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REIF Bjørn A P, ANDREASSEN Øyvind

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8) ABSTRACT

This report examines the postulate of local isotropy in stratified homogeneous turbulence from a theoretical point of view. The study is based on a priori analysis of the evolution equations governing single-point turbulence statistics that are formally consistent with the Navier-Stokes equations. The Boussinesq approximation has been utilized to account for the effect of buoyancy – a simplifying assumption that constitutes an excellent approximation in the case considered here. The study concludes that the hypothesis of local isotropy is formally inconsistent with the Navier-Stokes equations in homogeneous stratified turbulence. An estimate is provided that suggests that local isotropy may constitute only a physically justifiable approximation in the limit of a clear-cut separation between the time scales associated with the imposed buoyancy and the turbulent eddy-turnover time scale. This is unlikely to happen in most flows, at least those not too far from equilibrium. The results also suggest that the dynamical dependence of the small-scale turbulence on large-scale anisotropies associated with imposed density stratification is significantly stronger than that caused by an imposed mean straining. This report has in a revised form been published in SIAM Journal of Applied Mathematics, 2003, Vol. 64, No. 1, pp. 309-321.

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CONTENTS

A THEORETICAL STUDY OF SMALL SCALE TURBULENCE IN STRATIFIED TURBULENT SHEAR FLOWS

1 INTRODUCTION

Fluid dynamical processes in virtually all practical applications are strongly dominated by a rapid temporal and spatial variation of velocity and pressure; this is something we usually refer to as a turbulent motion. Turbulence is present in almost all fluid flow configurations, be it in air or water, and when it is present it tends to dominate the dynamics. The success of computational modelling of for instance dispersion of toxins, or other substances, in air or water is crucially dependent on the ability to predict the significantly enhanced mixing caused by turbulence. The ability to predict turbulence is equally crucial in aero- and hydrodynamical applications, as well as in atmospheric and oceanographic processes.

The problem of turbulence is not a problem of physical law; it is a problem of description. To some extent we are now able to compute turbulence *exactly*[1], that is to numerically solve the set of fundamental equataions that governs turbulent fluid flow (so-called “Direct Numerical Simulations”). Although we have experienced a dramatic increase in computer speed over the last decade or so, however, the computer power is still far from being sufficient; one could even argue if it ever will be in order to compute ’real’ turbulent flows. The reason it is so difficult to predict turbulence lies in its very nature; the enormous range of scales. In the atmosphere for instance, it can range from scales in the order of kilometer all the way down to a fraction of a meter, or even centimeter.

Direct numerical simulations invitable generates an enormous amount of data. These data sets are so large that we not only have diffulties managing within available computer resources, but also to extract useful information from. In order to compress the huge amount of information we usually rely on a statistical description of turbulence. This is the approch adopted here.

The present study is motivated by a need to gain a deeper understanding of the small scale dynamics of turbulence in a sheared and stratified environement. In particular, the dynamics of the smallest scales of motion in a turbulence field affected by shearing and buoyant forces imposed by mean velocity and temperature gradients, respectively. This need stems from problems related to the modeling of startified shear flows; virtually all present models for small-scale turbulence are fundamentally flawed.

The terminology ’small scales’ alludes to the scales at which kinetic energy is transformed into internal energy by the action of viscosity. The rate at which energy is dissipated in this small scale regime is traditionally believed to be closely related to the rate at which energy is transfered from the very largest, energy supplying, scales to sequentially smaller and

[1] but only within the errors of our computer model.

smaller scales, until it finally reaches the smallest ones where it is dissipated into heat. This concept is usually referred to as the turbulent energy cascade and it plays a crucial role in our present understanding of turbulence. An edited version of this report has been published in SIAM Journal of Applied Mathematics, 2003, Vol. 64, No. 1, pp. 309-321.

1.1 Local isotropy

It is well established that the imposition of density stratification and mean straining significantly promotes anisotropy on the energetic large-scale turbulence motion. It is frequently also argued that the small-scale motion would remain virtually unaffected by the large-scale anisotropy at sufficiently high Reynolds number (*Re*). This view inherently assumes that any direct effects of the large-scale motion on the smallest scales would be negligible at high enough *Re*, and that large-scale anisotropies would not mediate across the spectral gap fast enough to overcome the nonlinear scrambling of the cascade process. Small-scale turbulence is therefore expected to be statistical independent of the large-scale motion at sufficiently high *Re*. This is essentially the postulate of local isotropy put forward by A. N. Kolmogorov [6] more than 70 years ago; a postulate that has been enormously influential in turbulence research.

The conjecture of locally isotropic turbulence is sometimes also based on the notion of a clear-cut separation of characteristic time scales; since the limiting behavior of the small-to-large scale time-scale ratio asymptotes to $\tau/T \sim Re^{-1/2} \to 0$ as $Re \to \infty$, it is believed that small-scale turbulence would have sufficiently long time to interact with itself, and to establish a state of directional independence, or local isotropy.

The terminology 'local isotropy' alludes to statistical isotropy of the smallest, dissipative scales of motion; i.e.scales much smaller than the energetic large-scale motion. Mathematically, 'isotropy' implies that any statistical measure must display invariance to arbitrary reflections and rotations. Local isotropy is, however, not only a concept of theoretical interest. It is in fact widely used for instance by experimentalists to infer the rate of viscous dissipation of turbulent kinetic energy (formally defined as $\varepsilon \equiv 2\nu s'_{ij} s'_{ij}$) by only conducting measurements of one of the 6 independent components of the fluctuating rate-of-strain tensor s'_{ij} (defined in (2.8)). In particular, by imposing the assumption of local isotropy the number of derivative correlations that must be determined can be reduced from twelve to just a single one, e.g. $\varepsilon \sim \overline{(\partial_1 u_1)^2}$, see e.g. [5].

