Adv. Studies Theor. Phys., Vol. 6, 2012, no. 13, 629 - 641

On the Big Bang Cosmology as a Kinematic Event in Newtonian Cosmology Modified by Dark Energy

John F. Moxnes

Department for Protection
Norwegian Defence Research Establishment
P.O. Box 25, 2007 Kjeller, Norway
john-f.moxnes@ffi.no

Kjell Hausken

Faculty of Social Sciences
University of Stavanger
4036 Stavanger, Norway
kjell.hausken@uis.no

Abstract

The concepts dark matter and dark energy are commonly used to explain the universe. The dark energy can be conceived as a property of vacuum or the ether. However, a quite different but also common approach is to modify the equations of gravity. Besides the cosmological constant, modifying the Einstein equations is mathematically challenging within the traditional framework of metric theories of gravity. Such an approach is mathematically much easier within the Newtonian framework such as for example utilized for MOND; i.e. a specific modification of the Newtonian dynamics whenever the gravitational acceleration falls below a critical value. In this article we apply that the vacuum energy (LIVE) in addition to applying a negative pressure of the universe also modifies gravity. We modify gravity from the well known concept of Newtonian cosmology of an isotropic and homogeneous expanding universe. We modify gravity by applying an S^3 space instead of E^3 as background geometry in accordance with some newly published quasi metric theories.

Keywords: Cosmology, dark matter, dark energy, gravity, Einstein equation, cosmological constant, hyper space, gravitation

1 Introduction

The inflation theory says that the present universe should be extreme flat (that means Euclidean). Indeed, observations of the temperature variations in the cosmic background radiation indicate that space is extreme flat. However, an estimate based on the number of galaxies and their average mass only takes a value of about 4% of the mass necessary to achieve this flatness. The observational evidence for the existence of dark matter (DM) is based on the interpretations of astronomical data coming from rotational galaxies, galactic hopes and gravitational lensing to account for 26% of the missing mass (Jullo et al. 2010). The concept of dark energy (DE) is a more recent well known approach for explaining the universe on a cosmological scale. DE is considered to constitute the missing 70% (100-4-26) of the energy or mass (energy = m c^2) in the universe. Recent experimental evidence from supernovas of type 1a indicates that the universe indeed expands faster and faster (http:// www-supernova.1bl.gov/). DE will explain this also.

Although the concept of DM is an acknowledged part of the modern science world view, some particular galactic phenomenology, including spiral galaxy rotation curves seems to challenge the DM hypothesis. DE provides negative pressure (stretch) in the universe. This motivates our approach trying to modify the equations of gravity due to vacuum energy. However, beside the cosmological constant, modifying the Einstein equations is hard to do within the traditional framework of metric theories of gravity (Aguirre et al. 2001, Østvang 2007). Such an approach is mathematically much easier within the Newtonian approach such as for example utilized for MOND; i.e. a specific modification of the Newtonian dynamics whenever the gravitational acceleration falls below a critical value.

In a homogeneous and isotropic universe local knowledge is global knowledge. Our approach in this article is to modify gravity from the well known and so called Newtonian cosmology. This cosmology is conceptually based on the Robertson–Walker expanding universe for an isotropic and homogeneous universe (Bondi 1961). This could indeed be a fruitful exercise since Narlicar (1994) showed that the equivalence of the cosmological and gravitational spectral shift with the pure Newtonian Doppler interpretation can be established provided one parallel transfers the source four vector velocity along the null geodesic to the observer. Obviously so far the concept Newtonian cosmology is applicable, and the red shift of the expanding universe can be considered as Doppler. However, Bunn and Hogg (2009) argued that even in the context of the relativistic theory, a free falling co-moving observer corresponds to the case where the cosmological expansion can be considered as Doppler.

