

# From F-16 to F-35: Optimizing the Training of Pilots in the Royal Norwegian Air Force

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The Norwegian Armed Forces will soon make the largest investment in its history. It plans to spend \$8 billion to replace the Royal Norwegian Air Force's existing fleet of F-16 aircraft with 52 new F-35 aircraft. The Norwegian Defence Research Establishment (FFI) has undertaken multiple analyses to support the Norwegian Defence Logistics Organisation (NDLO) with the transition planning. During the transition period, Norway must maintain adequate fighter capability. The rate at which it can train new pilots and convert F-16 pilots to fly the F-35 will influence the cost and length of the transition period and the Air Force's operational-readiness status. Although finding the optimal training rate for pilots is a difficult planning problem, we achieved substantial success by using an integer linear program to generate optimal plans. By using this model, we were able to investigate multiple scenarios for pilot training. We determined the earliest year that the Air Force could reach a fully operational F-35 fighter capability, the optimal ratio of new to converted pilots, and the number of pilots that FFI should train each year during the transition phase. Our results enabled FFI to generate a training plan that resulted in large savings and operational advantages compared to the previous solution, which did not employ operations research techniques for planning.

*Keywords:* integer linear programming; military planning; operations research; pilot training.

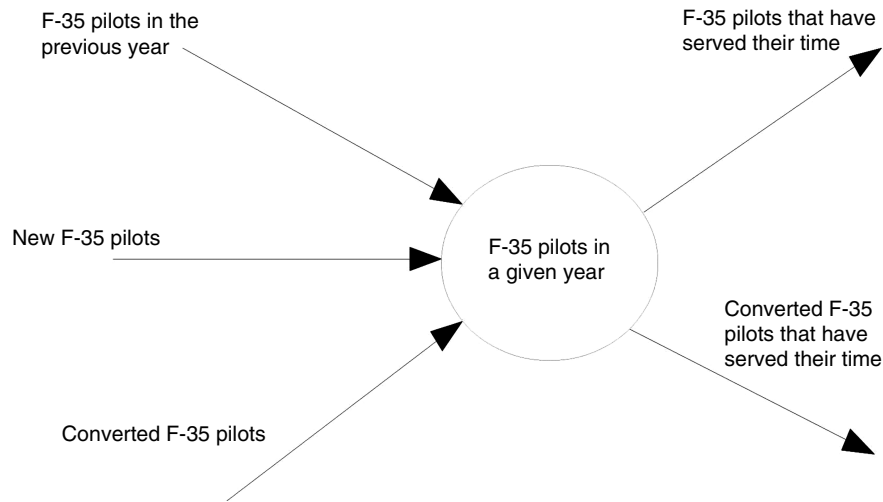
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The Norwegian Armed Forces will soon make the largest investment in its history. It plans to spend \$8 billion to replace the Royal Norwegian Air Force's existing fleet of F-16 aircraft with 52 new F-35 aircraft. The Norwegian fighter aircraft capability is the backbone in the Norwegian Armed Forces and is part of NATO's defence in the North of Europe. Therefore, maintaining this capability throughout the transition phase from the F-16 to the F-35 is crucial.

Planning for the new Norwegian fighter aircraft started more than two decades ago. Scientists from the Norwegian Defence Research Establishment (FFI), as a part of the Norwegian Defence Logistics Organisation's (NDLO) F-35 acquisition team, studied many problems related to the transition process. FFI is the chief adviser on defence-related science and technology to the Norwegian Armed Forces and the Ministry of Defence. The FFI scientists brought their knowledge of both operations research and the military to the acquisition team. The work we describe in this paper was part of FFI's efforts during the past five years, when the acquisition team started to study

specific plans for pilot training. The authors were among the scientists involved in the acquisition team; therefore, we will use the terms "FFI" and "we" interchangeably throughout this paper.

The transition period from the F-16 to the F-35 began when the first F-35 fighters were delivered, and will end when the F-35 fighter capability is fully operational. During this time, the Norwegian Air Force must operate the two very different fighter systems. One of the Norwegian Armed Force's goals in the transition phase is to minimize the length of this period, while ensuring that the level of operational readiness is acceptable at all times. The set of tasks a fighter system can perform simultaneously defines the system's level of operational readiness. This level is therefore a delicate interplay between maintenance capacity, number of aircraft, and number of pilots. Both the number of F-16 aircraft and the rate of delivery of the new F-35 aircraft are given; therefore, the level of operational readiness depends on the training of the pilots. During the transition phase, Norway must maintain a sufficient number of combat-ready



**Figure 1:** The number of F-35 pilots in a given year depends on the number of F-35 pilots the previous year, the number of new and converted F-35 pilots, and the number of F-35 pilots that quit, retire, or are no longer available.

pilots to be able to participate in NATO missions and maintain national sovereignty.