There exist several hundred articles and papers on the concept of locally isotropic turbulence. Among the pioneering ones are due to Kolmogorov [6] and Obukhov [10], to only mention a few. Monin and Yaglom [8] provides an extensive review on the early developments of the topic whereas more recent reviews are provided by Nelkin [9], Frisch [3], Sreenivasan and Antonia [14], and Warhaft [20]. Among the many studies there are a growing number of theoretical, experimental and numerical investigations that suggest that the concept of local isotropy is somewhat dubious. Townsend [17] and Uberoi [18] were probably among the first to suggest that there exists a direct effect of large scale anisotropy on the dissipative scales of motion, in addition to the indirect influence through the cascading process. This view was supported by e.g. Durbin and Speziale [2] who demonstrated that, as a formally consistent consequence of the Navier-Stokes equations, there must indeed exist a direct effect of mean straining on the dissipative scales. They concluded that local isotropy is a physically implausible argument in turbulence affected by mean straining.

Brasseur and Wei [1], and Yeung *et al.* [22] conducted numerical studies of the triadic interactions in forced turbulence. These studies demonstrated that triadic interactions between widely disparate scales directly modified the structure of the smallest scales in accordance with the structure of the large energetic ones. Experimental results in uniform turbulent shear flow [12] also imply a direct coupling between the large- and small-scales in strained turbulence. They further concluded, fully in line with Durbin and Speziale [2] that the hypothesis of local isotropy in isothermal turbulent shear flows seems untenable even in the limit of infinite *Re*.

Sreenivasan [13] reviewed experimental work on local isotropy of passive scalar fields, and suggested that local isotropy is not a natural concept for scalar fields in shear flows, except perhaps for such extreme *Re* that are of no practical use on earth. Van Atta [19] analyzed experimental data in stably stratified turbulence and noted that the effects are surprisingly rapid, destroying the directional independence of the smallest scales as soon as buoyancy forces become dynamically important. This was essentially confirmed by the enormous numerical simulations of Werne and Fritts [21] who studied a stratified shear layer. They found that turbulence affected by mean straining tends to develop a state of local streamwise axisymmetry, as opposed to local isotropy. The concept of locally axisymmetric turbulence in strained homogeneous flows has been theroretically and experimentally considered by George and Hussain [5] who concluded that a theory of local axisymmetry provides more credibility to the numerous measurements that have failed to confirm local isotropy. These findings, along with many more not mentioned here, add to the body of literature that shed new light on the consept of locally isotropic turbulence.

The present study examines local isotropy from a theoretical point of view. It extends the approach suggested in [2] to homogeneous flows affected by both density stratification and mean straining. The methodology is based on an examination of the dynamical equations governing single-point turbulence correlations that are characteristic of small-scale turbulence; these equations are formally consistent with the Navier-Stokes equations. The objective of the study is to provide insight of whether or not the hypothesis of local isotropy is a formally consistent concept in stratified flows, and if not, to also provide an estimate under what circumstances it would constitute a physically plausible approximation. The practical implications are related to the development of semi-empirical models intended to describe the statistical coupling between large- and small-scale turbulence; a development which is crucial for improved turbulence model formulations.

2 THE EVOLUTION OF SINGLE-POINT TURBULENCE STATISTICS

The present analysis is based on the incompressible Navier-Stokes equations in the limit of homogeneous turbulence, and to cases where the Boussinesq approximation constitutes a reasonable assumption. The latter assumption is not believed to be a severe limitation in the present context; the Boussinesq approximation represents a first order pertubation of the fluid density. In cases where this approximation fails, an even stronger effect of buoyancy is expected.

Single-point turbulence statistics allude to correlations of fluctuating quantities evaluated at the same position in space and time. Dynamical equations governing these statistics can be rigorously derived from the conservation equations for mass, momentum (Navier-Stokes) and energy:

$$\partial_i \tilde{u}_i = 0, \tag{2.1}$$

$$\partial_t \tilde{u}_i + \tilde{u}_k \partial_k \tilde{u}_i = -\partial_i \tilde{p} + \nu \nabla^2 \tilde{u}_i + \frac{\rho}{\rho_0} g_i, \tag{2.2}$$

$$\partial_t \tilde{\theta} + \tilde{u}_k \partial_k \tilde{\theta} = \kappa \nabla^2 \tilde{\theta} + 2\frac{\nu}{c_v} \tilde{s}_{ij} \tilde{s}_{ij}. \tag{2.3}$$

Repeated indices imply summation, e.g. $\tilde{u}_k \partial_k \tilde{\theta} = \tilde{u}_1 \partial_1 \tilde{\theta} + \tilde{u}_2 \partial_2 \tilde{\theta} + \tilde{u}_3 \partial_3 \tilde{\theta}$. The superscript $\tilde{}$ denotes instantaneous quantities, the subscript $_0$ denotes a constant reference state, and $\tilde{s}_{ij} \equiv \frac{1}{2}\left(\partial_i \tilde{u}_j + \partial_j \tilde{u}_i\right)$ is the instantaneous rate-of-strain tensor. Spatial and temporal differentiation are denoted $\partial_m \equiv \partial/\partial x_m$ and $\partial_t \equiv \partial/\partial t$, respectively, and $\nabla^2 = \partial^2_{mm} \equiv \partial^2/(\partial x_m \partial x_m)$. Here, $\nu = \mu/\rho_0$ is the kinematic viscosity, and $\kappa = \alpha/(\rho_0 c_v)$ the thermal diffusivity, where μ, α and c_v denote the dynamic viscosity, thermal conductivity and specific heat, respectively. $\mathbf{g}$ is the gravitational acceleration. According to the the Boussinesq approximation, the density ratio ρ/ρ_0 in (2.2) varies according to

$$\frac{\rho}{\rho_0} = 1 - \beta(\tilde{\theta} - \Theta_0) \tag{2.4}$$

where $\beta \equiv [-\partial \log(\rho)/\partial\theta]_\Theta$ defines the thermal expansion coefficient at fixed mean temperature $\Theta(\mathbf{x}, t)$.

