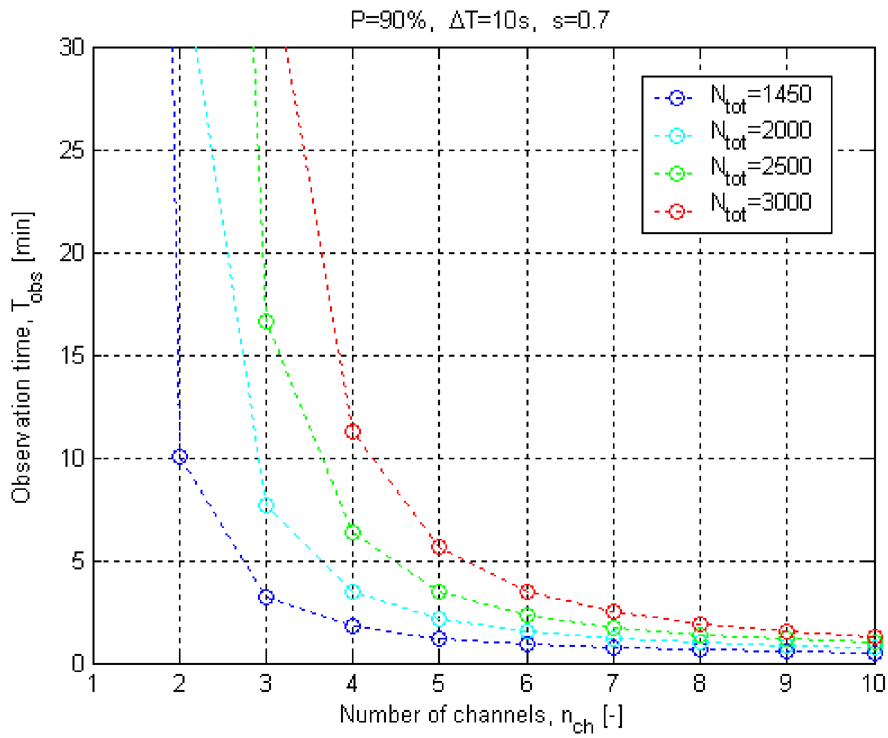

Figure 5.21 Observation time as a function of number of channels for different numbers of ships when the ship detection probability is 90% and the ship reporting interval is 10 s.

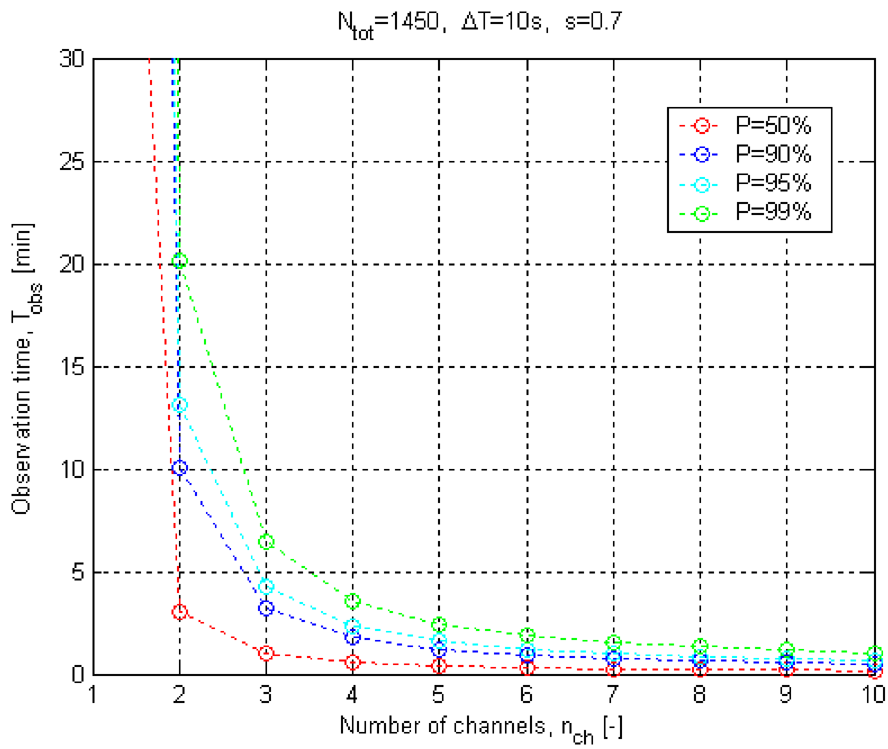

Figure 5.22 Observation time as a function of number of channels for different ship detection probabilities when the number of ships is 1450 and the ship reporting interval is 10 s.

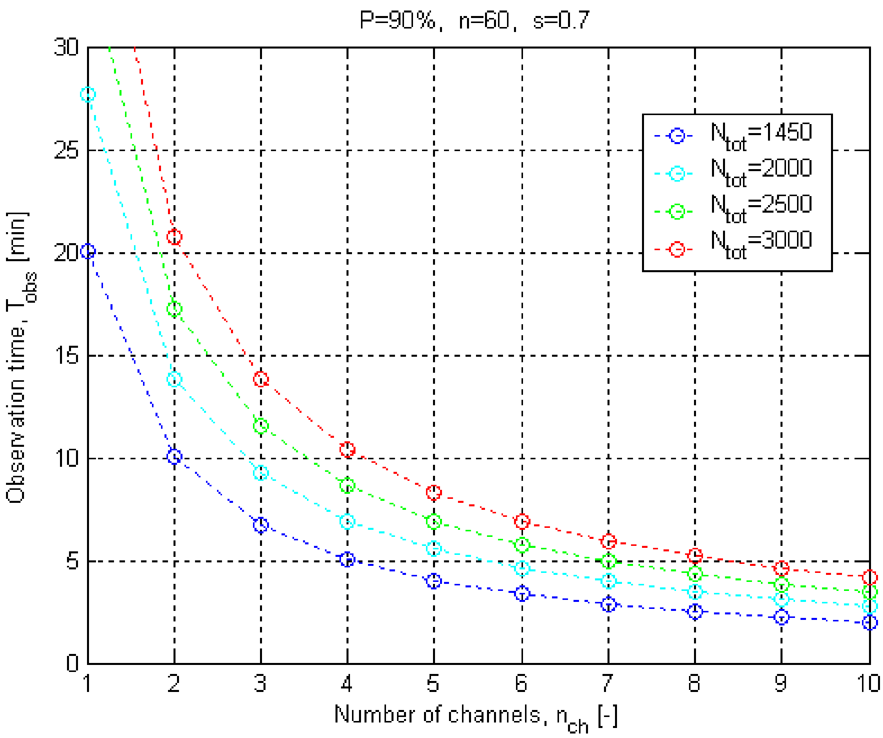

Figure 5.23 Observation time as a function of number of channels for different numbers of ships when the ship detection probability is 90% and the number of reports is 60.

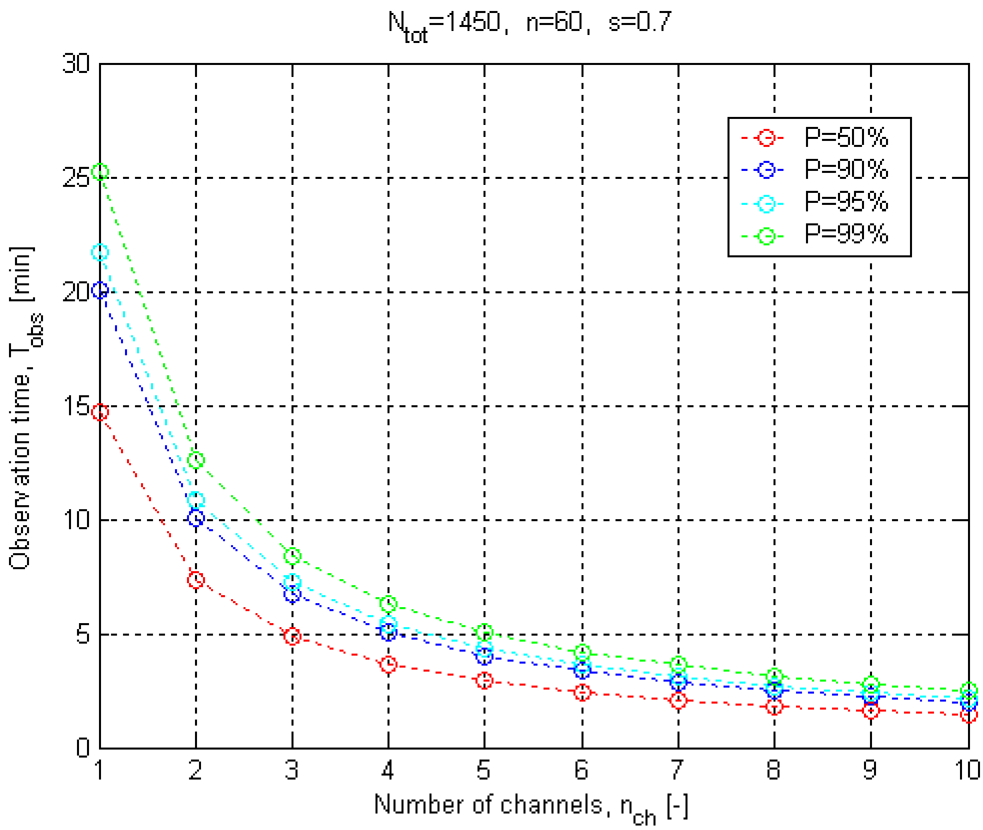

Figure 5.24 Observation time as a function of number of channels for different ship detection probabilities when the number of ships is 1450 and the number of reports is 60.

5.3 Number of ships

5.3.1 Number of ships vs ship detection probability

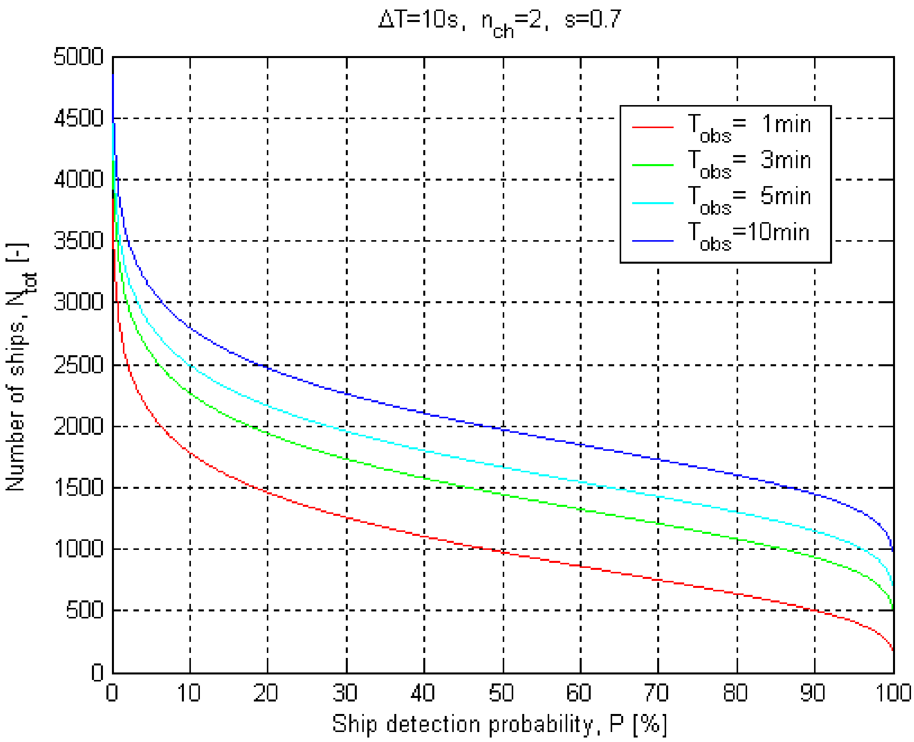

Figure 5.25 Number of ships as a function of ship detection probability for different observation times when the ship reporting interval is 10 s.

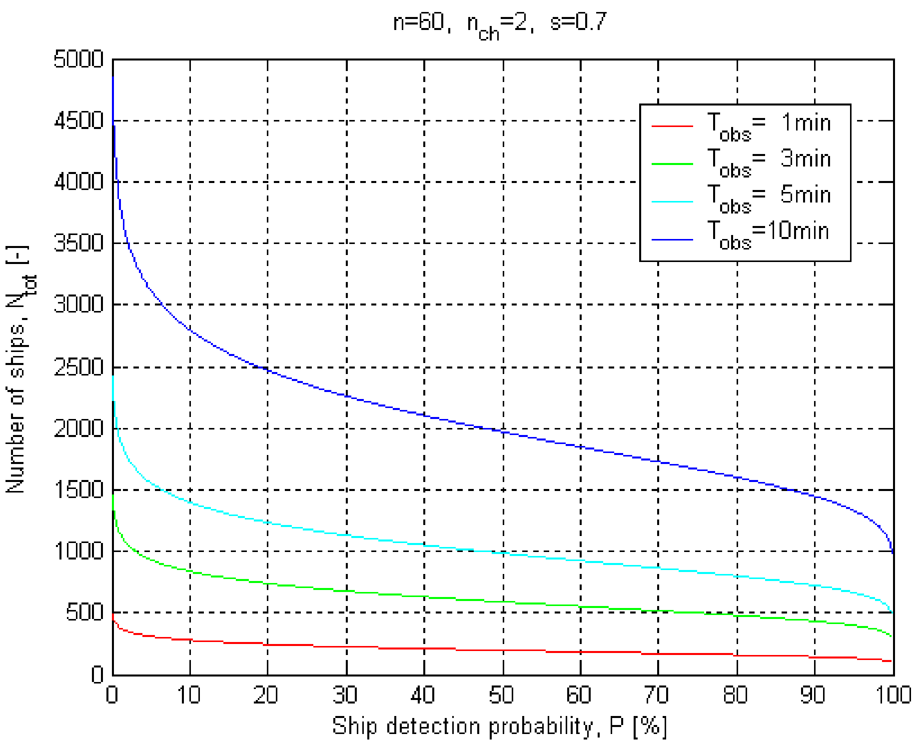

Figure 5.26 Number of ships as a function of ship detection probability for different observation times when the number of reports is 60.