The pilot training center (PTC) is responsible for pilot training. Figure 1 shows the factors that determine the number of available F-35 pilots in any given year. A pilot who converts from flying an F-16 to an F-35 requires less education from the PTC than a new F-35 pilot. Therefore, a high rate of converting F-16 pilots may seem tempting to decision makers; however, a complicating factor is that converted pilots are closer to the end of their careers than newly trained pilots, and will need to be replaced sooner. A high conversion rate will provide a steep ramp up to the goal of full operational readiness, because it will result in an immediate high level of experience among F-35 pilots. Although this might be favorable in the short term, it might also result in a corresponding low level of experience when the experienced pilots have served their time. Having the correct mix of converted and new F-35 pilots is important if the Air Force is to have a good balance between age (i.e., longer period until retirement) and experience from the start of the transition (Holloway 2010). Additionally, a defined training rate will give the current F-16 pilots predictable career development.

Initially, the military staff at the NDLO planned the F-35 pilot training using spreadsheets, studying the

interaction between the number of pilots, PTC's educational capacity, the expected length of pilot active duty, and the necessity to maintain an adequate level of operational readiness. This trial-and-error approach was inefficient and time consuming, and provided only a minimal chance of determining the best training plan. As a part of the acquisition team, FFI suggested an operations research (OR) approach to study various aspects of the transition process. As a result, the team decided to develop a model to search for cost-effective solutions when converting from one fighter system to another. To this end, we developed an integer linear programming model to generate optimal pilot training plans, and embedded this model into the existing model.

Optimization models and algorithms are commonly used to solve personnel-related problems, such as workforce planning problems (Brucker et al. 2011, Castillo-Salazar et al. 2016, Holz and Wroth 1980). Other methods that are used for this purpose include statistical models and simulation models (Daniels 1967, Wang 2005). In AIMMS (2014), the planning of flight attendant training is addressed as an integer linear programming problem. Holloway (2010) optimizes pilot conversion to the joint strike fighter (F-35) in the U.S. Marine Corps by choosing career paths for each pilot based on rank and experience. Other

examples of manpower planning using optimization of career paths are given in Baumgarten (2000), Jasper (1999), and Matar (2012). Davidson (2011) uses a simulation model to look at factors that affect a pilot's training time at the PTC.

In this paper, we describe the integer linear programming model that we developed to solve the pilot training problem. Our model resembles the flight attendant model in AIMMS (2014); however, the assumptions made in that paper were too simplistic for our problem. In particular, we needed to model the relationships between converted and new F-35 pilots, the restrictions on educational capacity, the necessity to maintain an adequate level of operational readiness, and the requirement for full operational readiness within a specified number of years.

Introducing OR techniques into planning the transition from the F-16 to the F-35 had major implications for both the planning process and for the results of this process. In particular, we achieved the following:

- Our pilot training model, embedded in the larger cost model, generated a training plan that saved tens of millions of dollars (the precise number is classified) over the original plan.
- Because solving the integer program is much faster than using manually generated solutions, we were able to investigate many more scenarios by changing the input parameters and constraints. This allowed us to understand the problem in more depth, and thus to develop a more realistic and cost-effective plan.

Other organizations would benefit from using our approach when planning the transition from one system to another, especially if the following criteria are true.

- The system must not fall below a specified minimum operational level during the transition phase.
- Only a few staff members can be trained simultaneously, because training capacity is limited.
  - The cost of transitioning from one system to the other is (very) expensive.
  - The old and new systems differ fundamentally.

These criteria may, for example, apply to the following types of personnel: military pilots; military command and control personnel; medical personnel

who have specialized skills or use highly specialized equipment; and engineering specialists at off-shore installations.

## Problem Description

Our integer linear program determines the optimal training plan for F-16 and F-35 pilots during a 20-year planning horizon in a manner such that it minimizes the total number of F-35 pilot years. We define a pilot year as a year in which one pilot is combat ready.

Maintaining the combat-ready status of F-35 pilots is the pivotal cost driver in peacetime; therefore, the number of combat-ready F-35 pilots should not exceed the required number for a longer period than necessary to obtain operational readiness. The total number of F-35 pilot years is the sum of the number of combat-ready F-35 pilots each year during the planning horizon. For example, if two, four, and six pilots are combat ready over a period of three years, the total number of F-35 pilot years for this three-year period is 12.