Equations governing fluctuating quantities can systematically be derived using the following procedure:

1. Decomposing the instantaneous velocity, pressure and temperature fields into mean and fluctuating parts, i.e. $\tilde{a}(\mathbf{x}, t) = A(\mathbf{x}, t) + a(\mathbf{x}, t)$.

2. Average to obtain the dynamical equation for the mean field; $A(\mathbf{x}, t) \equiv \overline{\tilde{a}(\mathbf{x}, t)}$, since $\overline{a(\mathbf{x}, t)} \equiv 0$ by definition.

3. Obtaining the evolution equations for the fluctuating fields $a(\mathbf{x}, t)$ by subtracting 2 from 1.

Using this procedure, the evolution of the $i-$th component of the fluctuating velocity $u_i(\mathbf{x},t)$ for an incompressible fluid can then be written as

$$\partial_t u_i + U_k \partial_k u_i + u_k \partial_k U_i + u_k \partial_k u_i + \overline{u_k \partial_k u_i} = -\frac{1}{\rho_0}\partial_i p + \nu \nabla^2 u_i - \beta g_i \theta, \tag{2.5}$$

$$\partial_i u_i = 0. \tag{2.6}$$

Here, $\mathbf{U}(\mathbf{x},t)$ denotes the mean velocity field, and $\theta(\mathbf{x},t)$ is the fluctuating temperature field. The corresponding dynamical equation governing the evolution of the fluctuating temperature field $\theta(\mathbf{x},t)$ reads:

$$\begin{aligned} \partial_t \theta + U_m \partial_m \theta &= -u_m \partial_m \Theta - u_m \partial_m \theta + \kappa \nabla^2 \theta + 4\frac{\nu}{c_v} S_{ij} s'_{ij} \\ &\quad + 2\frac{\nu}{c_v}\left(s'_{ij}s'_{ij} - \overline{s'_{ij}s'_{ij}}\right). \end{aligned} \tag{2.7}$$

Here,

$$s'_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) \tag{2.8}$$

and

$$S_{ij} = \frac{1}{2}(\partial_i U_j + \partial_j U_i) \tag{2.9}$$

denote the fluctuating and mean rate-of-strain tensor, respectively.

Transport equations governing suitable turbulence correlation can now be constructed from (2.5) – (2.7), and the results are formally consistent with the incompressible Navier-Stokes equations in the limit of the Boussinesq approximation. The assumption of homogeneity constitutes the only additional simplification and it implies that statistical measures of the flow must be translational invariant, i.e. single-point correlations are spatially constant.

The fluctuating pressure field $p(\mathbf{x},t)$ in (2.5) is the solution to a Poisson equation which can be obtained by taking the divergence of (2.5). Invoking the incompressibility and homogeneity constraints then gives

$$\nabla^2 p = -\rho_0 \partial_i u_k \partial_k u_i - 2\rho_0 \partial_k u_i \partial_i U_k - \rho_0 \beta g_i \partial_i \theta \tag{2.10}$$

which represents nonlocal effects on single-point statistics [2]. The fluctuating momentum and temperature equations, (2.5) and (2.7), can symbolically be written on operator form as $\mathcal{R}u_i = 0$ and $\mathcal{R}\theta = 0$, respectively. The transport equation governing the second-order moments, $\tau_{ij} = \overline{\tau'_{ij}} \equiv \overline{u_i u_j}$, is readily obtained by multiplying (2.5) by u_j, adding the result to itself with i and j interchanged, and finally averaging. This can symbolically be written as $\overline{u_j \mathcal{R} u_i + u_i \mathcal{R} u_j} = 0$. The result for homogeneous turbulence reads

$$\begin{aligned} d_t \tau_{ij} &= -\frac{1}{\rho_0}\left(\overline{u_j \partial_i p} + \overline{u_i \partial_j p}\right) - (\tau_{ik}\partial_k U_j + \tau_{jk}\partial_k U_i) \\ &\quad - \varepsilon_{ij} - \beta\left(g_i \overline{u_j \theta} + g_j \overline{u_i \theta}\right) \end{aligned} \tag{2.11}$$

[2] It is interesting to note that the solution of (2.10) shows that the evolution of single-point moments implicitely depend on two-point correlations, i.e. correlations of velocity components evaluated at different position in space, see e.g. Rotta [11] for more details.

where $d_t \equiv d/dt$ is the local time derivative. Recall that all spatial derivatives of turbulence correlations are zero in homogeneous turbulence. The second-order viscous dissipation rate tensor is given by

$$\varepsilon_{ij} = 2\nu\overline{\varepsilon'_{ij}} \equiv 2\nu\overline{\partial_m u_i \partial_m u_j}. \tag{2.12}$$

The evolution equation governing the turbulent kinetic energy, $k \equiv \frac{1}{2}\tau_{ii}$, is obtained by taking the trace of (2.11), and multiplying by $\frac{1}{2}$;

$$d_t k = -\tau_{ik}\partial_k U_i - \varepsilon - \frac{1}{2}\beta g_i \overline{u_i\theta}, \tag{2.13}$$

where $\varepsilon \equiv \frac{1}{2}\varepsilon_{ii}$ is the rate of turbulent energy dissipation.