As pointed out by Ehrenfest (1917) neither classical nor planetary orbits can be stable in space with dimensions larger than three, and traditional quantum atoms can not be stable either (Tangherlini 1963). Many superstring theories have several stable states that constitute different effective low-energy theories with different space-time dimensionalities (Albrecht 1994).

There are inflationary models predicting a universe consisting of parts of exponentially large size having different dimensionality (Linde and Zelnikov 1988). In response to the persistent problem of why the Planck scale is 16 orders of magnitude higher than the electroweak scale, the existence of extra dimensions has been suggested. These new dimensions are suggested to be as large as millimeters if one supposes that the field of matter lives in a 3 (space) +1 (time) dimensional hyper surface of the 3-brane and that only gravity can befit from new dimensions. The possibility of extra dimensions would be beneficial for the production of black holes since the Planck scale is reduced to accessible values and the Schwarzschild radius is significantly increased (Arkani-Hamed et al. 1998, Barrau et al. 2004). However, it has been argued that all but 3 (space) +1 (time) dimensional space times might correspond to "dead worlds" devoid of observers (Tegmark 1997) due to stability and predictability.

Recently a non metric space time theory has been presented (Østvang 2002, 2005, 2006, 2007). The theory is based on an $R \times S^3$ background rather than a Minkowski background ($R \times E^3$) as the geometry of the universe without matter. In the quasi metric theory the mathematical modeling of the Hubble expansion is indeed different from those in metric theories. Within the quasi metric theory the canonical description of space time is taken. Some field equations were postulated, but they follow naturally from the conception of the Newton-Cartan theory. Somewhat similar to the Newtonian theory the field equations were only partially coupled to the space time geometry. In this paper we follow a quasi metric approach. We build the theory on the concept of Newtonian (-Cartan) cosmology. Analogous to Østvang we use the $R \times S^3$ instead of $R \times E^3$ as a background geometry for gravitation.

In section 2 we as an introduction recapture some facts related to the cosmological constant and the Friedman models. In section 3 we present the concept of the Newtonian cosmology. In section 4 we expand the Newtonian cosmology to account for $R \times S^3$ instead for $R \times E^3$. Section 5 concludes.

2 The Einstein Equations and the Friedman cosmological models

The Einstein equation is obtained from the principle of least action applied on the Hilbert action. The matter action S_m is given by

$$S_m \overset{def}{=} \frac{1}{c}\int \Lambda\, g^{1/2} d\Omega \tag{2.1}$$

where "def" means definition. The integration is over all time and space. c is the speed of light. Λ is the Lagrangian density. g_{ij} is the metric tensor. g is the determinant of the metric tensor. It follows that

$$\begin{aligned}\delta S_m &= \frac{1}{c}\int \delta\left(g^{1/2}\Lambda\right) d\Omega = \frac{1}{c}\int \left(\frac{\partial\left(g^{1/2}\Lambda\right)}{\partial g^{ik}}\delta g^{ik} + \frac{\partial\left(g^{1/2}\Lambda\right)}{\partial\left(\partial g^{ik}/\partial x^l\right)}\delta\left(\partial g^{ik}/\partial x^l\right)\right) d\Omega \\ &= \frac{1}{2c}\int T_{ij}\left(-g\right)^{1/2}\delta g^{ij} d\Omega, \frac{1}{2}\left(-g\right)^{1/2} T_{ik} \overset{def}{=} \frac{\partial\left(g^{1/2}\Lambda\right)}{\partial g^{ik}} - \frac{\partial}{\partial x^l}\frac{\partial\left(g^{1/2}\Lambda\right)}{\partial\left(\partial g^{ik}/\partial x^l\right)}\end{aligned} \tag{2.2}$$

where T_{ij} is the energy-momentum tensor.

For macroscopic bodies we have

$$T_{ij} \overset{mod}{=} (e+p)u_i u_j - p\, g_{ij} \tag{2.3}$$

where "mod" means model assumption. u_i is the unit four vector, e is the energy density and p is the pressure. Pure Lorentz invariant vacuum energy (LIVE) is defined by $T_{ij} = -p\, g_{ij}$. This gives from equation (2.3) that $e = -p$ and then it follows that $T_{ij} = e\, g_{ij}$.