Through some yearly rate of converting F-16 pilots to F-35 pilots and training new F-35 pilots, the level of operational readiness for F-16 and F-35 pilots decreases and increases, respectively. During this transition phase, the minimum requirement for the level of operational readiness for the Norwegian fighter capacity must be satisfied at all times.

Part of the planning process at the NDLO was to determine the minimum acceptable level of operational readiness (MLOR) during the transition phase. We studied four MLORs and examined the influence of each on the pilot training plan and the length of the transition phase. As Table 1 illustrates, we defined the levels of operational readiness as ranges of numbers of combat-ready pilots.

Number of combat-ready pilots	Level of operational readiness
5–9	1
10–14	2
15–19	3
20–24	4
25–30	5

**Table 1: The number of combat-ready pilots indicates the level of operational readiness. We use illustrative data, because the actual data are classified.**

In the model, we say that when the MLOR is a specific number (e.g., three), any combination of levels for the F-16 and the F-35 that sum to that number satisfies the requirement. For example, if F-16 is at Level 1 and F-35 is at Level 2 or higher, then MLOR 3 is satisfied. Full operational capability corresponds to Level 5 of operational readiness in our model.

We set the length of the transition phase (i.e., the number of years until the F-35 must reach full operational capability) in the model's input data. By running the model with different values for this parameter, we were able to find the shortest transition phase possible without violating the model's constraints.

The bottleneck in our model is the capacity at the PTC. Pilots in training must fly a specified number of hours per year. The PTC capacity is defined by the total number of flight hours it can provide to the pilots in training. The number of training hours available each year depends on the number of Norwegian aircraft that are stationed at the PTC, which is specified in the Norwegian F-35 acquisition plan. Although pilots who are being converted to the F-35 need fewer flight hours than new F-35 pilots, this training requires two years for both types of pilots. The number of years an F-35 pilot is operative after training varies from pilot to pilot; however, in the model, we assume one mean value for all converted pilots and one mean value for all new F-35 pilots. This was sufficient for our purposes, because we were not investigating individual pilot careers.

In summary, the objectives of our study are to explore:

- the earliest possible year that the F-35 could attain full operational capability, given the desired input data values defined by the NDLO for the parameters described below (base case);
- the optimal number of new F-35 pilots and pilots converting from the F-16 to the F-35 in the base case; and
- the impact on the above two objectives of changing the values of some input data (i.e., variations from the base case).

The objective of the optimization model is to minimize the total number of F-35 pilot years over the planning horizon. The input parameters in the model are as follows.

1. The predefined number of levels of operational readiness and the number of pilots needed at each level.

2. The minimum acceptable level of operational readiness during the transition phase.

3. The training capacity at the PTC (i.e., number of available flight hours for training).

4. The number of training hours necessary at the PTC for converting pilots and new F-35 pilots.

5. The length-of-service time for F-16 pilots, converted pilots, and new F-35 pilots (we estimated these values based on the pilots' expected lengths of active duty).

6. The year by which the F-35 must be fully operational.

In addition, we include some technical input parameters for the development of the F-16 pilot pool. To study the problem, we varied parameters 2, 3, and 6, respectively, and left all other parameters fixed.

The optimization model includes the following constraints.

- Several constraints are required to handle the flow of pilots (Figure 1).

- The number of F-35 pilots cannot fall below the level at which the F-35 reaches full operational capability.

- The capacity at the PTC cannot be exceeded.

- When the F-35 reaches the MLOR, F-16 pilots should no longer be converted to F-35 pilots.

- When the F-35 reaches full operational readiness, no new F-16 pilots will be trained.

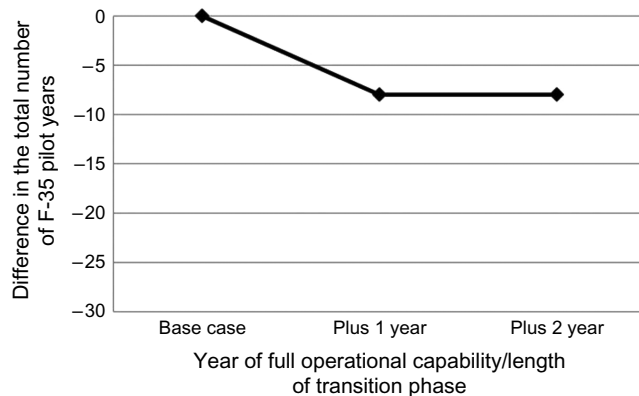
- The levels of operational readiness for the F-16 and F-35 in combination must always be higher than the MLOR.