By first writing (2.11) as $\overline{\mathcal{R}\tau'_{ij}} = 0$, the corresponding transport equation for the third-order moments, $\tau_{ijk} \equiv \overline{\tau'_{ijk}} \equiv \overline{u_i u_j u_k}$, can then be derived as $\overline{u_k \mathcal{R}\tau'_{ij} + \tau'_{ij}\mathcal{R}u_k} = 0$. The result can be written as

$$\begin{aligned} d_t\tau_{ijk} \quad = \quad & -\frac{1}{\rho_0}\left(\overline{\tau'_{ij}\partial_k p} + \overline{\tau'_{ik}\partial_j p} + \overline{\tau'_{jk}\partial_i p}\right) \\ & - \left(\tau_{mij}\partial_m U_k + \tau_{mik}\partial_m U_j + \tau_{mjk}\partial_m U_i\right) \\ & -\varepsilon_{ijk} - \beta\left(g_k\overline{\tau'_{ij}\theta} + g_j\overline{\tau'_{ik}\theta} + g_i\overline{\tau'_{jk}\theta}\right) \end{aligned} \tag{2.14}$$

where

$$\varepsilon_{ijk} \equiv 2\nu\left(\overline{u_i\varepsilon'_{jk}} + \overline{u_j\varepsilon'_{ik}} + \overline{u_k\varepsilon'_{ij}}\right) \tag{2.15}$$

denotes the third-order viscous dissipation rate tensor.

The equation governing the transport of turbulent heat flux ($\overline{u_i\theta}$) can readily be derived as $\overline{\theta\mathcal{R}u_i + u_i\mathcal{R}\theta} = 0$:

$$d_t\overline{u_i\theta} = -\frac{1}{\rho_0}\overline{\theta\partial_i p} + \mathcal{P}_{i\theta} - \varepsilon_{i\theta} - \beta g_i\overline{\theta^2} + 4\frac{\nu}{c_v}S_{kj}\overline{u_i s'_{kj}} + 2\frac{\nu}{c_v}\overline{u_i s'_{kj}s'_{kj}} \tag{2.16}$$

where

$$\varepsilon_{i\theta} \equiv (\kappa + \nu)\,\overline{\partial_m\theta\partial_m u_i} \tag{2.17}$$

and

$$\mathcal{P}_{i\theta} = -\left(\overline{u_m\theta}\partial_m U_i + \tau_{mi}\partial_m\Theta\right) \tag{2.18}$$

represent the rate of dissipation and production of turbulent heat flux, respectively.

To this end the dynamical equations governing the turbulent heat flux ($\overline{u_i\theta}$), and second- and third-order velocity moments (τ_{ij} and τ_{ijk}) have been derived. This rather limited choice of basic single-point correlations suffices to assess the validity of the local isotropy postulate in stratified turbulence, and to provide an estimate of when this hypothesis may constitute a physically plausible approximation. It should be noted however, that the abovementioned correlations are characteristic for the large-scale energetic part of the turbulence spectrum. In order to study the dynamics of the dissipative scales, on the other hand, correlations characteristic for these scales must be considered. In particular the dynamical equations governing the dissipation rate tensors ε_{ij}, ε_{ijk} and $\varepsilon_{i\theta}$ appearing in (2.11), (2.14) and (2.16), respectively. These tensors comprise correlations between fluctuating gradients and characterise therefore the high wave-number part in spectral space, or the small scales in physical space.

The transport equation for dissipation rate $\varepsilon_{i\theta}$ of turbulent heat flux can be derived as $(\kappa+\nu)\overline{\partial_m\theta\partial_m(\mathcal{R}u_i)} + (\kappa+\nu)\overline{\partial_m u_i\partial_m(\mathcal{R}\theta)} = 0$ and the result can be written as

$$\begin{aligned} d_t\varepsilon_{i\theta} &= -\varepsilon_{k\theta}\partial_k U_i - 2\mathcal{E}_{mk\theta i}\partial_m U_k + 4\frac{\nu}{c_v}S_{kj}\mathcal{J}_{ikj} \\ &\quad -\frac{1}{2}\left(1+Pr^{-1}\right)\varepsilon_{ik}\partial_k\Theta - \frac{1}{2}\beta\left(1+Pr\right)g_i\varepsilon_\theta + \mathcal{F}_{i\theta} \end{aligned} \tag{2.19}$$

for homogeneous turbulence where $Pr \equiv \nu/\kappa$ is the Prandtl number, $\mathcal{E}_{mk\theta i} = \overline{\partial_m\theta\partial_k u_i}$, and $\mathcal{J}_{ikj} = \overline{\partial_m u_i \partial_m s'_{kj}}$. The dissipation rate of temperature variance $\overline{\theta^2}$ is defined as

$$\varepsilon_\theta \equiv 2\kappa\overline{\partial_m\theta\partial_m\theta} \tag{2.20}$$

whereas the last term in (2.19) is

$$\begin{aligned} \mathcal{F}_{i\theta} &= -\frac{\kappa+\nu}{\rho_0}\overline{\partial_i\theta\nabla^2 p} + (\kappa+\nu)\left(\overline{u_n\partial_n u_i\nabla^2\theta} + \overline{u_n\partial_n\theta\nabla^2 u_i}\right) \\ &\quad + (\kappa+\nu)^2\,\overline{\nabla^2\theta\nabla^2 u_i} + 2\frac{\nu}{c_v}\overline{\partial_m u_i\partial_m(s'_{kj}s'_{kj})}. \end{aligned} \tag{2.21}$$

The evolution equations for ε_{ij} is derived as $\mathcal{R}\varepsilon_{ij} = 2\nu[\overline{u_i\partial_m(\mathcal{R}u_j)} + \overline{u_j\partial_m(\mathcal{R}u_i)}] = 0$ and the result reads

$$d_t\varepsilon_{ij} = \mathcal{H}_{ij} - 2\mathcal{E}_{mkij}\partial_m U_k - \left(\varepsilon_{jk}\partial_k U_i + \varepsilon_{ik}\partial_k U_j\right) - \beta\left(g_i\overline{\varepsilon'_{j\theta}} + g_j\overline{\varepsilon'_{i\theta}}\right) \tag{2.22}$$

where $\mathcal{E}_{ijkm} \equiv 2\nu\overline{\partial_i u_k\partial_j u_m}$ and

$$\begin{aligned} \mathcal{H}_{ij} &= -4\nu^2\overline{\partial^2_{mk}u_i\partial^2_{mk}u_j} - 2\nu\left(\overline{\varepsilon'_{jk}\partial_k u_i} + \overline{\varepsilon'_{ik}\partial_k u_j}\right) \\ &\quad -\frac{2\nu}{\rho_0}\overline{\left(\partial_j u_i + \partial_i u_j\right)\nabla^2 p}. \end{aligned} \tag{2.23}$$