5.3.2 Number of ships vs observation time

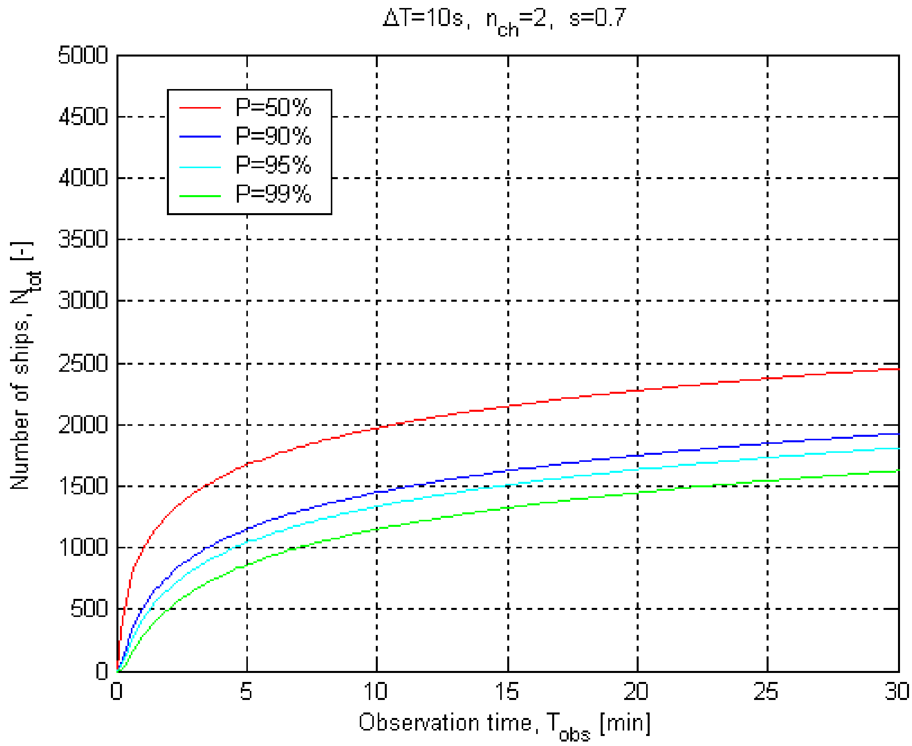

Figure 5.27 Number of ships as a function of observation time for different ship detection probabilities when the ship reporting interval is 10 s.

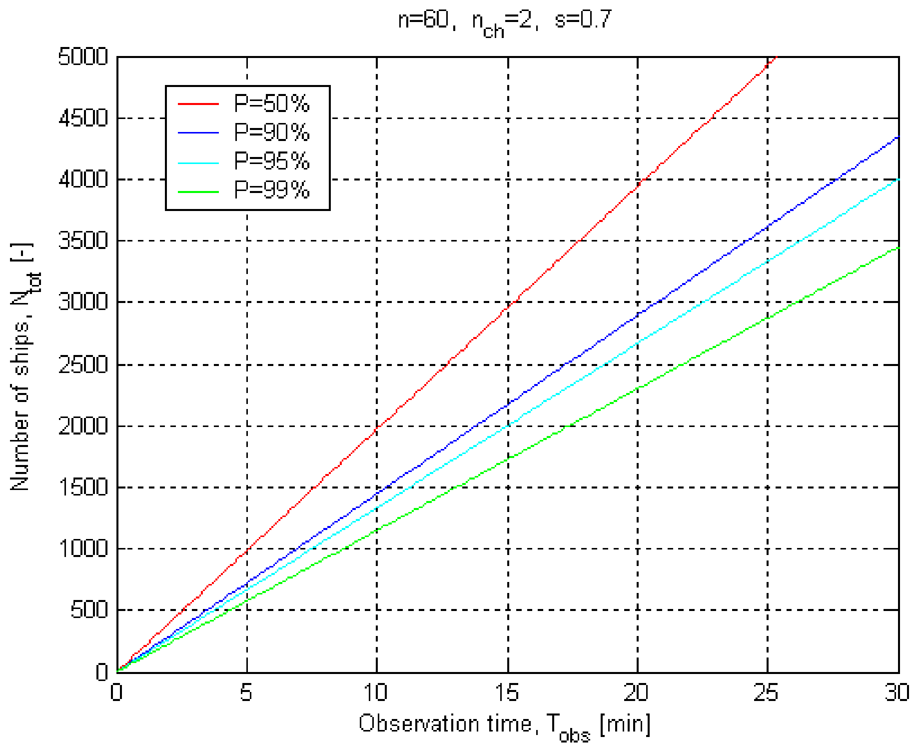

Figure 5.28 Number of ships as a function of observation time for different ship detection probabilities when the number of reports is 60.

5.3.3 Number of ships vs ship reporting interval

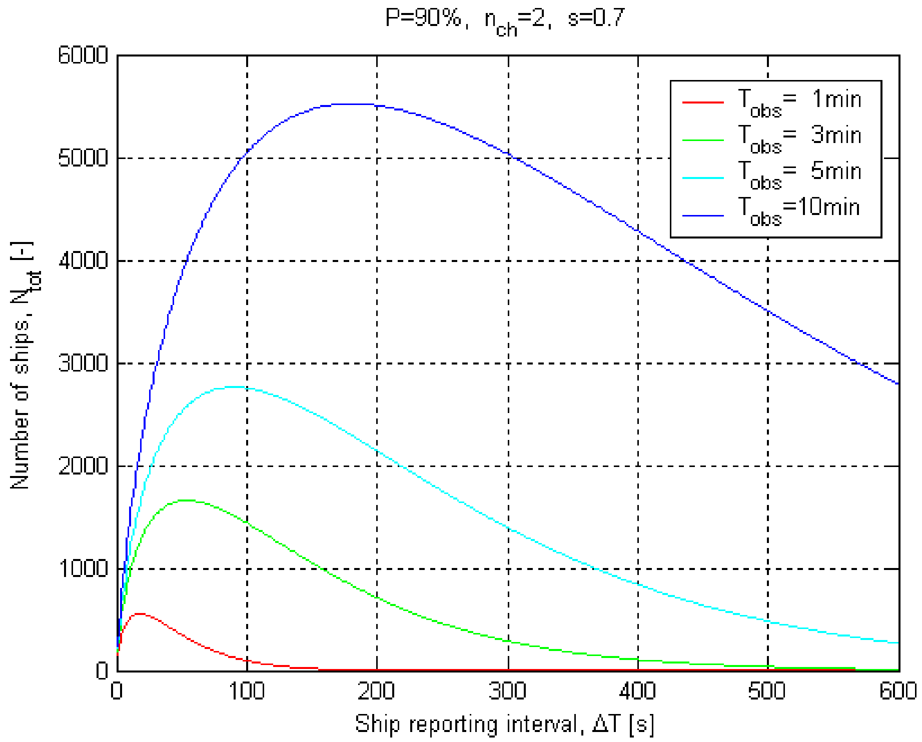

Figure 5.29 Number of ships as a function of ship reporting interval for different observation times when the ship detection probability is 90%.

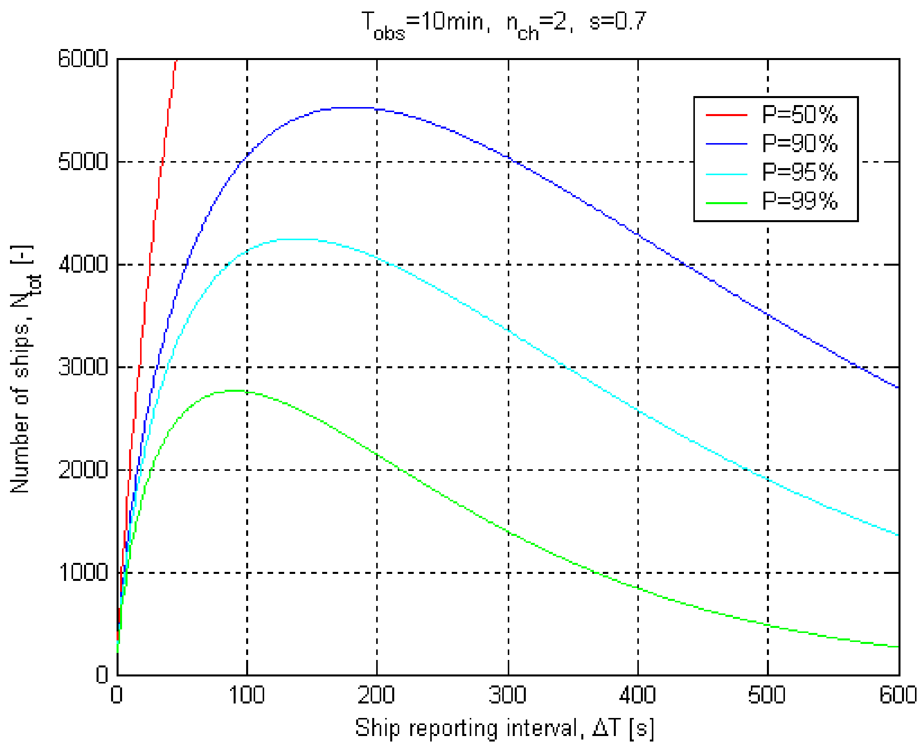

Figure 5.30 Number of ships as a function of ship reporting interval for different ship detection probabilities when the observation time is 10 min.

5.3.4 Number of ships vs number of reports

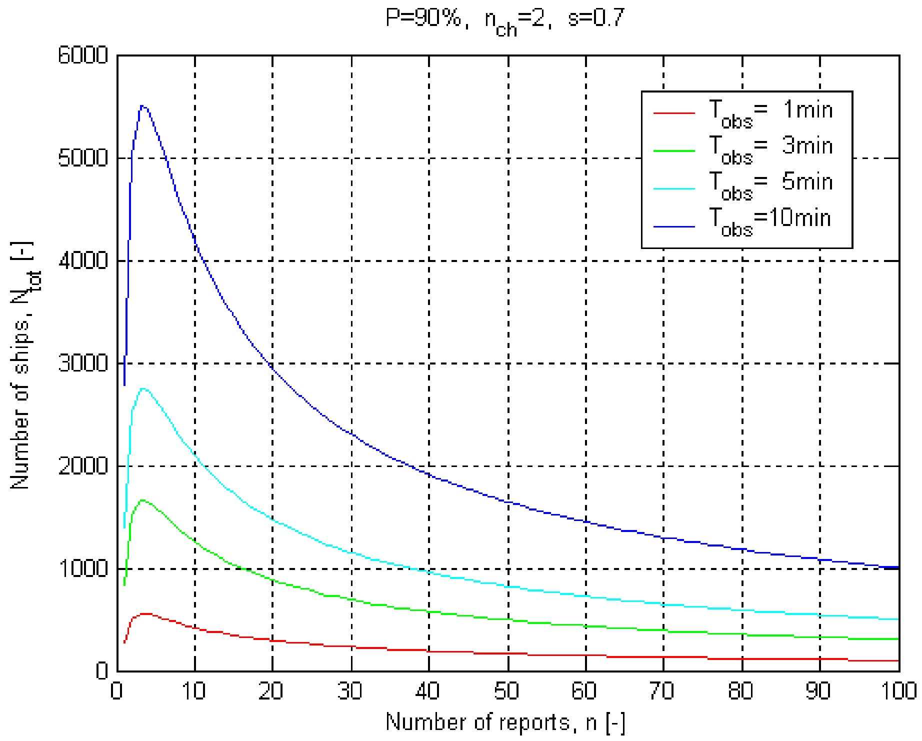

Figure 5.31 Number of ships as a function of number of reports for different observation times when the ship detection probability is 90%.

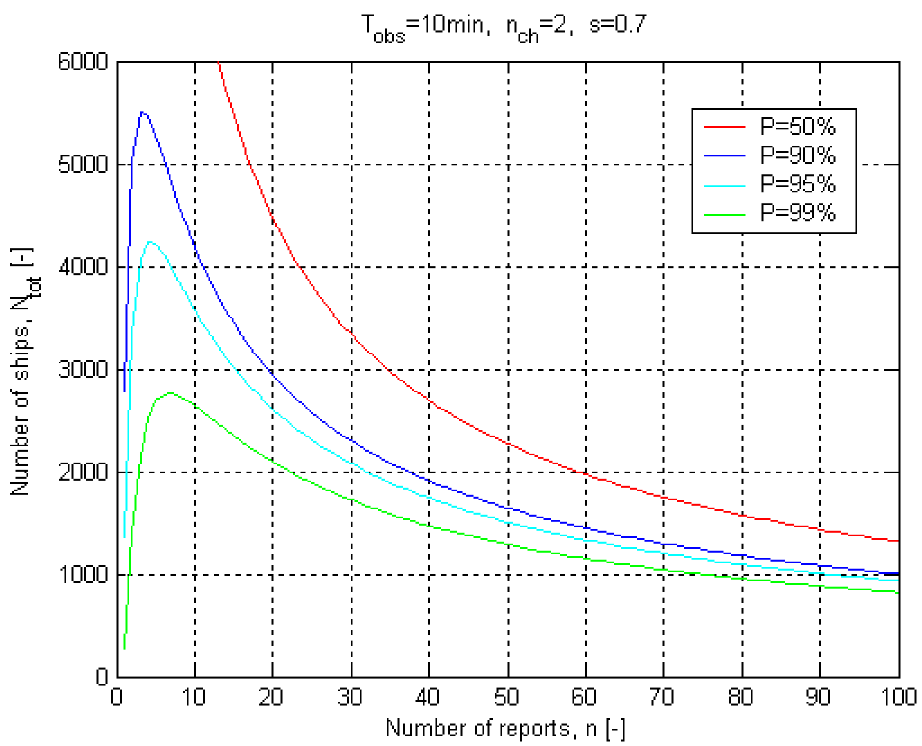

Figure 5.32 Number of ships as a function of number of reports for different ship detection probabilities when the observation time is 10 min.