For the gravitational field the action is given by

$$S_g \overset{def}{=} -\frac{c^3}{16\pi G}\int R(-g)^{1/2} d\Omega \tag{2.4}$$

G is the gravitational constant and R is the scalar curvature of space-time. The variation gives that (Landau and Lifshitz 1980)

$$\delta S_g = -\frac{c^3}{16\pi G}\int \left(R_{ij} - \frac{1}{2}g_{ij}R\right)\left(-g\right)^{1/2}\delta g^{ij} d\Omega \tag{2.5}$$

Then from $\delta S_g + \delta S_m \overset{mod}{=} 0$ the Einstein equation follows as

$$R_{ij} - \frac{1}{2}g_{ij}R = \frac{8\pi G}{c^4}T_{ij} \tag{2.6}$$

The Einstein equation is equivalent to a coupled system of non-linear second order partial differential equations for the space-time metric components. One must solve simultaneously for the space-time metric and the matter distribution.

In order to handle problems in cosmology there have been many attempts to modify the Einstein equation. The most popular version is to add a cosmological constant term in the action integral for the field equation. This was initially suggested by Einstein to save Mach's principle (or more precisely to "save" Mach's principle as he interpreted it). Following Mach's lead Einstein expected that the mass distribution should set inertial frames. However, the Minkowski static space-time solution on the Einstein equations without matter and cosmological constant was indeed the most clearly anti-Machian space-time possible[1]. However, then de Sitter (1917) showed that in fact a static solution could be found without matter even with a cosmological constant (de Sitters universe).

The gravitation action with the cosmological constant is

$$S_g = -\frac{c^3}{16\pi G}\int \left(R+2\lambda\right)(-g)^{1/2}d\Omega \qquad (2.7)$$

This gives the Einstein equation with the cosmological constant, to read

$$R_{ij} - \frac{1}{2}g_{ij}R = \frac{8\pi G}{c^4}T_{ij} + \lambda g_{ij} \qquad (2.8)$$

If one ascribes a small value to the cosmological constant, the presence of this term will not significantly affect gravitation over too large distances of space time, but will lead to the appearance of new types of cosmological solutions which could describe the universe as a whole.

Friedman (1924) found a class of cosmological models. The models are simple because they assume at the outset that the universe is isotropic and spatially homogenous. The line element can be written as

$$ds^2 = c^2dt^2 - a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2\left(Sin^2\theta d\varphi^2 + d\theta^2\right)\right) \qquad (2.9)$$

where k=1 for a closed spherical model. For the open model k=-1 and for the flat model k=0.

The universe expands or contracts as the curvature radius a(t) increases or decreases. A change in a(t) leads to a change in all distances between bodies in space. Thus as a(t) increases, the bodies in such a space move away from each other. From the point of view of an observer located on one of the bodies, it will appear as if this moving away at a given time is proportional to the separation of the bodies. This is in agreement with Hubble's law.

Inserting the metric in equation (2.9) into equation (2.8) one readily finds that (Landau and Lifshitz 1980)

---

[1] Einstein in a letter to de Sitter of March 24, 1917: In my opinion it would be dissatisfying, it there were a conceivable world without matter. The g_{ij} field should rather be determined by the matter, and not able to exist without matter. This is the heart of what I understand by the demand for the relativity of inertia. One could just as well speak of the "material conditionedness of geometry". As long as this demand was not fulfilled, for me the goal of general relativity was not yet completely achieved. This was first achieved through the introduction of the λ term. (See Barbour and Pfister (1995) for more details.)