The corresponding evolution equation for third-order dissipation rate tensor ε_{ijk} (2.15) is obtained as $\mathcal{R}\varepsilon_{ijk} = 2\nu(\mathcal{L}_{kij} + \mathcal{L}_{jik} + \mathcal{L}_{ijk}) = 0$ where $\mathcal{L}_{kij} = \overline{(u_k\mathcal{R}\varepsilon'_{ij} + \varepsilon'_{ij}\mathcal{R}u_k)}$. After some algebra, the final result can be symbolically written as

$$d_t\varepsilon_{ijk} = \mathcal{P}_{ijk} + \mathcal{U}_{ijk} + \mathcal{G}_{ijk} + \mathcal{N}_{ijk} \tag{2.24}$$

where

$$\mathcal{P}_{ijk} = P_{ijk} + P_{jik} + P_{kij} \tag{2.25}$$
$$\mathcal{U}_{ijk} = U_{ijk} + U_{jik} + U_{kij} \tag{2.26}$$
$$\mathcal{G}_{ijk} = G_{ijk} + G_{jik} + G_{kij} \tag{2.27}$$
$$\mathcal{N}_{ijk} = N_{ijk} + N_{jik} + N_{kij} \tag{2.28}$$

and

$$
\begin{aligned}
P_{kij} \;=\; & -\frac{2\nu}{\rho_0}\left(\overline{(u_i\nabla^2 u_j + u_j\nabla^2 u_i)\partial_k p}\right) \\
& -\frac{2\nu}{\rho_0}\left(\overline{(u_i\nabla^2 u_k + u_k\nabla^2 u_i)\partial_j p}\right) \\
& -\frac{2\nu}{\rho_0}\left(\overline{(u_k\nabla^2 u_j + u_j\nabla^2 u_k)\partial_i p}\right) \\
& -\frac{4\nu}{\rho_0}\overline{(\partial_i(u_j u_k) + \partial_j(u_i u_k) + \partial_k(u_i u_j))\nabla^2 p},
\end{aligned}
\tag{2.29}
$$

$$
\begin{aligned}
U_{kij} \;=\; & -2\nu\left(\overline{u_k\left(\partial_m u_i \partial_n u_j + \partial_m u_i \partial_n u_j\right)}\right)\partial_m U_n \\
& -\varepsilon_{nij}\partial_n U_k - \varepsilon_{knj}\partial_n U_i - \varepsilon_{kin}\partial_n U_j,
\end{aligned}
\tag{2.30}
$$

$$
G_{kij} = -2\nu\beta\left(g_i\overline{u_k\varepsilon'_{j\theta}} + \overline{u_k\varepsilon'_{i\theta}} + \overline{u_i\varepsilon'_{j\theta}}\right),
\tag{2.31}
$$

$$
\begin{aligned}
N_{kij} \;=\; & 2\nu\left(\overline{u_k\partial_m u_n\left(\partial_n u_i \partial_m u_j + \partial_n u_j \partial_m u_i\right)} - \overline{u_n\partial_n u_k \partial_m u_i \partial_m u_j}\right) \\
& -2\nu^2\left(\overline{\partial_m u_i \partial_m u_j \nabla^2 u_k} + \overline{\partial_m u_k\left(\partial_m u_j \nabla^2 u_i + \partial_m u_i \nabla^2 u_j\right)}\right) \\
& -4\nu^2\overline{u_k\nabla^2 u_i \nabla^2 u_j}.
\end{aligned}
\tag{2.32}
$$

It follows directly from (2.29) – (2.32) that $P_{kij} = P_{kji}$, $U_{kij} = U_{kji}$, $G_{kij} = G_{kji}$ and $N_{kij} = N_{kji}$. Consequently $\mathcal{P}_{ijk}$, $\mathcal{U}_{ijk}$, $\mathcal{G}_{ijk}$ and $\mathcal{N}_{ijk}$ are symmetric for any permutation of indices, see (2.28). This property is obviously required by the definition of ε_{ijk} (2.15).

3 IMPOSING LOCAL ISOTROPY *A PRIORI*

The theory of isotropic turbulence is essentially based on the fact that all statistical measures of the flow must display invariance to arbitrary reflections and rotations. The properties of isotropic tensors can here be put to good use in order to establish if the postulate is formally consistent with the Navier-Stokes equations. This methodology was first used by Durbin and Speziale [2] where it was applied to the second-order dissipation rate equations (2.12) to investigate the impact of mean straining on the small scales. The objective here is not only to elucidate the impact of density stratification on small scale turbulence, but also to relate it to the impact of mean straining.