5.3.5 Number of ships vs number of channels

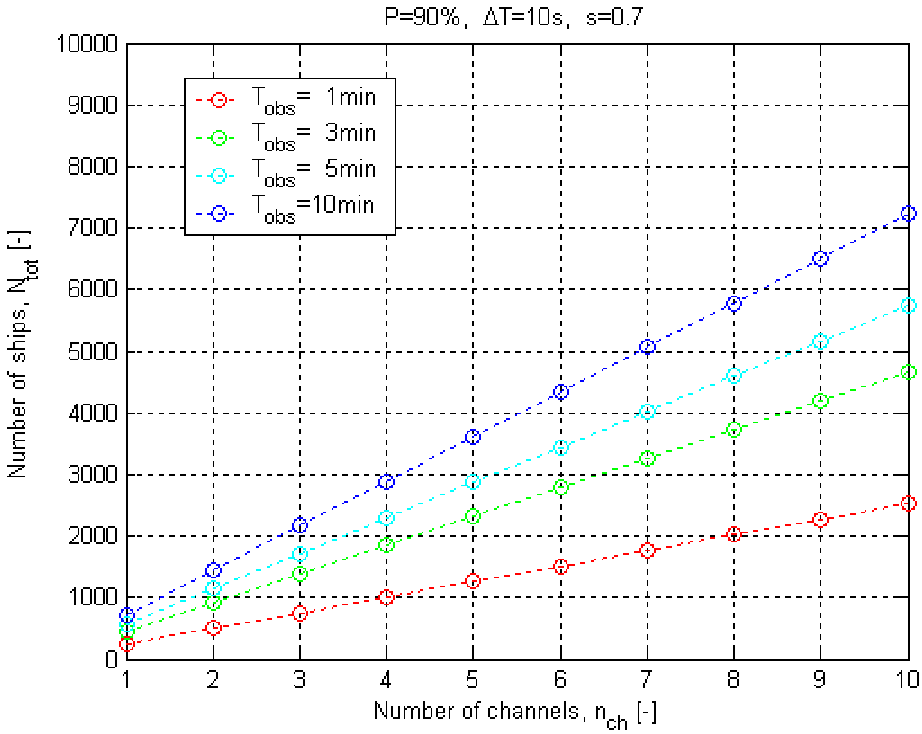

Figure 5.33 Number of ships as a function of number of channels for different observation times when the ship detection probability is 90% and the ship reporting interval is 10 s.

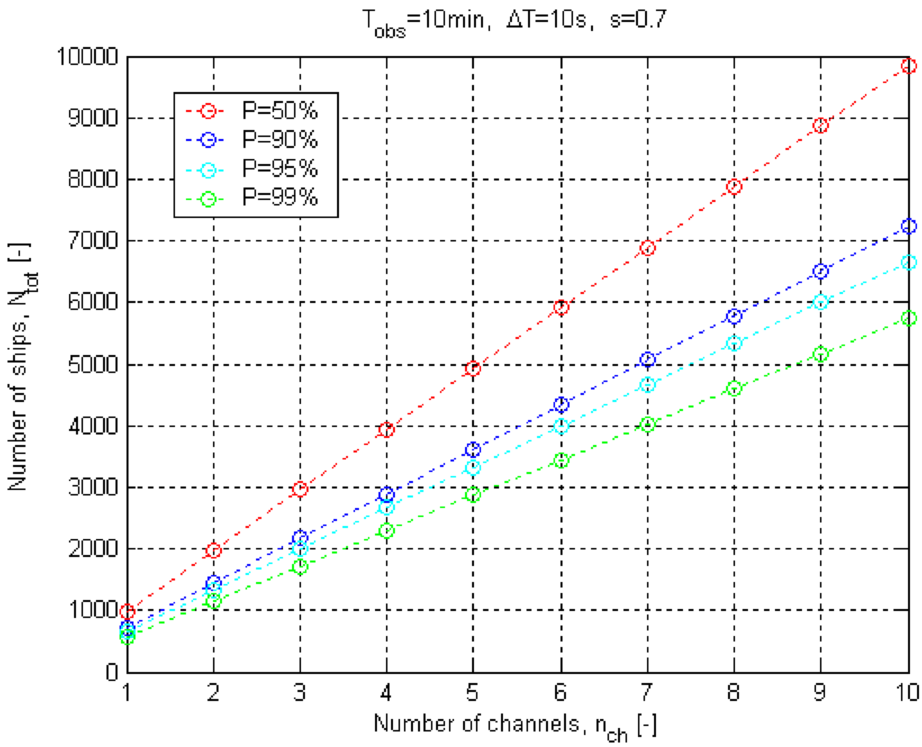

Figure 5.34 Number of ships as a function of number of channels for different ship detection probabilities when the observation time is 10 min and the ship reporting interval is 10 s.

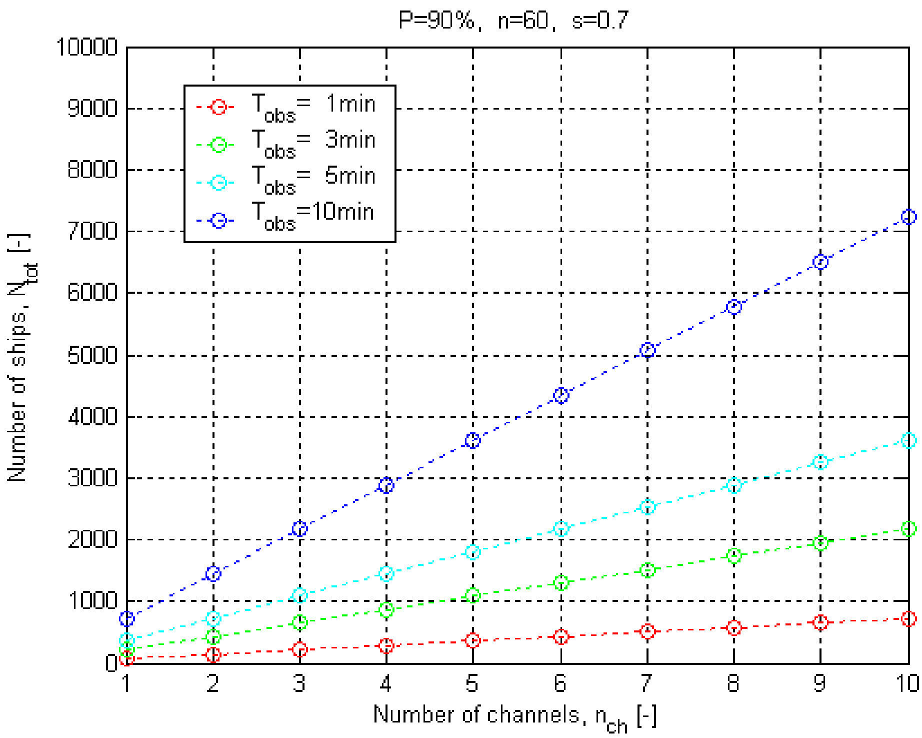

Figure 5.35 Number of ships as a function of number of channels for different observation times when the ship detection probability is 90% and the number of reports is 60.

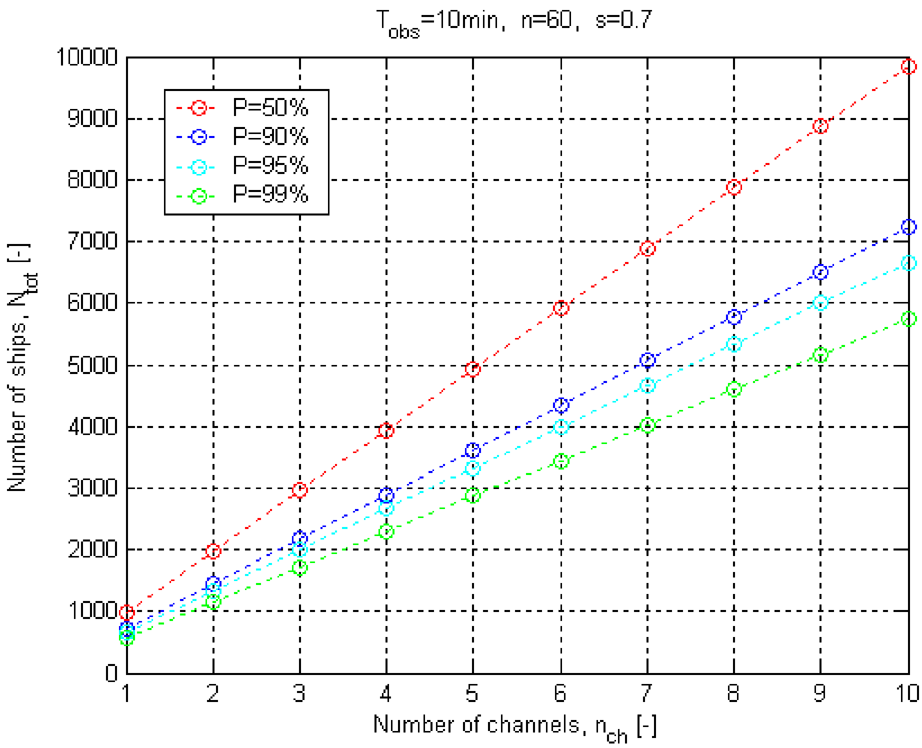

Figure 5.36 Number of ships as a function of number of channels for different ship detection probabilities when the observation time is 10 min and the number of reports is 60.

5.4 Intersection point for the ship detection probability curves - different ship reporting intervals

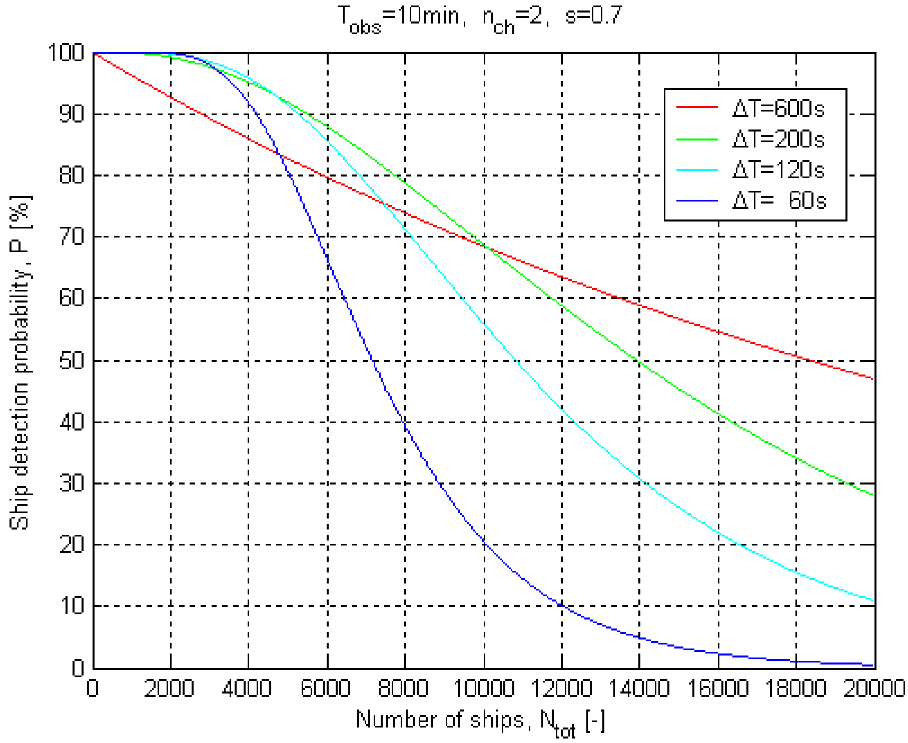

Figure 5.37 Ship detection probability as a function of number of ships for different ship reporting intervals when the observation time is 10 min.