$$\frac{1}{2}\dot{a}(t)^2-\frac{4}{3}\pi\, a(t)^2 e(t)\frac{G}{c^2}=-\frac{k\,c^2}{2}+\frac{\lambda\,c^2}{6}a(t)^2,(a)$$
$$\dot{e}(t)=-\frac{3\left(e+p\right)\dot{a}(t)}{a(t)},(b) \tag{2.10}$$

One can consider different types of models (or combinations of models) for the universe, to read

$$\begin{aligned}
&a)\ Dust: p \overset{mod}{=} 0\\
&b)\ Relativistic\ gas: p \overset{mod}{=} \frac{1}{3}e\\
&c)\ Higgs\,energy: p \overset{mod}{=} w\,e\\
&d)\ Dark\,energy(LIVE): p \overset{mod}{=} -e
\end{aligned} \tag{2.11}$$

Inserting into equation (2.10b) gives that

$$\begin{aligned}
&Dust: p=0 \Rightarrow \dot{e}(t)=-3e\frac{\dot{a}(t)}{a(t)} \Rightarrow e(t)a(t)^3=const.\\
&Relativistic\ gas: p=\frac{1}{3}e \Rightarrow e(t)a(t)^4=const.\\
&Higgs\,energy: p=we \Rightarrow \dot{e}(t)=-\frac{e(t)\left(3+3w\right)\dot{a}(t)}{a(t)} \Rightarrow e(t)a(t)^{3+3w}=const.\\
&Dark\ \ energy(LIVE): p=-e \Rightarrow e=e_0=const.
\end{aligned} \tag{2.12}$$

Thus for LIVE $T_{ij}=e_0\,g_{ij}$. This has the same effect as adding a term $8\pi\, e_0\,\frac{G}{3c^2}$ to the Einstein equation to achieve an effective cosmological constant; i.e. $\lambda_{eff}\overset{def}{=}\frac{8\pi\,G}{c^4}e_0+\lambda$ as seen in equation (2.8) or (2.10). Equivalently one could say that the Einstein cosmological constant contributes a term $e_0=\frac{\lambda\,c^4}{8\pi\,G}$ to the vacuum energy (Weinberg 1989).

The Hubble constant is defined by $H\overset{def}{=}\dot{a}(t)/a(t)$. Inserting into equation (2.10a) gives

$$H^2=\frac{\dot{a}(t)^2}{a(t)^2}=-\frac{k\,c^2}{a(t)^2}+\frac{8\pi G}{3c^2}e(t)+\frac{\lambda\,c^2}{3}\Rightarrow\frac{k\,c^2}{a(t)^2}=\frac{8\pi G}{3c^2}e(t)+\frac{\lambda\,c^2}{3}-H^2 \tag{2.13}$$

We observe that the curvature of space is negative or positive according to the term $\frac{8\pi G}{3c^2}e(t)+\frac{\lambda c^2}{3}-H^2$. The difference comes to zero when $\frac{8\pi G}{3c^2}e(t)+\frac{\lambda c^2}{3}-H^2=0$. This gives the critical energy e_k as

$$e_k \stackrel{def}{=} \frac{3c^2}{8\pi G}H^2-\frac{\lambda c^4}{8\pi G} \tag{2.14}$$

The experimental value of the Hubble constant H is hampered by the uncertainty in establishing a scale of cosmic distances suitable for distant galaxies. The latest value is H= 0.25 10^(-17)/s. This gives a critical energy of $c^2 e_k = 1.0\,10^{-23} kg/m^3$. However, an estimate based on the number of galaxies and their average mass only takes a value of about 4% of this value. The observational evidence for DM is based on the interpretations of astronomical data coming from rotational galaxies, galactic hopes and gravitational lensing to account for 26% of the missing energy (Jullo et al. 2010).