It is well known that, at any given order, a general isotropic tensor can be written as a linear combination of a set of linearly independent isotropic tensors. The number of independent isotropic tensors depends on the order of the tensor itself. Here we will consider tensors up to fourth rank. The most general isotropic forms of *any* first, second-, third- and fourth-order isotropic tensor [3] can be written as

$$
\begin{aligned}
\mathcal{X}_i &= 0 && (3.1)\\
\mathcal{X}_{ij} &= \alpha_0\delta_{ij} && (3.2)\\
\mathcal{X}_{ijk} &= \alpha_1\epsilon_{ijk} = 0 && (3.3)\\
\mathcal{X}_{ijkl} &= \alpha_2\delta_{ij}\delta_{kl} + \alpha_3\delta_{ik}\delta_{jl} + \alpha_4\delta_{il}\delta_{jk} && (3.4)
\end{aligned}
$$

where the fundamental isotropic tensor of rank 2 is the Kronecker delta:

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j; \\ 0, & \text{otherwise} \end{cases} \tag{3.5}$$

and of rank 3 the Levi-Civita alternating tensor:

$$\epsilon_{ijk} = \begin{cases} 1, & \text{if } ijk \text{ is from the sequence } 12312; \\ -1, & \text{if } ijk \text{ is from the sequence } 32132; \\ 0, & \text{otherwise,} \end{cases} \tag{3.6}$$

As already alluded to, the implications of the small-scale isotropy postulate can be elucidated by writing the evolution equations (2.12), (2.15) and (2.19) on their most general isotropic forms using (3.1) – (3.4).

3.1 First order velocity-temperature correlations

Let us first consider the equation governing the dissipation rate of turbulent heat flux. The isotropic form of (2.19) is obtained by substituting

$$
\begin{aligned}
\varepsilon_{i\theta}^{ISO} &= 0 && (3.7)\\
\mathcal{F}_{i\theta}^{ISO} &= 0 && (3.8)\\
\varepsilon_{ik}^{ISO} &= \frac{1}{3}\varepsilon_{mm}\delta_{ik} && (3.9)\\
\mathcal{E}_{km\theta i}^{ISO} &\sim \epsilon_{kmi} = 0 && (3.10)\\
\mathcal{J}_{ikj}^{ISO} &\sim \epsilon_{ikj} = 0, && (3.11)
\end{aligned}
$$

which follows from (3.1) – (3.4). The last two results follows from the symmetry properties $\mathcal{E}_{km\theta i} = \mathcal{E}_{mk\theta i}$ and $\mathcal{J}_{ikj} = \mathcal{J}_{ijk}$, where the former only applies to homogeneous turbulence. The isotropic form of (2.19) then becomes

$$0 = -\frac{2}{3}\varepsilon\partial_i\Theta - \varepsilon_\theta g_i\beta Pr \tag{3.12}$$

[3] These are not specific to turbulence correlation tensors, but general valid for first through fourth order tensors.

where $\varepsilon \equiv \frac{1}{2}\varepsilon_{mm}$ is the dissipation rate of turbulent kinetic energy. According to (3.12), isotropy would firstly require that the gravitation (g_i) must act in the direction of the mean temperature gradient $\partial_i\Theta$, which obviously not is genrally true. Secondly, *if* the direction of the gravitational acceleration happens to coincide with the mean temperature gradient, e.g. i=2, the resulting relationship $2\varepsilon\partial_2\Theta = -3\varepsilon_\theta \beta g_2 Pr$ seems far too stringent to be generally true. The implication of local isotropy, i.e. that $\varepsilon_{i\theta} = 0$, is therefore formally inconsistent with the Navier-Stokes equations. In fact, a closer examination of the evolution equation governing the third-order 'generalized' dissipation tensor $\mathcal{E}_{km\theta i}$ yields the additional constraints: $\varepsilon\partial_i\Theta = 0$ if $m \neq k = i$. For $m = k \neq i$, (3.12) is recovered. The terminology 'generalized' alludes to the relation $(\kappa + \nu)\mathcal{E}_{mm\theta i} \equiv \varepsilon_{i\theta}$. Another interesting observation that can be made from (2.19) is that mean straining does not formally conflict with the assumption of local isotropy on this particular level of velocity-temperature correlation.

3.2 Second-order velocity moments

Local isotropy on the second-order moment level requires (2.22) to balance in the isotropic limit (3.4). The terms in (2.22) are replaced by their most general isotropic counterparts, and the result is:

$$\varepsilon_{i\theta}^{ISO} = 0 \tag{3.13}$$

$$\varepsilon_{ij}^{ISO} = \frac{2}{3}\varepsilon\delta_{ij} \tag{3.14}$$

$$\mathcal{H}_{ij}^{ISO} = \frac{1}{3}\mathcal{H}_{mm}\delta_{ij} = \frac{2}{3}\mathcal{H}\delta_{ij} \tag{3.15}$$

$$\mathcal{E}_{mkij}^{ISO} = \varepsilon\left(\alpha_2\delta_{ij}\delta_{kl} + \alpha_3\delta_{ik}\delta_{jl} + \alpha_4\delta_{il}\delta_{jk}\right) \tag{3.16}$$

where the coefficients $\alpha_2 - \alpha_4$ are determined by imposing (i) homogeneity ($\mathcal{E}_{mkij} = \mathcal{E}_{kmij}$); (ii) continuity ($\mathcal{E}_{mkmj} = 0$) and (iii) the definition $\mathcal{E}_{mmij} = 2\varepsilon$. These constraints yield $\alpha_1 = 4/15$, $\alpha_2 = -1/15 = \alpha_3$ The resulting isotropic form of (2.12) can then be written as

$$d_t\varepsilon\delta_{ij} = \mathcal{H}\delta_{ij} - \frac{2}{5}\varepsilon S_{ij}, \tag{3.17}$$

where $\mathcal{H} \equiv \frac{1}{2}\mathcal{H}_{mm}$. This is the equation derived by Durbin and Speziale [2] which proves that the assumption of local isotropy is formally inconsistent with the Navier-Stokes equation on the second-order moment level when mean straining is imposed, i.e. when $i \neq j$. Clearly, the imposition of buoyancy does not render the local isotropy assumption formally invalid on the second-order velocity-moment level. It should further be noted that the implicit dependence on the stratification contained in the fluctuating pressure term in (2.23) does not contribute to the scalar $\mathcal{H}$ in incompressible flows.