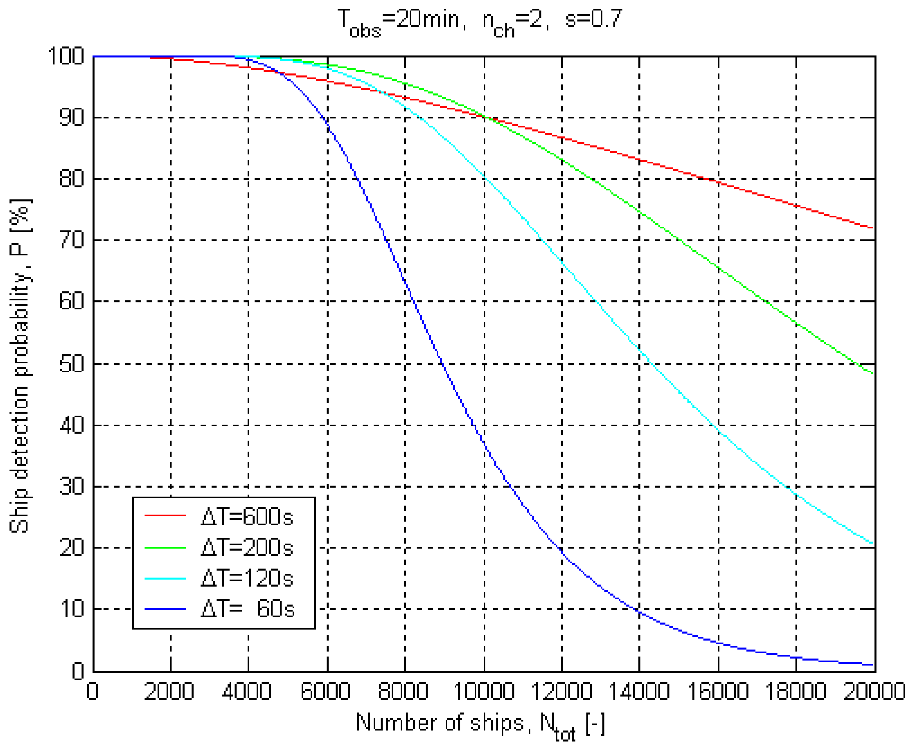

Figure 5.38 Ship detection probability as a function of number of ships for different ship reporting intervals when the observation time is 20 min.

ΔT_a [s]	ΔT_b [s]	$_cN^{\Delta T}_{tot}$ [-]	$_cP_{\Delta T}$ [%]	
			T_{obs}=10min	T_{obs}=20min
600	200	10 118	68.2	89.9
600	120	7 444	75.5	94.0
600	60	4 771	83.5	97.3
200	120	4 691	93.0	99.5
200	60	3 171	97.3	99.9
120	60	2 548	99.2	100

Table 5.1 Number of ships and ship detection probability at the intersection point for different ship reporting interval pairs.

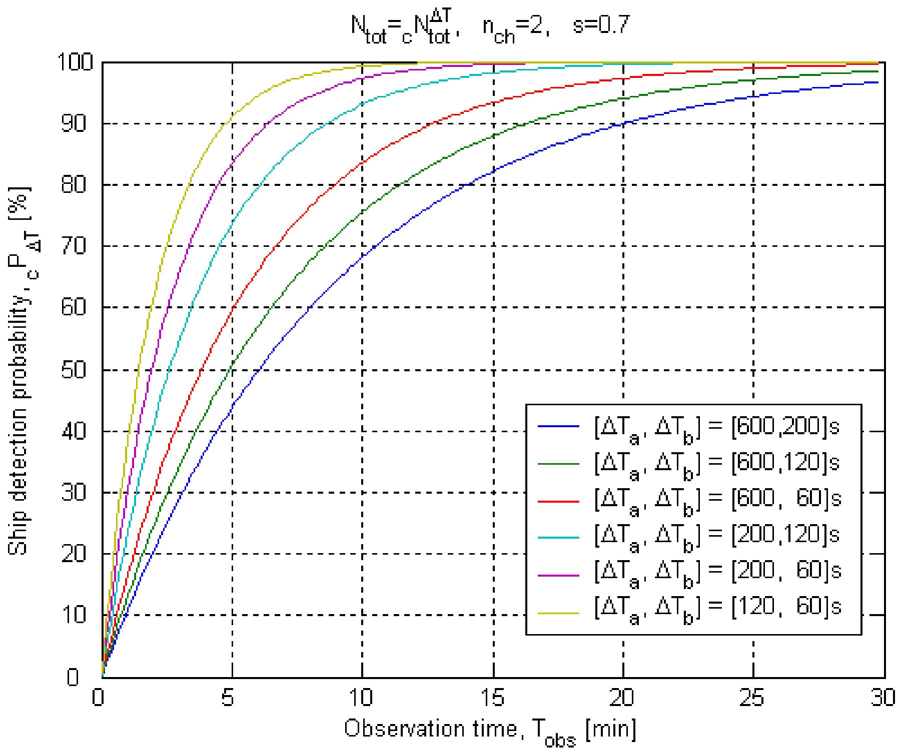

Figure 5.39 Ship detection probability at the intersection point as a function of observation time for different ship reporting interval pairs.

5.5 Intersection point for the ship detection probability curves - different numbers of reports

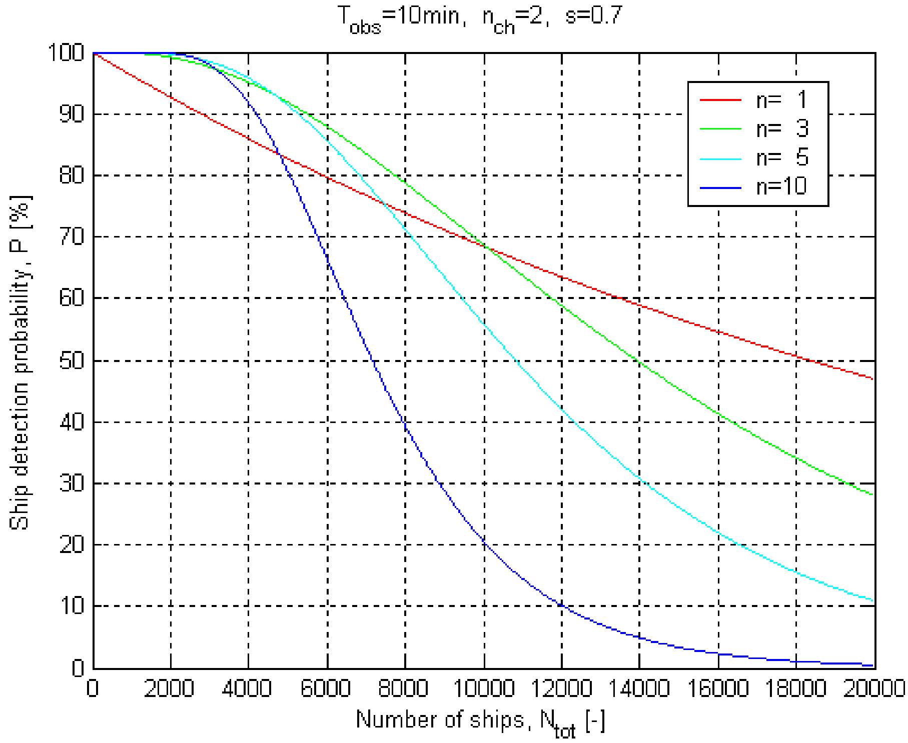

Figure 5.40 Ship detection probability as a function of number of ships for different numbers of reports when the observation time is 10 min.

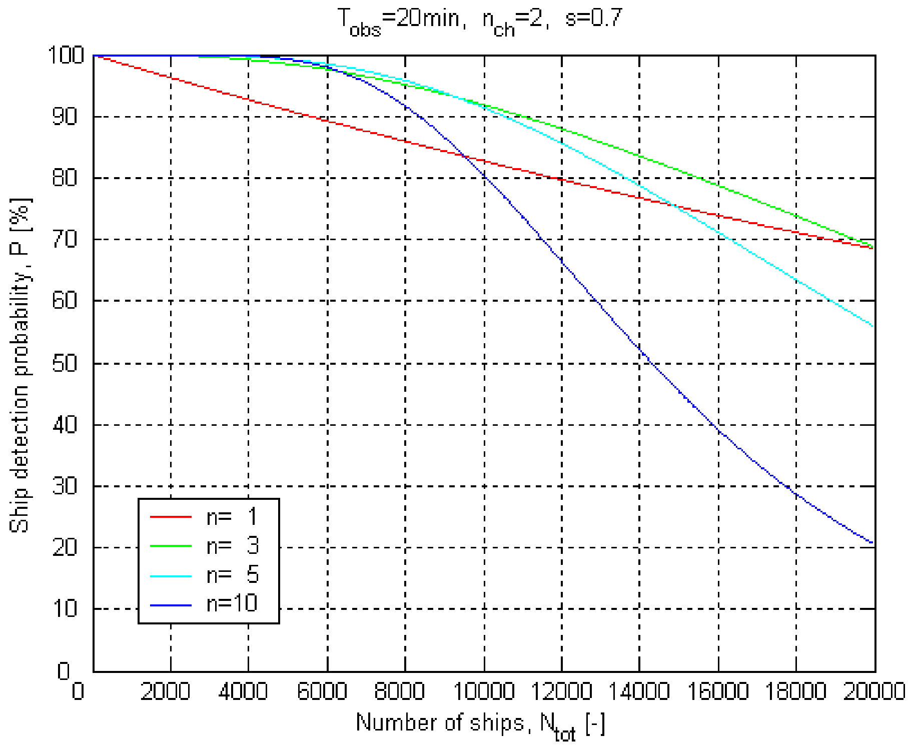

Figure 5.41 Ship detection probability as a function of number of ships for different numbers of reports when the observation time is 20 min.

n_a	n_b	$_cP_n$ [%]	$_cN^n{}_{tot}$ [-]	
			T_{obs}=10min	T_{obs}=20min
1	3	68.2	10 114	20 228
1	5	75.5	7 442	14 884
1	10	83.5	4 772	9 544
3	5	93.0	4 698	9 395
3	10	97.2	3 175	6 349
5	10	99.2	2 560	5 121

Table 5.2 Ship detection probability and number of ships at the intersection point for different number of reports pairs.

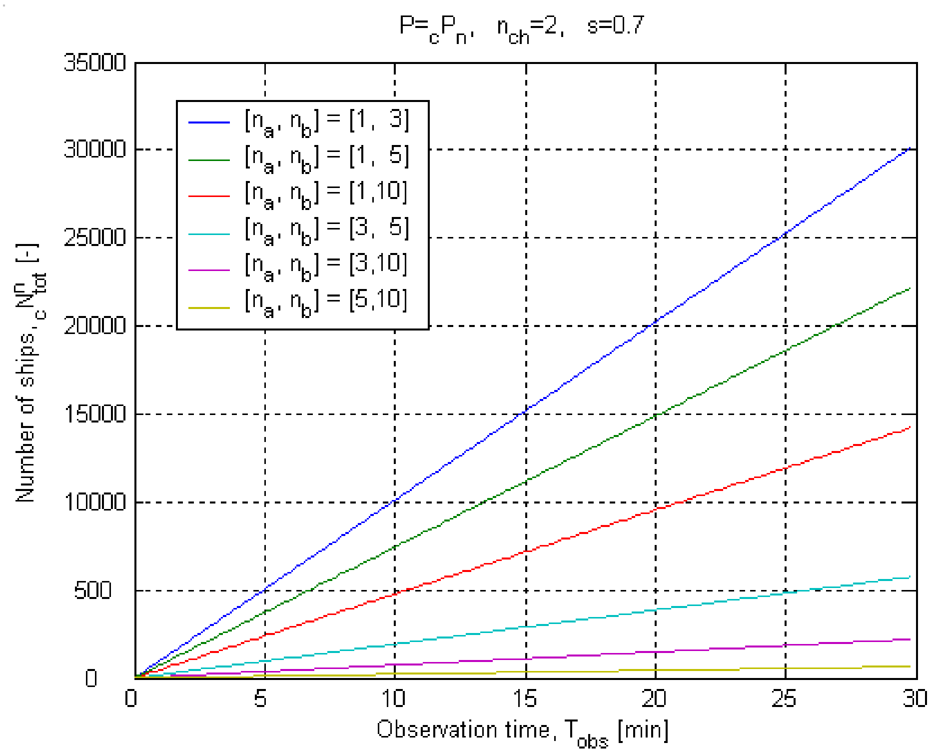

Figure 5.42 Number of ships at the intersection point as a function of observation time for different number of reports pairs.

5.6 Optimum number of reports

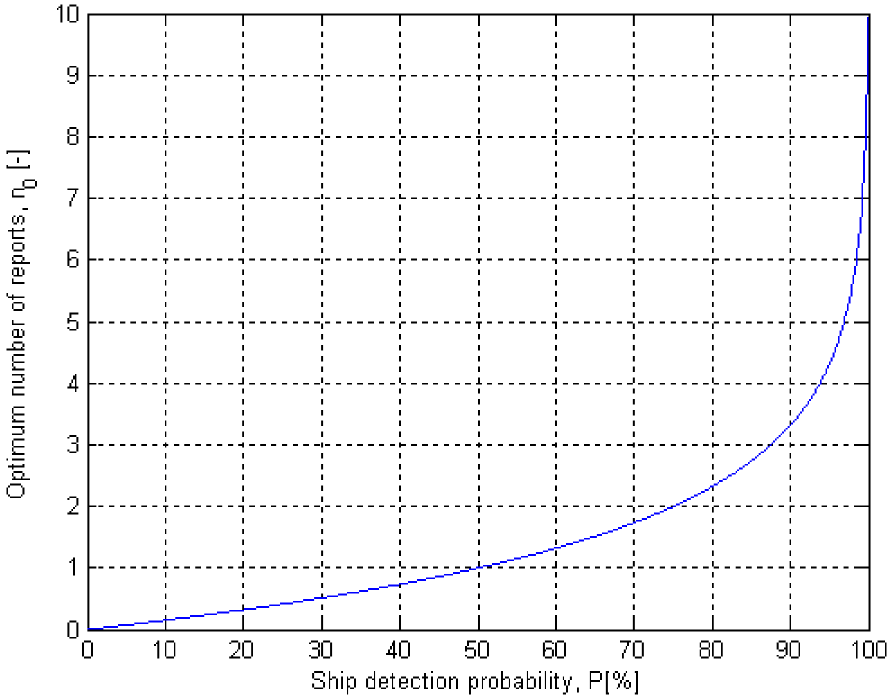

Figure 5.43 Optimum number of reports as a function of ship detection probability.