The Friedman models possess some problems. First t=0 could be a singular point of the energy density[2]. Another problem is the so called horizon problem: The horizon radius was much less than the radius of the universe when photons of the background radiation became free. Thus the current observed homogeneity and isotropy of the cosmic background radiation has no natural explanation. A final problem is the flatness problem. From the Friedman solutions it follows that $(e - e_k) / e_k \sim 1$. Observations today give that $(e - e_k) / e_k \sim 1$. The age of the universe is around 10^60 Planck lengths ($t_P = (8\pi Ge/3c^2)^{-1/2}$). The model does not explain why the density initially was so incredibly near the critical one.

The inflation theory can solve these problems. The inflation theory assumes that for the early phase of the universe e is constant. This gives an exponential expansion rate of the universe. The energy density is not singular initially. However, the initial value of the expansion rate for a=0 is problematic unless k=0 or k=-1. Any exponential expansion leads to a rapid increase in the cosmological horizon over which causal signals can propagate. Thus the present isotropy and homogeneity of the universe could be explained since the distance which causality can propagate has been exponentially increased at an early moment. Finally any period of exponential expansion during for example 100 Planck times with constant e will result in the curvature term k c^2/2 becoming negligible with respect to the second term of the left side of equation (2.10a). This gives an explanation for the flatness of the universe.

The inflation theory a concrete prediction: that the present universe should be

---

[2] This can be “solved” by assuming that for times less than the Planck time quantum effects must be

extreme flat (Euclidean). Observations of the temperature variations in the cosmic background radiation indicate that space is extreme flat. However, the estimate based on the number of galaxies and their average mass only takes a value of about 4% of the mass necessary to achieve this flatness. The observational evidence for the existence of DM is based on the interpretations of astronomical data coming from rotational galaxies, galactic hopes and gravitational lensing to account for 26% of the missing energy (Jullo et al. 2010). DE is hypothesized to constitute the missing 70% (100-4-26) of the gravitational energy (energy = m c^2) in the universe. Recently experimental evidence from supernovas of type 1a indicates that the universe expands faster and faster (http://www-supernova.1bl.gov/). To account for this it is assumed that the cosmological constant is representative for DE of the type LIVE. It was fond by gravitational lensing that $-1.04 \le w \le -0.88$. This suggests that DE of the type LIVE is indeed possible.

3 Newtonian cosmology on $\boldsymbol{R \times E^3}$

Narlicar (1994) showed that the equivalence of the cosmological and gravitational spectral shift and the classical Doppler interpretation can be established provided one parallel transfers the source four vector velocity along the null geodesic to the observer. Bunn and Hogg (2009) argued that the commonly chosen free falling co-moving observers in cosmology correspond to the case where the cosmological expansion can be considered as Doppler. Equation (2.10) can also be given a pure non relativistic classical interpretation picture.

It is convenient to define $M(t) \overset{def}{=} (4/3)\pi a(t)^3 e(t)/c^2$. Equation (2.10a) can be written as

$$\frac{1}{2}\dot{a}(t)^2 - \frac{GM(t)}{a(t)} = -\frac{k\,c^2}{2} + \frac{\lambda\,c^2}{6}a(t)^2 \tag{3.1}$$

Equation (2.10) or (3.10) can be given a classical interpretation within the Newtonian cosmology: kinetic + gravitational (potential) energy in the universe per unit mass is equal to $-\frac{kc^2}{2} + \frac{\lambda c^2}{6}a(t)^2$. When λ =0 and with dust (M is constant) the kinetic + potential energy would be constant. When λ =0 we see that the kinetic + potential energy is negative when k is positive. That means a closed solution of the universe and also a finite expansion of the universe. Notice that (3.1) per se fails to explain whether the universe is open or closed. When k is negative the universe is open and will expand for all times. Newtonian cosmology fails to provide an explanation for why the term on

taken into account.

the right side in equation (3.1) must be like this. For dust we see that the kinetic energy term $\frac{1}{2}\dot{a}(t)^2$ approaches infinity when $\dot{a}(t)$ approaches zero. For the inflation theory when e is constant we find

$$\frac{1}{2}\dot{a}(t)^2 - \frac{4}{3}\pi\, a(t)^2 e\frac{G}{c^2} = -\frac{k\,c^2}{2} + \frac{\lambda\, c^2}{6}a(t)^2, a(t) \to 0 \Rightarrow \frac{1}{2}\dot{a}(t)^2 \to -\frac{k\,c^2}{2} \tag{3.2}$$

This gives and expansion rate of c if k =-1.