Based on the theoretical arguments in the previous section, $\varepsilon_{i\theta}^{ISO} \neq 0$ in general. If we retain $\varepsilon_{i\theta}^{ISO} \neq 0$ and the assumption of local isotropy for rank 2 tensors, however, (3.17) becomes

$$d_t\varepsilon\delta_{ij} = \mathcal{H}\delta_{ij} - \frac{2}{5}\varepsilon S_{ij} - \underbrace{\frac{3}{2}\beta\left(g_i\varepsilon_{j\theta} + g_j\varepsilon_{i\theta}\right)}_{\mathcal{B}_{ij}}. \tag{3.18}$$

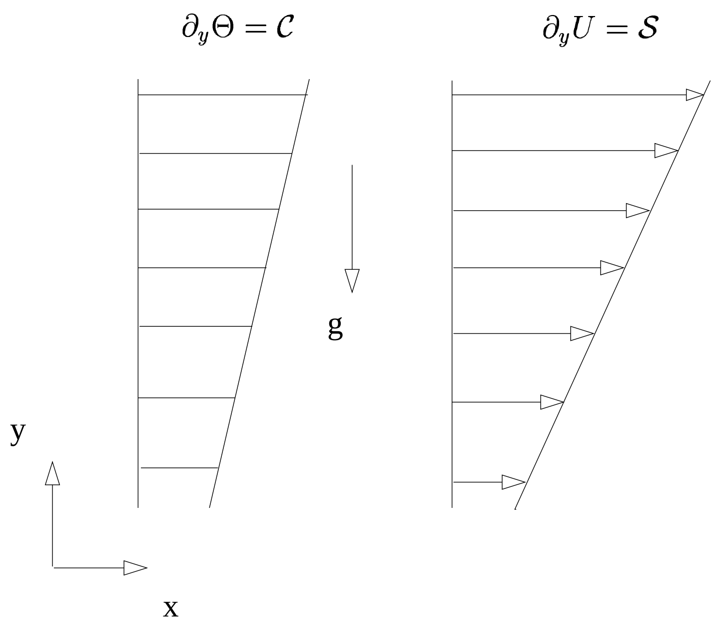

Figure 3.1: Homogeneous shear flow.

Equation (3.18) then provides us with another fact that strongly supports our assumption that $\varepsilon_{i\theta}^{ISO} \neq 0$ should be true; it implies that the rate of decay of ε, in the absence of mean shear ($S_{ij} = 0$) *should be unaffected by any imposed density stratification if the small scale turbulence were truly isotropic*. However, there are no numerical or experimental evidence that this should be the case! On the contrary, it has been observed that even the slightest effect of buoyancy significantly alters the evolution of ε ($d_t\varepsilon$), see e.g. Thoroddsen and Van Atta [16].

In order to provide an estimate of the nonlinear term $\mathcal{H}$ in (3.18) let us consider decaying grid turbulence unaffected by mean straining and stratification. The evolution equation for the turbulent time scale k/ε is readily obtained by combining (2.13) and (3.17). The results reads

$$d_t\left(\frac{k}{\varepsilon}\right) = -\left(1 + \frac{k}{\varepsilon^2}\mathcal{H}\right). \tag{3.19}$$

There exist experimental evidence that grid-generated turbulent kinetic energy exhibits a power-law decay, i.e. $k \sim t^{-n}$, where the decay exponent is $n \approx 1.3$ in a large number of measurements reported in the literature, cf. e.g. [7]. The value of the decay exponent is reported to increase to $n \approx 5/2$ in the final period of decay. The power-law behavior of k requires $\varepsilon \sim t^{-(n+1)}$, and henceforth $k/\varepsilon \sim t$ and $d_t(k/\varepsilon) \sim O(1)$. With this, equation (3.19) provides the estimate

$$\mathcal{H} = \mathcal{O}\left(\frac{\varepsilon^2}{k}\right) \tag{3.20}$$

which are widely used, almost without exception, by turbulence modellers.

Let us now consider homogeneous shear flow with $\partial_y U = \mathcal{S} > 0$, $\partial_y \Theta = \mathcal{C} > 0$ and $\mathbf{g} = [0, -g, 0]$, see figure 3.1. The assumption of local isotropy, in terms of an imposed density stratification, would then be a formally justified approximation if we can neglect $\mathcal{B}$ as compared to $\mathcal{H}$ in (3.18), i.e. if $||\mathcal{B}|| \ll ||\mathcal{H}||$, or equivalently if

$$k\mathcal{C} \gg |Ri_g \varepsilon_{2\theta}| \left(\frac{\mathcal{S}k}{\varepsilon}\right)^2 \tag{3.21}$$

by using the estimate (3.20). The gradient Richardson number $Ri_g \equiv \mathcal{N}^2/\mathcal{S}^2$ and $\mathcal{N}^2 \equiv \beta g \mathcal{C}$ is the Brunt-Väisala frequency. If we consider a flow close to equilibrium, it is reasonable to assume that $\mathcal{P}_{2\theta}/\varepsilon_{2\theta} = \mathcal{O}(1)$ in (2.16), where $\mathcal{P}_{2\theta} \equiv -\tau_{22}\mathcal{C} = -\frac{2}{3}k\mathcal{C}$. The last equality is obtained by substituting the isotropic value $\tau_{22} = \frac{2}{3}k$. Equation (3.21) can then be written as

$$|Ri_g| = \left|\frac{\mathcal{N}^2}{\mathcal{S}^2}\right| \ll \left(\frac{\varepsilon}{\mathcal{S}k}\right)^2 = \mathcal{O}(0.1). \tag{3.22}$$

The right hand side of (3.22) has been evaluated using $\mathcal{S}k/\varepsilon \sim 6$ which typically is reached in physical and numerical experiments of homogeneous shear flows near equilibrium [15]. The constraint (3.22) thus implies that local isotropy constitutes a justifiable approximation only at very small Richardson numbers; in fact so small that buoyancy effect can not essentially be present in practise. The inequality also suggests that the imposition of density stratification exerts a significantly stronger effect on the dissipative scales than an imposed mean straining.