5.7 Optimum ship reporting interval

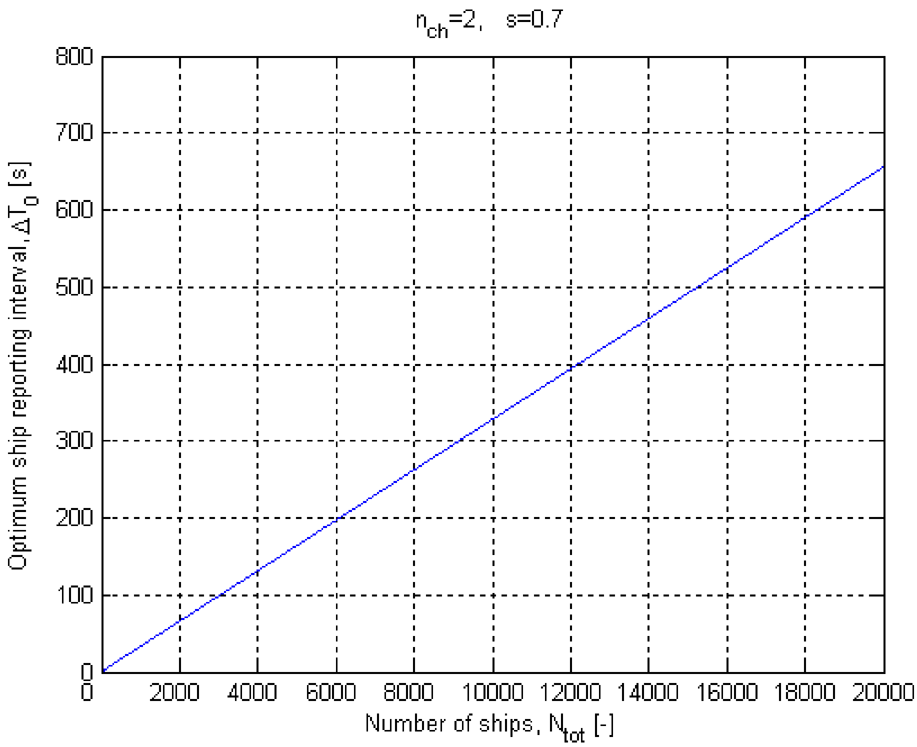

Figure 5.44 Optimum ship reporting interval as a function of number of ships.

5.8 Optimum message density

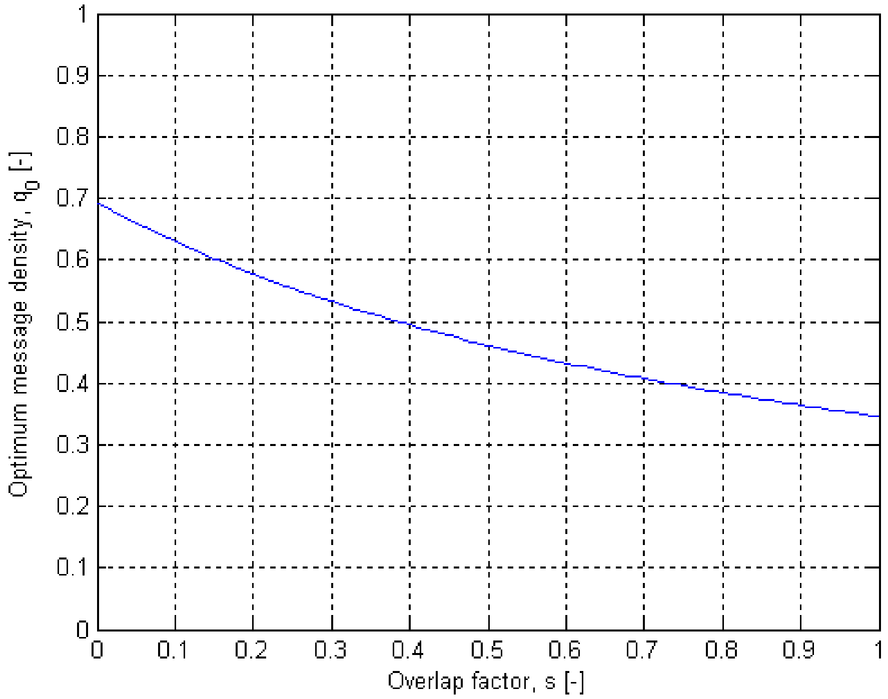

Figure 5.45 Optimum message density as a function of overlap factor.

5.9 Optimization of the system

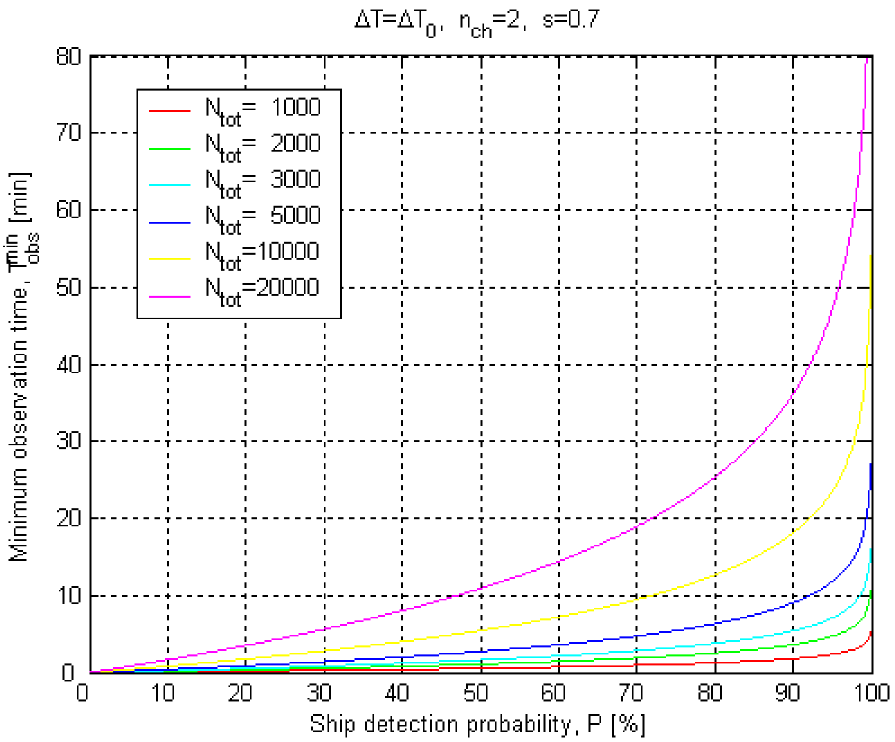

Figure 5.46 *Minimum observation time as a function of ship detection probability for different numbers of ships when the optimum ship reporting interval is used.*

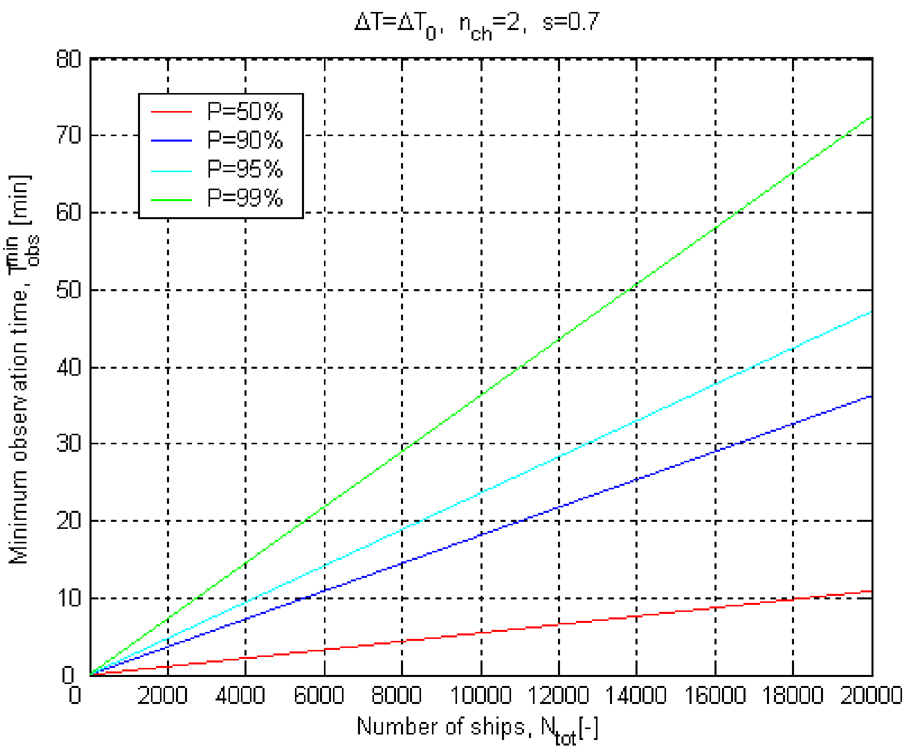

Figure 5.47 *Minimum observation time as a function of number of ships for different ship detection probabilities when the optimum ship reporting interval is used.*

N_{tot}	ΔT_0		T_{obs}^{min}							
			***P*=99%** (n_0=6.6)		***P*=95%** (n_0=4.3)		***P*=90%** (n_0=3.3)		***P*=50%** (n_0=1.0)	
1000	33s	(0.55min)	217s	(3.6min)	141s	(2.4min)	109s	(1.8min)	33s	(0.55min)
2000	66s	(1.1min)	435s	(7.3min)	283s	(4.7min)	217s	(3.6min)	65s	(1.1min)
3000	98s	(1.6min)	652s	(11min)	424s	(7.1min)	326s	(5.4min)	98s	(1.6min)
5000	164s	(2.7min)	1086s	(18min)	707s	(12min)	543s	(9.1min)	164s	(2.7min)
7000	229s	(3.8min)	1521s	(25min)	989s	(16min)	760s	(13min)	229s	(3.8min)
10 000	328s	(5.5min)	2173s	(36min)	1413s	(24min)	1086s	(18min)	327s	(5.5min)
15 000	491s	(8.2min)	3259s	(54min)	2120s	(35min)	1630s	(27min)	491s	(8.2min)
20 000	655s	(11min)	4345s	(72min)	2827s	(47min)	2173s	(36min)	654s	(11min)

Table 5.3 Optimum ship reporting interval and minimum required observation time for different numbers of ships and ship detection probabilities. The number of channels is 2 and the overlap factor is 0.7.

6 DISCUSSION

We will in this chapter discuss the results that were presented in Chapter 5.

Sections 6.1-6.3 discuss the behaviour of the ship detection probability, the observation time, and the number of ships with respect to the other parameters in the system. Sections 6.4-6.5 discuss the properties of the intersection point for the ship detection probability curves, and Sections 6.6-6.9 discuss the results from the optimization of the system. Finally, in Section 6.10 a summary is given.

6.1 Ship detection probability

This section discusses the behaviour of the ship detection probability with respect to the number of ships (Section 6.1.1), the observation time (Section 6.1.2), the ship reporting interval (Section 6.1.3), the number of reports (Section 6.1.4), and the number of channels (Section 6.1.5).

6.1.1 Ship detection probability vs number of ships

Figure 5.1 and Figure 5.2 show the ship detection probability P as a function of the number of ships N_{tot} for different observation times T_{obs}. The figures show that the ship detection probability *decreases* with *increasing* number of ships. This can also be seen directly from Equations (4.2) and (4.3). Figure 5.3, Figure 5.4, Figure 5.6, Figure 5.8, Figure 5.10, and Figure 5.12 also illustrate this behaviour.

Comparison of Figure 5.1 and Figure 5.2 shows that the dark blue curve ($T_{obs} = 10$ min) is identical in the two figures. This is as expected since a ship reporting interval of $\Delta T = 10$ s (Figure 5.1) corresponds to a number of reports of $n = 60$ (Figure 5.2) when the observation time is $T_{obs} = 10$ min.

Note that Figure 5.1 and Figure 5.2 correspond to Figure 5.25 and Figure 5.26 (discussed in Section 6.3.1) with the axes interchanged.

6.1.2 Ship detection probability vs observation time

Figure 5.3 and Figure 5.4 show the ship detection probability P as a function of the observation time T_{obs} for different numbers of ships N_{tot}. The figures show that the ship detection probability *increases* with *increasing* observation time. This can also be seen directly from Equations (4.2) and (4.3). Figure 5.1, Figure 5.2, Figure 5.5, Figure 5.7, Figure 5.9, and Figure 5.11 also illustrate this behaviour.

Comparison of Figure 5.3 and Figure 5.4 shows that corresponding curves (same value for N_{tot}) in the two figures give the same value for the ship detection probability P when the observation time is $T_{obs} = 10$ min, i.e.,

$$P_{\Delta T=10\mathrm{s}}(T_{obs}=10\mathrm{min}) = P_{n=60}(T_{obs}=10\mathrm{min}) \quad (6.1)$$

This is as expected since a ship reporting interval of $\Delta T = 10$ s (Figure 5.3) corresponds to a number of reports of $n = 60$ (Figure 5.4) when the observation time is $T_{obs} = 10$ min.