Equation (2.10b) can also be given a classical interpretation. It simply expresses that the universe expands isentropically. Let dU be the change in energy of a system. V is the volume. S is the entropy and T is the temperature of the universe. We have that

$$dU = -pdV + TdS, e \overset{def}{=} U/V \tag{3.3}$$

For isentropic expansion we have that dS =0. Then from equation (3.3)

$$de = dU/V - U/V^2 dV = -pdV/V - U/V^2 dV = -\left(p+e\right)dV/V \tag{3.4}$$

The volume of a space of negative or zero curvature is infinite. However we apply in general that

$$dV/V \overset{mod}{=} 3da/a \tag{3.5}$$

Thus

$$\begin{aligned} &dV/V = 3da/a \Rightarrow de = -3\left(p+e\right)da/a \\ &de = -\left(p+e\right)dV/V = -3\left(p+e\right)da/a \Rightarrow \dot{e} = -3\left(p+e\right)\dot{a}/a \end{aligned} \tag{3.6}$$

The isentropic expansion equation can indeed be developed directly from the Einstein equation by applying that ${T^i}_{0;j} = 0$.

4 Newtonian cosmology expanded to $R \times S^3$

Recently a non metric space time framework has been presented (Østvang 2002, 2005, 2006, 2007). The theory is based on a $R \times S^3$ background rather than a Minkowski background as the geometry of the universe without matter. We follow the concept of a quasi metric theory. We build our theory on the concept of Newtonian cosmology. In particular we use the S^3 instead of E^3 to construct gravitational energy. Equation (2.10a) can be written as

$$\underbrace{\frac{1}{2}\dot{a}(t)^2}_{\substack{Kinetic\\ energy\\ per\ mass}} - \underbrace{\frac{4}{3}\pi a(t)^3}_{Volume} \underbrace{\frac{G}{c^2}\frac{e(t)}{a(t)}}_{\substack{Gravitational\\ energy\ per\\ volume\ per\ mass}} = -\frac{k\,c^2}{2} + \frac{\lambda\, c^2}{6}a(t)^2 \tag{4.1}$$

The gravitational energy shows the 1/a dependency and the term with the cosmological constant shows a^2 dependency. We can form the following familiar differential equation for the gravitational potential, to read

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial r^2} + \frac{2}{r}\frac{\partial\varphi}{\partial r} = -\delta(r) \Rightarrow \varphi = \varphi(r) = \frac{1}{r} \tag{4.2}$$

where $\delta(r)$ is the Dirac delta function such that $\int 4\pi r^2\delta(r)dr = 1$. We now write

$$\begin{aligned} &\frac{1}{2}\dot{a}(t)^2 - \frac{4}{3}\pi a(t)^3 \frac{G}{c^2} e(t)\ \varphi\big(e(t), a(t)\big) = -\frac{k\,c^2}{2} + \frac{\lambda\,c^2}{6} a(t)^2, (a) \\ &\dot{e}(t) = -\frac{3\big(e(t) + p(t)\big)\dot{a}(t)}{a(t)}, (b) \end{aligned} \tag{4.3}$$

We seek an equation for $\varphi(\ )$ which would close the equation set if an equation of state $p = p\big(e, a\big)$ is given. The concept of LIVE means to apply that $p = -e_0$. This gives the cosmological constant $\lambda = \frac{8\pi G}{c^4} e_0$ in equation (4.1). However we apply that LIVE also modifies equation (4.2).