Durbin and Speziale [2] further demonstrated, in the absence of density stratification, that

$$\frac{\mathcal{S}k}{\varepsilon} \ll \mathcal{O}(1) \tag{3.23}$$

is a necessary condition for local isotropy to constitute a formally justified approximation in absence of density stratification. This relation is readily obtained by requiring $||\mathcal{H}|| \gg ||\varepsilon\mathcal{S}||$ in (3.17). Using this and (3.22) yields the combined constraint

$$\left|\frac{\mathcal{N}^2 k^2}{\varepsilon^2}\right| \ll \frac{\mathcal{S}^2 k^2}{\varepsilon^2} \ll \mathcal{O}(1). \tag{3.24}$$

This result implies that the time-scales associated with buoyancy and mean shear must be much larger than the integral turbulent time scale in order for the local isotropy hypothesis to constitute a formally justified approximation. In the absence of mean straining, the magnitude of the Brunt-Väisala frequency is thus required to be much smaller that integral scale turbulent frequency in order for the hypothesis to constitute a physically plausible approximation. This is not feasible in homogeneous flows, at least for flows relatively close to equilibrium.

We can also recast (3.22) in terms of the buoyancy and shear Reynolds numbers frequently used in the literature;

$$Re_B = \left|\frac{\varepsilon}{\nu N^2}\right| = \frac{\varepsilon}{\nu|\beta g \mathcal{C}|}, \qquad \text{and} \qquad Re_S = \frac{\varepsilon}{\nu \mathcal{S}^2} \tag{3.25}$$

by noting that $Ri_g = Re_S/Re_B$. The result (3.24) can then be written as

$$\frac{1}{Re_B} \ll \frac{1}{Re_S} \ll \frac{1}{Re} \tag{3.26}$$

where $Re \equiv k^2/(\varepsilon\nu)$ is the integral scale turbulent Re or, equivalently.

3.3 Third-order velocity moments

Let us finally focus our attention on the evolution of the third-order dissipation rate tensor (ε_{ijk}). If small-scale turbulence on the third-order velocity-moment level were truly isotropic, the evolution equation (2.15) must be fully consistent with the mathematical properties (3.4). The terms $\mathcal{P}_{ijk}, \mathcal{U}_{ijk}$ and $\mathcal{N}_{ijk}$ are all third-rank tensors of the fluctuating velocity field. Since these terms must be symmetric for any permutation of indices, it follows from (3.6);

$$\varepsilon_{ijk}^{ISO} = \mathcal{P}_{ijk}^{ISO} = \mathcal{U}_{ijk}^{ISO} = \mathcal{N}_{ijk}^{ISO} = 0, \qquad \forall \ i, j, k. \tag{3.27}$$

The term $\mathcal{G}_{ijk} \sim g_i d_{j\theta k}$ differs from the other terms in (2.24) in that it only compromises a second-rank tensor of fluctuating quantities, i.e. $d_{j\theta k}$. The most general isotropic form of $d_{j\theta k}$ can then be written as

$$d_{j\theta k}^{ISO} = \frac{1}{3}\overline{u_m \varepsilon'_{m\theta}}\delta_{jk} \neq 0, \tag{3.28}$$

according to (3.2). The result on the third-order velocity-moment level shows that the assumption of local isotropy is formally consistent with Navier-Stokes equations on the third-moment level if, and only if, $\overline{u_m \varepsilon'_{m\theta}} \equiv 0$. This requirement is on the other hand generally not fulfilled.

4 CONCLUDING REMARKS

The present study has demonstrated that the hypothesis of local isotropy is formally inconsistent with the Navier-Stokes equations in homogeneous stratified turbulence, irrespectively whether the stratification is stable or not. The imposition of a mean temperature gradient is shown to essentially affect the small-scale turbulence in the same manner as an imposed mean shear, but with a significant stronger impact.

George [4] has suggested, based on experimental findings, that the small-scale motion remain closely linked to the large-scale coherent motion. Anisotropies of the large scales would thus be reflected over the entire spectral range. These findings are consistent with the results presented herein.

The outcome of the present analysis is also very similar to the findings of Yeung *et al.* [22] although their approach is rather different. They considered the effect of anisotropic large-scale turbulence on the small-scale anisotropy whereas the present study focuses on the imposition mean-flow anisotropy. Despite this difference, both cases reach the same conclusion, namely that the imposition of large-scale anisotropy, be it related to turbulence or the mean-flow, does not show up on all levels of small-scale velocity-moments.

In particular, density stratification does not formally conflict with the local isotropy hypothesis on the second-order level, whereas it shows up for the first- and third-order correlations examined here. Similarly, mean shear does not formally conflict with the isotropy assumption on the first- and third-order levels, whereas it is formally inconsistent on the second-order level. It is however sufficient to show anisotropy on *any* small-scale statistics in order for the local isotropy hypothesis to be violated. This was pointed out by Yeung *et al.* [22] who also argued that the converse is not true; a single statistical measure that displays a state of local isotropy is a necessary but not a sufficient condition to guarantee small-scale isotropy.

A qualitative analysis of the second-order dissipation rate transport equation has indicated that local isotropy constitutes a physically justifiable approximation, at this particular level of single-point moments, only if the imposed time scale associated with buoyancy, or mean straining, is much larger than the integral turbulent time scale. It can therefore be concluded that local isotropy does not seem to be a physically plausible argument in flows relatively close to equilibrium since the imposed and the eddy turnover time scales usually are of the same order. A successful continuation in the development of predictive methods for turbulent flows relies heavily upon the ability to characterise small-sclae turbulence in terms of the large scales. The theoretical outcome of this study has shown that it seems necessary to include information of the *mean* flow field in models for the small-scale turbulence in order to retain some consistency with the Navier-Stokes equations.

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