We further notice that for $T_{obs} < 10$ min the ship detection probability is larger when the ship reporting interval is kept constant at $\Delta T_0 = 10$ s (Figure 5.3) than when the number of reports is kept constant at $n = 60$ (Figure 5.4). For $T_{obs} > 10$ min the situation is opposite. This can be expressed as

$$P_{\Delta T=10\mathrm{s}}(T_{obs}<10\mathrm{min}) \ > \ P_{n=60}(T_{obs}<10\mathrm{min}) \quad (6.2)$$

and

$$P_{\Delta T=10\mathrm{s}}(T_{obs}>10\mathrm{min}) \ < \ P_{n=60}(T_{obs}>10\mathrm{min}) \quad (6.3)$$

Note that Figure 5.3 and Figure 5.4 correspond to Figure 5.13 and Figure 5.14 (discussed in Section 6.2.1) with the axes interchanged.

6.1.3 Ship detection probability vs ship reporting interval

Figure 5.5 and Figure 5.6 show the ship detection probability P as a function of the ship reporting interval ΔT for different observation times T_{obs} and numbers of ships N_{tot}. The figures show that the ship detection probability first *increases* and then *decreases* for *increasing* ship reporting interval. This behaviour can also bee seen from Equation (4.2) where the ship reporting interval appears in two different places in the equation. In the first place, increasing the ship reporting interval increases the ship detection probability, while in the second place it decreases the ship detection probability. This shows that there exists an optimum ship reporting interval ΔT_0 that gives the highest possible ship detection probability for given number of ships and observation time.

Comparison of Figure 5.5 and Figure 5.6 shows that the dark blue curve is identical in the two figures. This is as expected since the number of ships and the observation time are the same in both cases ($N_{tot} = 1450$, $T_{obs} = 10$ min).

We further notice that the optimum ship reporting interval ΔT_0 is *independent* of the observation time (Figure 5.5), and that it *increases* with *increasing* number of ships (Figure 5.6). This can also be seen directly from Equation (4.17).

6.1.4 Ship detection probability vs number of reports

Figure 5.7 and Figure 5.8 show the ship detection probability P as a function of the number of reports n for different observation times T_{obs} and numbers of ships N_{tot}. The figures show that the ship detection probability first *increases* and then *decreases* for *increasing* numbers of reports. This behaviour can also bee seen from Equation (4.3) where the number of reports appears in two different places in the equation. In the first place, increasing the number of reports decreases the ship detection probability, while in the second place it increases the ship detection probability. This shows that there exists an optimum number of reports n_0 that gives the highest possible ship detection probability for given number of ships and observation time. For numbers of ships $N_{tot} = 3000 - 10000$ and observation times $T_{obs} = 1 - 5$ min, the optimum number of reports n_0 lies in the range $n_0 = 1 - 10$.

Comparison of Figure 5.7 and Figure 5.8 shows that the dark blue curve is identical in the two figures. This is as expected since the number of ships and the observation time are the same in both cases ($N_{tot} = 1450$, $T_{obs} = 10$ min).

6.1.5 Ship detection probability vs number of channels

Figure 5.9-Figure 5.12 show the ship detection probability P as a function of the number of channels n_{ch} for different observation times T_{obs} and numbers of ships N_{tot}. The figures show that the ship detection probability *increases* with *increasing* number of channels. This can also be seen from Equations (4.2) and (4.3).

Comparison of Figure 5.9-Figure 5.12 shows that the dark blue curve is identical in all four figures. This is as expected since the number of ships and the observation time are the same in each case ($N_{tot} = 1450$, $T_{obs} = 10$ min).

6.1.6 Summary

Equations (4.2) and (4.3) and Figure 5.1-Figure 5.12 show the ship detection probability as a function of different parameters.

We have found that the ship detection probability *decreases* with *increasing* number of ships, and *increases* with *increasing* observation time and *increasing* number of channels.

We have further found that there exists an optimum ship reporting interval with corresponding optimum number of reports that gives the highest possible ship detection probability for given number of ships and observation time. The optimum ship reporting interval is *independent* of the observation time, and *increases* with *increasing* number of ships.

6.2 Observation time

This section discusses the behaviour of the observation time with respect to the ship detection probability (Section 6.2.1), the number of ships (Section 6.2.2), the ship reporting interval (Section 6.2.3), the number of reports (Section 6.2.4), and the number of channels (Section 6.2.5).

6.2.1 Observation time vs ship detection probability

Figure 5.13 and Figure 5.14 show the observation time T_{obs} as a function of the ship detection probability P for different numbers of ships N_{tot}. The figures show that the observation time *increases* with *increasing* requirement to the ship detection probability. This can also be seen directly from Equations (4.4) and (4.5). Figure 5.15, Figure 5.16, Figure 5.18, Figure 5.20, Figure 5.22, and Figure 5.24 also illustrate this behaviour.

Comparison of Figure 5.13 and Figure 5.14 shows that corresponding curves (same value for N_{tot}) in the two figures give the same value for the ship detection probability P when the observation time is $T_{obs} = 10$ min, i.e.,

$$P_{\Delta T=10\text{s}}(T_{obs} = 10\text{min}) = P_{n=60}(T_{obs} = 10\text{min}) \tag{6.4}$$

This is as expected since a ship reporting interval of $\Delta T = 10$ s (Figure 5.13) corresponds to a number of reports of $n = 60$ (Figure 5.14) when the observation time is $T_{obs} = 10$ min.

We further notice that the required observation time increases faster as a function of required ship detection probability when the ship reporting interval is kept constant at $\Delta T = 10$ s (Figure 5.13) than when the number of reports is kept constant at $n = 60$ (Figure 5.14). This is particularly noticeable for higher numbers of ships (red curves).

Note that Figure 5.13 and Figure 5.14 correspond to Figure 5.3 and Figure 5.4 (discussed in Section 6.1.2) with the axes interchanged.

6.2.2 Observation time vs number of ships

Figure 5.15 and Figure 5.16 show the observation time T_{obs} as a function of the number of ships N_{tot} for different ship detection probabilities P. The figures show that the observation time *increases* with *increasing* number of ships. This can also be seen directly from Equations (4.4) and (4.5). Figure 5.13, Figure 5.14, Figure 5.17, Figure 5.19, Figure 5.21, and Figure 5.23 also illustrate this behaviour. Note that when the number of reports is kept constant (Figure 5.16), the observation time *increases linearly* with *increasing* number of ships. This can also be seen from Equation (4.5).

Comparison of Figure 5.15 and Figure 5.16 shows that corresponding curves (same value for P) in the two figures give the same value for the number of ships N_{tot} when the observation time is $T_{obs} = 10\text{min}$, i.e.,

$$N_{tot}^{\Delta T=10\text{s}}(T_{obs} = 10\text{min}) = N_{tot}^{n=60}(T_{obs} = 10\text{min}) \tag{6.5}$$

This is as expected since a ship reporting interval of $\Delta T = 10$ s (Figure 5.13) corresponds to a number of reports of $n = 60$ (Figure 5.14) when the observation time is $T_{obs} = 10$ min.

We further notice that the observation time increases faster for increasing number of ships (except for small numbers of ships) when the ship reporting interval is kept constant at $\Delta T = 10$ s (Figure 5.15) than when the number of reports is kept constant at $n = 60$ (Figure 5.16).

Note that Figure 5.15 and Figure 5.16 correspond to Figure 5.27 and Figure 5.28 (discussed in Section 6.3.2) with the axes interchanged.

6.2.3 Observation time vs ship reporting interval

Figure 5.17 and Figure 5.18 show the observation time T_{obs} as a function of the ship reporting interval ΔT for different numbers of ships N_{tot} and ship detection probabilities P. The figures show that the observation time first *decreases* and then *increases* for *increasing* ship reporting interval. This behaviour can also be seen from Equation (4.4) where the ship reporting interval appears in two different places in the equation. In the first place, increasing the ship reporting interval increases the observation time, while in the second place it decreases the observation time. This shows that there exists an optimum ship reporting interval ΔT_0 that gives the shortest possible observation time for given number of ships and ship detection probability.

Comparison of Figure 5.17 and Figure 5.18 shows that the dark blue curve is identical in the two figures. This is as expected since the number of ships and the ship detection probability are the same in both cases ($N_{tot} = 1450$, $P = 90\%$).

We further notice that the optimum ship reporting interval ΔT_0 is *independent* of the ship detection probability (Figure 5.18), and *increases* with *increasing* number of ships (Figure 5.17). This can also be seen directly from Equation (4.17).

6.2.4 Observation time vs number of reports

Figure 5.19 and Figure 5.20 show the observation time T_{obs} as a function of the number of reports n for different numbers of ships N_{tot} and ship detection probabilities P. The figures show that the observation time first *decreases* and then *increases* for *increasing* numbers of reports. This behaviour can also bee seen from Equation (4.5) where the number of reports appears in two different places in the equation. In the first place, increasing the number of reports increases the required observation time, while in the second place it decreases the observation time. This shows that there exists an optimum number of reports n_0 that gives the shortest possible observation time for given number of ships and ship detection probability.

Comparison of Figure 5.19 and Figure 5.20 shows that the dark blue curve is identical in the two figures. This is as expected since the number of ships and the ship detection probability are the same in both cases ($N_{tot} = 1450$, $P = 90\%$).

We further notice that the optimum number of reports n_0 is *independent* of the number of ships (Figure 5.19), and *increases* with *increasing* requirement to the ship detection probability (Figure 5.20). This can also be seen directly from Equation (4.15). For numbers of ships $N_{tot} = 1450 - 10000$ and ship detection probabilities $P = 50 - 90\%$, the optimum number of reports n_0 lies in the range $n_0 = 1 - 10$.

6.2.5 Observation time vs number of channels

Figure 5.21-Figure 5.24 show the observation time T_{obs} as a function of the number of channels n_{ch} for different numbers of ships N_{tot} and ship detection probabilities P. The figures show that the required observation time *decreases* with *increasing* number of channels. This can also bee seen from Equations (4.4) and (4.5).

Comparison of Figure 5.21-Figure 5.24 shows that the dark blue curve gives $T_{obs} = 10$ min for $n_{ch} = 2$ in all four figures. This is as expected since the number of ships and the ship detection probability are the same in each case ($N_{tot} = 1450$, $P = 90\%$).

6.2.6 Summary

Equations (4.4) and (4.5) and Figure 5.13-Figure 5.24 show the observation time as a function of different parameters.

We have found that the observation time *increases* with *increasing* requirement to the ship detection probability and *increasing* number of ships, while *decreases* with *increasing* number of channels.

We have further found that there exists an optimum ship reporting interval with corresponding optimum number of reports that gives the shortest possible observation time for given number of ships and ship detection probability. The optimum ship reporting interval is *independent* of the ship detection probability, and *increases* with *increasing* number of ships. The optimum number of reports is *independent* of the number of ships and *increases* with *increasing* requirement to the ship detection probability.

6.3 Number of ships

This section discusses the behaviour of the number of ships with respect to the ship detection probability (Section 6.3.1), the observation time (Section 6.3.2), the ship reporting interval (Section 6.3.3), the number of reports (Section 6.3.4), and the number of channels (Section 6.3.5).

6.3.1 Number of ships vs ship detection probability

Figure 5.25 and Figure 5.26 show the number of ships N_{tot} that the system can handle as a function of the ship detection probability P for different observation times T_{obs}. The figures show that the number of ships that the system can handle *decreases* with *increasing* requirement to the ship detection probability. This can also be seen directly from Equations (4.6) and (4.7). Figure 5.27, Figure 5.28, Figure 5.30, Figure 5.32, Figure 5.34, and Figure 5.36 also illustrate this behaviour.

Comparison of Figure 5.25 and Figure 5.26 shows that the dark blue curve ($T_{obs} = 10$ min) is identical in the two figures. This is as expected since a ship reporting interval of $\Delta T = 10$ s (Figure 5.25) corresponds to a number of reports of $n = 60$ (Figure 5.26) when the observation time is $T_{obs} = 10$ min .

Note that Figure 5.25 and Figure 5.26 correspond to Figure 5.1 and Figure 5.2 (discussed in Section 6.1.1) with the axes interchanged.

6.3.2 Number of ships vs observation time

Figure 5.27 and Figure 5.28 show the number of ships N_{tot} that the system can handle as a function of the observation time T_{obs} for different requirements to the ship detection probability P. The figures show that the number of ships that the system can handle *increases* with *increasing* observation time. This can also be seen directly from Equations (4.6) and (4.7). Figure 5.25, Figure 5.26, Figure 5.29, Figure 5.31, Figure 5.33, and Figure 5.35 also illustrate this behaviour. Note that when the number of reports is kept constant (Figure 5.28), the number of ships that the system can handle *increases linearly* with *increasing* observation time. This can also be seen directly from Equation (4.7).

Comparison of Figure 5.27 and Figure 5.28 shows that corresponding curves (same value for P) in the two figures give the same value for the number of ships N_{tot} when the observation time is $T_{obs} = 10\text{ min}$, i.e.,

$$N_{tot}^{\Delta T=10\text{s}}(T_{obs} = 10\text{min}) = N_{tot}^{n=60}(T_{obs} = 10\text{min}) \qquad (6.6)$$

This is as expected since a ship reporting interval of $\Delta T = 10\text{ s}$ (Figure 5.27) corresponds to a number of reports of $n = 60$ (Figure 5.28) when the observation time is $T_{obs} = 10\text{ min}$.

We further notice that the number of ships that the system can handle increases faster for increasing observation time (except for short observation times) when the number of reports is kept constant at $n = 60$ (Figure 5.28) than when the ship reporting interval is kept constant at $\Delta T = 10\text{ s}$ (Figure 5.27).

Note that Figure 5.27 and Figure 5.28 correspond to Figure 5.15 and Figure 5.16 (discussed in Section 6.2.2) with the axes interchanged.