We formulate the Newtonian cosmology on $R \times S^3$ instead of the traditional $R \times E^3$. We solve the Poisson equation on S^3 of radius $1/\xi$, to read instead of equation (4.2)

$$\frac{\partial^2\varphi}{\partial r^2}(1 - \xi^2 r^2) + \frac{\partial\varphi}{\partial r}\left(-\xi^2 r + \frac{2}{r}\Big(1 - \xi^2 r^2\Big)\right) \overset{mod}{=} -\delta(r), \quad r\xi < 1 \tag{4.4}$$

$1/\xi$ is the radius of an S^3 sphere. When $\xi \to 0$ (i.e. without LIVE) we achieve equation (4.2). The solution of equation (4.4) is readily verified to be $\varphi(r) = \frac{1}{r}\Big(1 - \xi^2 r^2\Big)^{1/2}$. The gravitation 1/r potential is thus weakened by the term $\Big(1 - \xi^2 r^2\Big)^{1/2}$. This term applies as long as $\xi^2 r^2 < 1$. Thus we write

$$\varphi\big(a(t)\big) = \frac{\Big(1 - \xi^2 a(t)^2\Big)^{1/2}}{a(t)}, \quad \xi a(t) < 1 \tag{4.5}$$

We observe that when the cosmological radius $a(t)$ is comparable to $1/\xi$ this would modify the 1/a(t) relationship. We can write that

$$\begin{aligned} &\frac{1}{2}\dot{a}(t)^2 - \frac{4}{3}\pi a(t)^3 \frac{G}{c^2} e(t) \frac{\Big(1 - \xi^2 a(t)^2\Big)^{1/2}}{a(t)} = -\frac{k\,c^2}{2} + \frac{\lambda\,c^2}{6} a(t)^2, \xi a(t) < 1 \\ &\dot{e}(t) = -\frac{3\big(e(t) + p(t)\big)\dot{a}(t)}{a(t)} \end{aligned} \tag{4.6}$$

This gives a critical energy density as

$$e_k{}' = \frac{3c^2}{8\pi G\left(1-\xi^2 a^2\right)^{1/2}} H^2 - \frac{\lambda c^4}{8\pi G} \tag{4.7}$$

The concept of LIVE is based on an attempt to fit the critical energy density to empirics by applying the cosmological constant. However, by applying the parameter ξ we have an additional constant to fit. This will allow more flexibility.

For $\xi^2 a^2$ << 1, we could write that $\left(1-\xi^2 a^2\right)^{1/2} \approx 1 - 1/2\,\xi^2 a^2$. Thus

$$\begin{aligned} &\frac{1}{2}\dot{a}(t)^2 - \frac{GM(t)}{a(t)} \approx -\frac{kc^2}{2} + \frac{\lambda c^2}{6} a(t)^2 - \frac{GM(t)}{2}\xi^2 a(t) \\ &= -\frac{kc^2}{2} + \frac{\lambda_{eff}\, c^2}{6} a(t)^2, \lambda_{eff} \overset{def}{=} \lambda\left(1 - \frac{3GM(t)}{c^2 a(t)}\xi^2\right) \end{aligned} \tag{4.8}$$

This gives a cosmological constant that decreases with time for an increasing size of the universe.

The quasi metric approach cuts the link between the gravitational field and the metric of space time. Thus a space time universe with density above the critical density for small times or little expansion could adjust to under the critical density for larger times or expansions even for a constant energy of the universe [3]. We could also use ξ to fine tune the effective cosmological constant (Weinberg 1989).