6.3.3 Number of ships vs ship reporting interval

Figure 5.29 and Figure 5.30 show the number of ships N_{tot} that the system can handle as a function of the ship reporting interval ΔT for different observation times T_{obs} and ship detection probabilities P. The figures show that the number of ships that the system can handle first *increases* and then *decreases* for *increasing* ship reporting interval. This behaviour can also bee seen from Equation (4.6) where the ship reporting interval appears in two different places in the equation. In the first place, increasing the ship reporting interval increases the number of ships that the system can handle, while in the second place it decreases the number of ships. This shows that there exists an optimum ship reporting interval ΔT_0 that gives the highest possible number of ships that the system can handle for given observation time and ship detection probability.

Comparison of Figure 5.17 and Figure 5.18 shows that the dark blue curve is identical in the two figures. This is as expected since the observation time and the ship detection probability are the same in both cases ($T_{obs} = 10$ min, $P = 90\%$).

6.3.4 Number of ships vs number of reports

Figure 5.31 and Figure 5.32 show the number of ships N_{tot} that the system can handle as a function of the number of reports n for different observation times T_{obs} and requirements to the ship detection probability P. The figures show that the number of ships that the system can handle first *increases* and then *decreases* for *increasing* numbers of reports. This behaviour can also bee seen from Equation (4.7) where the number of reports appears in two different places in the equation. In the first place, increasing the number of reports decreases the number of ships that the system can handle, while in the second place it increases the number of ships. This shows that there exists an optimum number of reports n_0 that gives the highest possible number of ships that the system can handle for given observation time and ship detection probability.

Comparison of Figure 5.31 and Figure 5.32 shows that the dark blue curve is identical in the two figures. This is as expected since the observation time and the ship detection probability are the same in both cases ($T_{obs} = 10$ min, $P = 90\%$).

We further notice that the optimum number of reports n_0 is *independent* of the observation time (Figure 5.31), and *increases* with *increasing* requirement to the ship detection probability (Figure 5.32). This can also be seen directly from Equation (4.15). For observation times $T_{obs} = 1-10$ min and ship detection probabilities $P = 90-99\%$, the optimum number of reports n_0 lies in the range $n_0 = 1-10$.

6.3.5 Number of ships vs number of channels

Figure 5.33-Figure 5.36 show the number of ships N_{tot} that the system can handle as a function of the number of channels n_{ch} for different observation times T_{obs} and requirements to the ship detection probability P. The figures show that the number of ships that the system can handle *increases linearly* with *increasing* number of channels. This can also bee seen from Equations (4.6) and (4.7).

Comparison of Figure 5.33-Figure 5.36 shows that the dark blue curve gives $N_{tot} = 1450$ for $n_{ch} = 2$ in all four figures. This is as expected since the observation time and the ship detection probability are the same in each case ($T_{obs} = 10$ min, $P = 90\%$).

6.3.6 Summary

Equations (4.6) and (4.7) and Figure 5.25-Figure 5.36 show the number of ships that the system can handle as a function of different parameters.

We have found that the number of ships that the system can handle *decreases* with *increasing* requirement to the ship detection probability, *increases* with *increasing* observation time, and *increases linearly* with *increasing* number of channels.

We have further found that there exists an optimum ship reporting interval with corresponding optimum number of reports that gives the highest number of ships that the system can handle for given observation time and ship detection probability. The optimum number of reports is *independent* of the observation time and *increases* with *increasing* requirement to the ship detection probability.

6.4 Intersection point for the ship detection probability curves - different ship reporting intervals

We wanted to study the properties of the intersection point (${}_cN_{tot}^{\Delta T}$, ${}_cP_{\Delta T}$) for the ship detection probability curves for different ship reporting intervals. The following ship reporting intervals were used in the calculations; $\Delta T = [600, 200, 100, 60]$ s . This corresponds to numbers of reports of $n = [1,3,5,10]$ when the observation time is $T_{obs} = 10$ min . Figure 5.37 and Figure 5.38 show the ship detection probability P as a function of number of ships N_{tot} for the different ship reporting intervals ΔT when the observation time is $T_{obs} = 10$ min and $T_{obs} = 20$ min respectively. Table 5.1 gives the values for the intersection points (${}_cN_{tot}^{\Delta T}$, ${}_cP_{\Delta T}$) for different ship reporting interval pairs ($\Delta T_a, \Delta T_b$), while Figure 5.39 shows the ship detection probability ${}_cP_{\Delta T}$ at the intersection point as a function of the observation time T_{obs} for the different ship reporting interval pairs.

Equation (4.9), Figure 5.37, and Figure 5.38 show that the number of ships ${}_cN_{tot}^{\Delta T}$ at the intersection point is *independent* of the observation time T_{obs} . This means that for two ship reporting intervals ΔT_a and ΔT_b , the corresponding ship detection probability curves will always intersect at the same number of ships, independently of how long the observation time is. The corresponding ship detection probability ${}_cP_{\Delta T}$, however, depends on the observation time and *increases* with *increasing* observation time, see Equation (4.2) and Figure 5.37-Figure 5.39.

This behaviour with respect to number of ships and ship detection probability at the intersection point can be illustrated by an example. From Figure 5.37, Figure 5.38, and Table 5.1 we see that the ship detection probability curves for $\Delta T = 600$ s (red curve) and $\Delta T = 200$ s (green curve) intersect for ${}_cN_{tot}^{\Delta T} \approx 10000$ both when $T_{obs} = 10$ min (Figure 5.37) and when $T_{obs} = 20$ min (Figure 5.38). The ship detection probability ${}_cP_{\Delta T}$ at the intersection point, however, increases from ${}_cP_{\Delta T} \approx 68\%$ when $T_{obs} = 10$ min to ${}_cP_{\Delta T} \approx 90\%$ when $T_{obs} = 20$ min . This can also be seen from Figure 5.39 (dark blue curve).

From Figure 5.37 and Figure 5.38 we further notice that the longest ship reporting interval in a pair gives the highest ship detection probability P when $N_{tot} > {}_cN_{tot}^{\Delta T}$, while the opposite is true when $N_{tot} < {}_cN_{tot}^{\Delta T}$. If choosing between two ship reporting intervals, one should therefore choose the longest ship reporting interval if $N_{tot} > {}_cN_{tot}^{\Delta T}$, and the shortest ship reporting interval if $N_{tot} < {}_cN_{tot}^{\Delta T}$, in order to obtain the highest possible ship detection probability P.

6.5 Intersection point for the ship detection probability curves - different numbers of reports

We wanted to study the properties of the intersection point (${}_cN_{tot}^{n}$, ${}_cP_n$) for the ship detection probability curves for different numbers of reports. The following numbers of reports were used in the calculations; $n = [1,3,5,10]$. This corresponds to ship reporting intervals of $\Delta T = [600, 200, 100, 60]$ s when the observation time is $T_{obs} = 10$ min. Figure 5.40 and Figure 5.41 show the ship detection probability P as a function of number of ships N_{tot} for different numbers of reports n when the observation time is $T_{obs} = 10$ min and $T_{obs} = 20$ min respectively. Table 5.2 gives the values for the intersection points (${}_cN_{tot}^{n}$, ${}_cP_n$) for different number of reports pairs (n_a, n_b), while Figure 5.42 shows the number of ships ${}_cN_{tot}^{n}$ at the intersection point as a function of observation time T_{obs} for the different number of reports pairs.

Equation (4.11), Figure 5.40, and Figure 5.41 show that the ship detection probability ${}_cP_n$ at the intersection point is *independent* of the observation time T_{obs}. This means that for two numbers of reports n_a and n_b, the corresponding ship detection probability curves will always intersect at the same ship detection probability, independently of how long the observation time is. The corresponding number of ships ${}_cN_{tot}^{n}$, however, depends on the observation time and *increases linearly* with *increasing* observation time, see Equation (4.7) and Figure 5.40-Figure 5.42.

This behaviour with respect to ship detection probability and number of ships at the intersection point can be illustrated by an example. From Figure 5.40, Figure 5.41, and Table 5.2 we see that the ship detection probability curves for $n = 1$ (red curve) and $n = 3$ (green curve) intersect for ${}_cP_n \approx 68\%$ both when $T_{obs} = 10$ min (Figure 5.40) and when $T_{obs} = 20$ min (Figure 5.41). The number of ships ${}_cN_{tot}^{n}$ at the intersection point, however, increases from ${}_cN_{tot}^{n} \approx 10000$ when $T_{obs} = 10$ min to ${}_cN_{tot}^{n} \approx 20000$ when $T_{obs} = 20$ min. This can also be seen from Figure 5.42 (dark blue curve).

From Figure 5.40 and Figure 5.41 we further notice that the highest number of reports in a pair gives the highest ship detection probability P when $N_{tot} < {}_cN_{tot}^{n}$, while the opposite is true when $N_{tot} > {}_cN_{tot}^{n}$. If choosing between two numbers of reports, one should therefore choose the highest number of reports if $N_{tot} < {}_cN_{tot}^{n}$, and the smallest number of reports if $N_{tot} > {}_cN_{tot}^{n}$, in order to obtain the highest possible ship detection probability P.

6.6 Optimum number of reports

Figure 5.43 shows the optimum number of reports n_0 as a function of ship detection probability P. The figure shows that the optimum number of reports *increases* with *increasing* requirement to the ship detection probability.

We see that for ship detection probabilities $P \leq 50\%$ the optimum number of reports is $n_0 \leq 1$. For ship detection probabilities $50\% < P < 90\%$ the optimum number of reports is $1 < n_0 < 5$, and for $90\% \leq P \leq 100\%$ the optimum number of reports is $4 < n_0 <\sim 10$. Thus we can conclude that the optimum number of reports lies in the range $0 < n_0 <\sim 10$.

Note that the optimum number of reports is *independent* of both the observation time T_{obs} and the number of ships N_{tot}, as well as the overlap factor s and the number of channels n_{ch}. The optimum number of reports depends only on the ship detection probability P (see Equation (4.15)), i.e.,

$$n_0 = n_0(P) \tag{6.7}$$

The optimum number of reports n_0 will therefore be the same for given requirement to the ship detection probability independently of how long the observation time is. However, increasing the observation time will increase the number of ships that the system can handle, see Equation (4.7).

6.7 Optimum ship reporting interval

Figure 5.44 shows the optimum ship reporting interval ΔT_0 as a function of number of ships N_{tot}. The figure shows that the optimum ship reporting interval *increases linearly* with *increasing* number of ships.

From Figure 5.44 we see that for 3000 ships within the field of view the optimum ship reporting interval is $\Delta T_0 = 100$ s, increasing to $\Delta T_0 = 650$ s for 20 000 ships within the field of view.

Note that the optimum ship reporting interval is *independent* of both the observation time T_{obs} and the ship detection probability P. The optimum ship reporting interval depends only on the number of ships N_{tot} (in addition to the number of channels n_{ch} and the overlap factor s which have fixed values for a given system, see Equation (4.17)), i.e.,

$$\Delta T_0 = \Delta T_0(N_{tot}) \tag{6.8}$$

The optimum ship reporting interval ΔT_0 will therefore be the same for given number of ships independently of how long the observation time is. However, increasing the observation time will increase the ship detection probability for the system, see Equation (4.2).

6.8 Optimum message density

For a system where all ships are within communication range of each other, and the message transmissions are coordinated according to the SOTDMA algorithm, the optimum message density is 1 message per slot per channel, i.e., $q_0 = 1$ (assuming an overlap factor of $s = 0$, which is the case when the ships are close enough to be within communication range of each other).

For a space-based AIS system, which effectively behaves as if the ships transmit independently of each other, the optimum message density is 0.693 messages per slot per channel, i.e., $q_0 = 0.693$ when the overlap factor is $s = 0$, see Equation (4.19). Note that this does *not* mean that 69.3% of the available slots are used for transmissions. Since the message transmissions are effectively unorganized, some slots will contain more than one message. The fraction of available slots that is actually used for transmissions will therefore normally be smaller than q_0.

For given overlap factor s, i.e., for given AIS sensor altitude and field of view, the optimum message density q_0 is a constant. The optimum message density is calculated from the number of ships N_{tot}, the optimum ship reporting interval ΔT_0, the number of channels n_{ch}, and the number of slots per second per channel α, see Equation (4.19). For a given system, n_{ch} and α are constant. The ratio $N_{tot}/\Delta T_0$ must therefore also be constant in order to keep q_0 constant. This means that if the number of ships increases the optimum ship reporting interval also increases. This is consistent with results found in Section 6.7.

Figure 5.45 shows the optimum message density as a function of the overlap factor. We see that for increasing overlap factor the optimum message density decreases. For $s = 0$ the optimum message density is $q_0 = 0.693$, decreasing to $q_0 = 0.408$ for $s = 0.7$ (corresponding to an AIS sensor altitude of about 850 km and a field of view to horizon) and to $q_0 = 0.347$ for $s = 1$.

6.9 Optimization of the system

The system can be optimized so that, for given requirement to ship detection probability and number of ships that the system must be able to handle, the necessary observation time is as short as possible.

Figure 5.46 and Figure 5.47 show the minimum required observation time $T_{obs}^{\min}$ as a function of ship detection probability P and number of ships N_{tot} when the optimum ship reporting interval ΔT_0 is used. Table 5.3 shows optimum ship reporting interval with corresponding minimum required observation time for different numbers of ships and ship detection probabilities.

Figure 5.46, Figure 5.47, and Table 5.3 show that the minimum required observation time *increases* with *increasing* requirement to the ship detection probability and *increases linearly* with *increasing* number of ships.