5 Conclusion

Besides the cosmological constant, modifying the Einstein equations is hard to do within the traditional framework of metric theories of gravity. Such an approach is mathematically much easier within the Newtonian approach such as for example utilized for MOND; i.e. a specific modification of the Newtonian dynamics whenever the gravitational acceleration falls below a critical value. In a homogeneous and isotropic universe local knowledge is global knowledge and Newtonian cosmology can fruitfully be used. Analogously to the recently published quasi metric theory, we modify gravity due to LIVE by applying $R \times S^3$ instead of $R \times E^3$ as the basic geometric structure. However, we build our theory on the concept of Newtonian cosmology. We find that a space time universe with density above the critical density for small times

[3] A possible interpretation could also be that that a space time universe which is closed would open up as it enlarges even for a constant energy of the space time universe.

could adjust to under the critical density for larger times even for a constant energy of the universe. We could also use the extra flexibility of S^3 as an effective cosmological constant. Effectively we thus can perceive a cosmological constant that decreases with time.

References

[1] A. Albrecht, The birth of the universe and fundamental forces, Ed. F. Occionero, Springer, Berlin, 1994.

[2] A. Aguirre, C.P. Burgess, A. Friedland and D. Nolte, Astrophysical Constraints on Modifying Gravity at Large Distances. Classical and Quantum Gravit., 18 (2001), R223-R232.

[3] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, New dimensions at a millimeter to a Fermi and superstrings at a TeV. Physics Letters B, 429 (1998) (3-4), 257–263.

[4] J. Barbour and H. Pfister, Mach’s Principle. From Newton’s Bucket to Quantum Geometry. Einstein Studies, volume 6, Birkhäuser, Boston, 1995.

[5] A. Barrau, J. Grain and S. Alexeev, Mini Black holes at the LHC: a delight for new physics! Physics Letters B, 584 (2004), 114-122.

[6] H. Bondi, Cosmology, 2ed. ed., Cambridge University Press, Cambridge, 1961.

[7] E.F. Bunn and D.W. Hogg, The kinematic origin of the cosmological red shift. American Journal of Physics, 77 (2009), 8, 688–694.

[8] W. de Sitter, Einstein's theory of gravitation and its astronomical consequences, Third Paper, Monthly Notices of the Royal Astronomical Society, 78 (1917), 3-28.

[9] P. Ehrenfest, In What Way Does It Become Manifest in the Fundamental Laws of Physics that Space Has Three Dimensions? Proceedings of the Amsterdam Academy, 20 (1917), 200-209.

[10] A. Friedman, Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes, Zeitschrift für Physik 21 (1924), 1, 326–332.

[11] E. Jullo, P. Natarajan, J.-P. Kneib, A. D'Aloisio, M. Limousin, J. Richard and C. Schimd, Cosmological constraints from strong gravitational lensing in clusters of galaxies. Science, 329 (2010), 924-927.

[12] L.D. Landau and E.M. Lifshitz, The classical theory of fields. Pergamon Press, England, 1980.

[13] A. Linde and M.I. Zelnikov, Inflationary universe with fluctuating dimension, Physics Letters B, 215 (1988), 1, 59-63.

[14] J.V. Narlicar, Spectral shifts in general relativity. American Journal of Physics 62 (1994), 903-907.

[15] D. Østvang, An explanation of the 'Pioneer effect' based on quasi-metric relativity, Classical and Quantum Gravity, 19 (2002), 15, 4131-4139.

[16] D. Østvang, Quasi-Metric Relativity, Gravitation and Cosmology, 11 (2005), 205-219.

[17] D. Østvang, Classical electrodynamics in quasi-metric space-time, Gravitation and Cosmology, 12 (2006), 262-272.

[18] D. Østvang, Spherically symmetric, metrically static, isolated systems in quasi-metric gravity. Gravitation and Cosmology, 1 (2007), 49, 1-15.

[19] F.R. Tangherlini, Schwarzschild field in n dimensions and the dimensionality of space problem, Nuvovo Cimento 27 (1963), 636-651.

[20] M. Tegmark, On the dimensionality of spacetime. Classical and Quantum Gravity, 14 (1997), L69-L75.

[21] S. Weinberg, The cosmological constant problem. Reviews of Modern Physics 61 (1989), 1, 1-23.

Received: January, 2012