Table 5.3 further shows that for 1000 ships within the field of view the optimum ship reporting interval is 33 s, increasing to 5.5 min for 10 000 ships and 11 min for 20 000 ships within the field of view. Corresponding observation times are (assuming ship detection probabilities in the range 90-99%) 1.8-3.6 min for 1000 ships, 18-36 min for 10 000 ships, and 36-72 min for 20 000 ships within the field of view. In principle, the system could be optimized to handle any number of ships if the appropriate ship reporting interval is chosen, and the observation time is sufficiently long.

6.10 Summary

Based on the analyses in this chapter we have found the following properties for the system:

The ship detection probability *decreases* with *increasing* number of ships, and *increases* with *increasing* observation time and *increasing* number of channels.

The observation time *decreases* with *increasing* number of channels, and *increases* with *increasing* number of ships and *increasing* requirement to the ship detection probability.

The number of ships that the system can handle *decreases* with *increasing* requirement to the ship detection probability, *increases* with *increasing* observation time, and *increases linearly* with *increasing* number of channels.

The intersection point for the ship detection probability curves (ship detection probability as a function of number of ships) for different ship reporting intervals has the following properties:

- The number of ships at the intersection point is *independent* of the observation time
- The ship detection probability at the intersection point *increases* with *increasing* observation time.

The intersection point for the ship detection probability curves (ship detection probability as a function of number of ships) for different numbers of reports has the following properties:

- The ship detection probability at the intersection point is *independent o*f the observation time
- The number of ships at the intersection point *increases linearly* with *increasing* observation time.

There exists an optimum message density for the system with corresponding optimum ship reporting interval and optimum number of reports. The optimum message density depends only on the overlap factor, and is a constant for a system with an AIS sensor at given altitude and field of view. The optimum ship reporting interval depends only on the number of ships and *increases linearly* with *increasing* number of ships. The optimum number of reports depends only on the ship detection probability and *increases* with *increasing* ship detection probability.

The system can be optimized so that, for given requirement to ship detection probability and number of ships that the system must be able to handle, the necessary observation time is as short as possible. For 1000 ships within the field of view the optimum ship reporting interval is 33 s with corresponding observation times 1.8-3.6 min (assuming ship detection probabilities in the range 90-99%), while for 10 000 ships within the field of view the optimum ship reporting interval is 5.5 min with corresponding observation times 18-36 min (assuming ship detection probabilities in the range 90-99%). In principle, the system could be optimized to handle any number of ships if the appropriate ship reporting interval is chosen, and the observation time is sufficiently long.

7 SUMMARY

The recently introduced Universal Shipborne Automatic Identification System (AIS) is a ship-to-ship and ship-to-shore reporting system based on broadcasting of messages in the maritime VHF band. The AIS messages could also be received from space, and this report has studied the behaviour of such a space-based AIS system with respect to the relevant parameters.

The necessary theory for studying the system was developed. Important parameters were found to be; the number of ships within the AIS sensor field of view (N_{tot}), the requirement to the ship detection probability (P), the observation time (T_{obs}), the ship reporting interval (ΔT), the number of slots per second per channel ($\alpha = 37.5$ s^{-1}), the number of channels used for the transmissions ($n_{ch} = 2$), and the overlap factor (s) which depends on the AIS sensor's altitude and field of view.

Based on the theory developed and the analyses performed, a method for optimizing the system was established. For given requirement to ship detection probability and number of ships that the system must be able to handle, the system can be optimized for the shortest possible observation time by choosing the optimum ship reporting interval ΔT_0, where

$$\Delta T_0 = \frac{1}{\ln 2} \cdot \frac{(1+s)}{\alpha \cdot n_{ch}} \cdot N_{tot} \tag{7.1}$$

The minimum required observation time $T_{obs}^{\min}$ for the system will then be

$$T_{obs}^{\min} = \frac{1}{\ln 2} \cdot \Delta T_0 \cdot \ln(1-P)^{-1} \tag{7.2}$$

The analyses have shown that the space-based AIS system can be optimized to handle 10 000 ships within the field of view with a ship detection probability of 99% if the ship reporting interval is set to 5.5 min, and the observation time is at least 36 min. In principle, the system could be optimized to handle any number of ships if the appropriate ship reporting interval is chosen, and the observation time is sufficiently long.

A THE OVERLAP FACTOR

The overlap factor s describes the overlap that occurs when AIS messages that are sent in adjacent time slots from ships in different parts of the observation area partly overlap due to differences in the signal path lengths between each of the ships and the AIS sensor. Its value depends on the AIS sensor's altitude and field of view. We will in the following show how the overlap factor can be calculated.

Figure A.1 shows the observation area as seen from the side and from above. We have assumed that the general observation area is doughnut shaped and symmetric about the central axis. A special case of this will be the *circular* observation area (when $\varphi_{\min} = 0$).

A ship at angular distance φ from the center of the observation area will have the same signal path length R_s to the AIS sensor as all other ships that are at the same angular distance. These ships lie within the dark red area (dA) in Figure A.1. The observation area can be divided into two main parts relative to this area:

Main area I : This area (A_I) is marked by pink colour in Figure A.1 and contains ships that transmit messages that do *not* overlap with messages from adjacent time slots from ships in the dark red area.

Main area II: This area (A_{II}) is marked by grey colour in Figure A.1 and contains ships that transmit messages that *overlap* with messages from adjacent time slots from ships in the dark red area.

The parameters in Figure A.1 have the following meaning:

H_{sat} - AIS sensor altitude

R - Earth radius

R_s - Slant range to the dark red area in the figure

$R_s^{horizon}$ - Slant range to the horizon

$R_s^{\min}$ - Slant range to the inner border of main area I

$R_s^{\max}$ - Slant range to the outer border of main area I

φ - Angular distance from the center of the observation area to the dark red area

$\varphi_{horizon}$ - Angular distance to the horizon

$\varphi_{\min}$ - Angular distance to the inner border of the observation area

$\varphi_{\max}$ - Angular distance to the outer border of the observation area

$\alpha_{\min}$ - Angular distance to the inner border of main area I

$\alpha_{\max}$ - Angular distance to the outer border of main area I

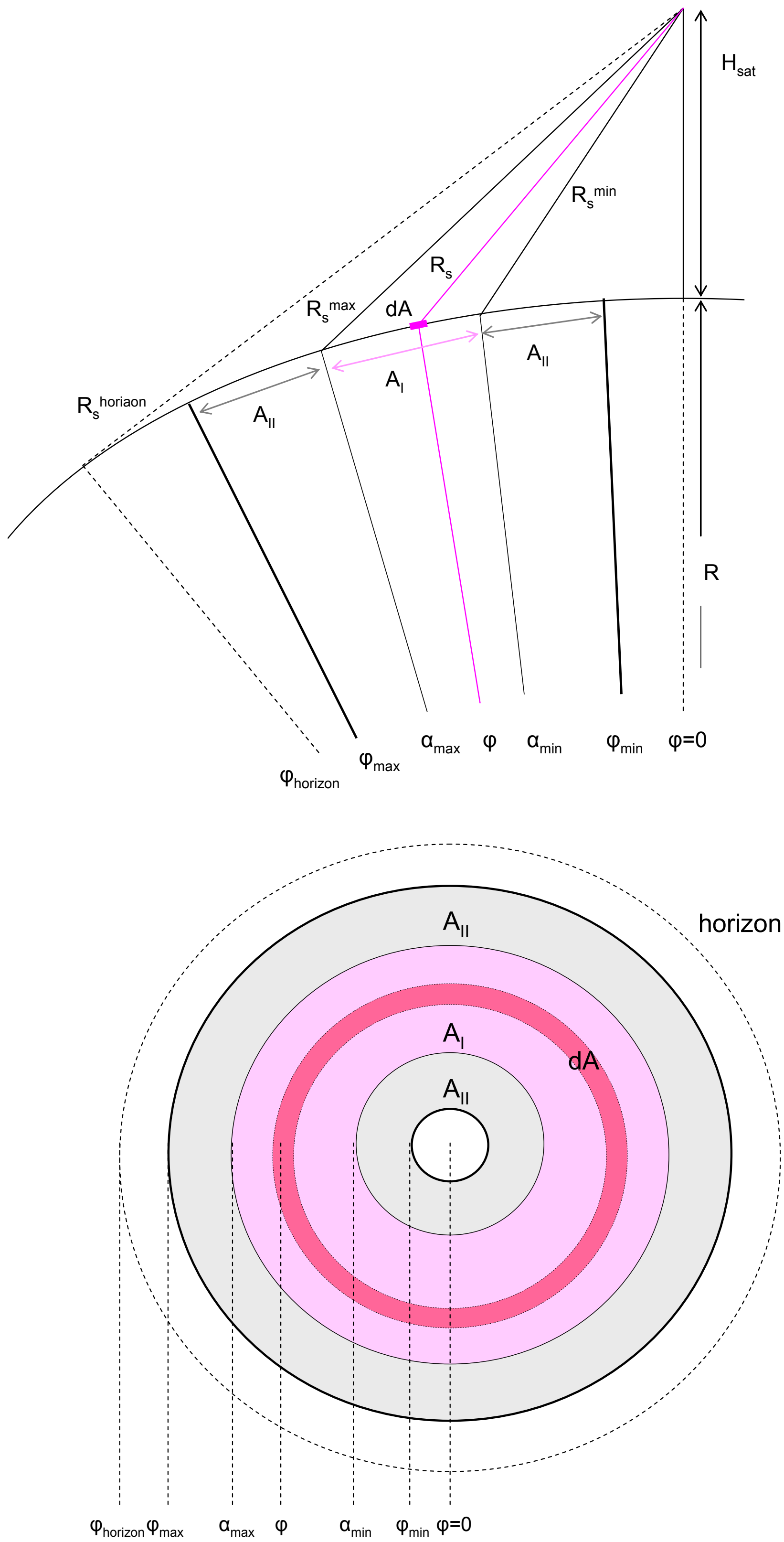

Figure A.7.1 The observation area seen from the side (upper figure) and from above (lower figure). See main text for a description of the different parameters.

If assuming an even ship distribution within the observation area, the overlap factor s can be written

$$s = \int \left(\frac{dA}{A_{tot}} \right) \cdot \left(\frac{A_{II}}{A_{tot}} \right) = \frac{1}{A_{tot}} \int \left(1 - \frac{A_I}{A_{tot}} \right) \cdot dA \tag{A.1}$$

where dA corresponds to the dark red area in Figure A.1, A_{tot} is the area of the observation area, A_I is the area of main area I, and A_{II} is the area of main area II . The last three parameters are connected through

$$A_{II} = A_{tot} - A_I \tag{A.2}$$

The area dA can further be calculated from

$$dA = 2\pi R^2 \sin\varphi \cdot d\varphi \tag{A.3}$$

while the area A_{tot} of the observation area can be found from

$$A_{tot} = 2\pi R^2 \int_{\varphi_{\min}}^{\varphi_{\max}} \sin\varphi \cdot d\varphi = 2\pi R^2 \left(\cos\varphi_{\min} - \cos\varphi_{\max} \right) \tag{A.4}$$

and the area A_I of main area I can be calculated from

$$A_I = 2\pi R^2 \int_{\alpha_{\min}(\varphi)}^{\alpha_{\max}(\varphi)} \sin\alpha \cdot d\alpha = 2\pi R^2 \left(\cos\left[\alpha_{\min}(\varphi)\right] - \cos\left[\alpha_{\max}(\varphi)\right] \right) \tag{A.5}$$

Substituting Equations (A.3)-(A.5) into Equation (A.1) gives the following equation for the overlap factor

$$\boxed{s = 1 - \frac{1}{\left(\cos\varphi_{\min} - \cos\varphi_{\max} \right)^2} \int_{\varphi_{\min}}^{\varphi_{\max}} \left(\cos\left[\alpha_{\min}(\varphi)\right] - \cos\left[\alpha_{\max}(\varphi)\right] \right) \sin\varphi \cdot d\varphi} \tag{A.6}$$

where

$$\alpha_{\min}(\varphi) = \arccos\left(\frac{R^2 + \left(R + H_{sat} \right)^2 - R_s^{\min}(\varphi)}{2R\left(R + H_{sat} \right)} \right) \tag{A.7}$$

$$\alpha_{\max}(\varphi) = \arccos\left(\frac{R^2 + \left(R + H_{sat} \right)^2 - R_s^{\max}(\varphi)}{2R\left(R + H_{sat} \right)} \right)$$

and

$$R_s^{\min}(\varphi) = \max\left[\left(R_s(\varphi) - \Delta R_s\right), R_s(\varphi_{\min})\right]$$
$$R_s^{\max}(\varphi) = \min\left[\left(R_s(\varphi) + \Delta R_s\right), R_s(\varphi_{\max})\right] \tag{A.8}$$

where ΔR_s (=202 nm) is the maximum difference in signal path lengths that can be tolerated without causing overlap and

$$R_s(\varphi) = \sqrt{R^2 + \left(R + H_{sat}\right)^2 - 2R\left(R + H_{sat}\right)\cos\varphi} \tag{A.9}$$

For numerical evaluation of the overlap factor the following expression can be used

$$\boxed{s = 1 - \frac{1}{\left(\cos\varphi_{\min} - \cos\varphi_{\max}\right)^2}\sum_{n=0}^{N}\left(\cos\left[\alpha_{\min}(\varphi_n)\right] - \cos\left[\alpha_{\max}(\varphi_n)\right]\right)\sin\varphi_n \cdot \Delta\varphi} \tag{A.10}$$

where

$$\varphi_n = \varphi_{\min} + n \cdot \Delta\varphi \tag{A.11}$$

and

$$\Delta\varphi = \frac{\varphi_{\max} - \varphi_{\min}}{N} \tag{A.12}$$

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