We developed a representative integer linear program, which we provide in the appendix. We used IBM ILOG CPLEX (IBM 2014) to solve our model, and set all CPLEX parameters to their default values.

## Results

Because many of our results are classified, we cannot fully discuss all of them; however, we can describe our major results.

Our major objective in studying the pilot transition was to determine the speed with which the fleet of F-35 aircraft could become fully operational, and to



**Figure 2:** The graph shows the decrease in the total number of F-35 pilot years as we increased the length of the transition phase by one and two years from the shortest possible length.

determine the best approach for converting F-16 pilots and training new F-35 pilots to achieve full operational capability. We gained two main insights from our analysis. In the base case, we set the MLOR in the transition phase to Level 3. In Figure 2, we see the changes in the total number of F-35 pilot years as we vary the length of the transition phase from the shortest possible length to one year, and then to two years or longer. By adding one year to the transition phase, compared to the base case, we found that we needed eight fewer F-35 pilot years during the transition phase. Adding one additional year to the transition phase did not affect the number of F-35 pilot years (i.e., we still needed eight fewer F-35 pilot years during the planning horizon). The total number of F-35 pilot years decreases when the transition phase is longer because waiting longer before training pilots is possible. We achieve this result when we increase the transition phase by one year; however, increasing the length of the transition phase by two or more years provides no additional benefit.

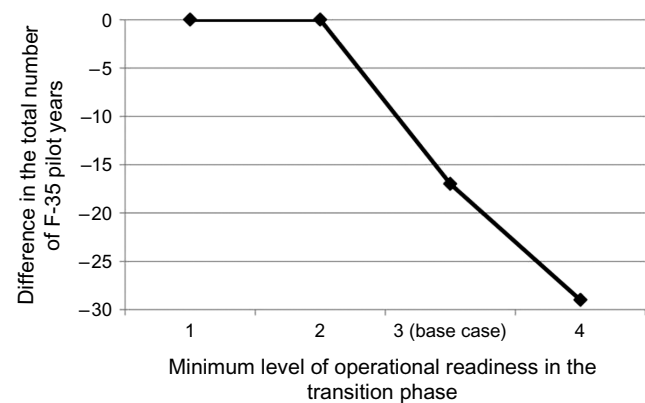
The base case has twice as many new F-35 pilots as F-16 pilots. Increasing the length of the transition phase did not affect this ratio. In summary, our analysis provided us with the earliest possible year of a fully operational F-35 capability and the optimal ratio between converted and new F-35 pilots. In addition, we found that (1) the length of the transition phase did not impact the optimal ratio between converted and new F-35 pilots, and (2) prolonging the transition

phase by one year over the base case resulted in a lower total number of F-35 pilot years, because pilots can be trained later.

Figure 3 shows the changes to the total number of F-35 pilot years as we varied the MLOR in the transition phase. Lower MLORs mean more F-35 pilot years over the entire planning period, because we stop converting F-16 pilots when the F-35 reaches the MLOR. Thus, the result of lower MLORs is fewer converted pilots and more new F-35 pilots. New F-35 pilots use more of the capacity at the PTC; therefore, more F-35 pilots must start their training earlier to allow the armed forces to reach a sufficient number of combat-ready F-35 pilots by the end of the transition phase. Another insight was that the selected MLOR does not influence the length of the transition phase—only the ratio of converted to new F-35 pilots.

When we select low MLORs, we are actually selecting low conversion rates. Many factors can impact the choice of MLOR (e.g., the national security situation, costs, pilot career policies, and the balance between new and experienced pilots); however, many of these factors were outside the scope of our study.

We also looked at influence of the PTC's capacity on the results of the model. This capacity is hard to adjust, because it depends on the number of Norwegian aircraft stationed at the PTC; however, we wanted to determine how flexible this bottleneck is. We found that increasing the PTC capacity by nine percent allowed us to achieve full operational F-35 capability one year earlier. Similarly, decreasing the



**Figure 3:** The graph shows the changes to the total number of F-35 pilot years as we varied the MLOR during the transition phase.

PTC capacity by 11 percent delayed full operational capability by one year.

FFI used the integer linear program results as input to the larger cost model to support its planning for the new fighter system. This model included both the costs associated with training the F-35 pilots, and the differences in operating costs between the two aircraft types. This analysis also considered pilot qualifications, which are attained through experience. Subject matter experts made adjustments to this cost model. The NDLO's proposed solution, prior to being given FFI's analysis, involved a longer and more expensive transition phase than the FFI solution. FFI proposed a solution that saved tens of millions of dollars, a considerable amount of money for a relatively small military organization. As the result of FFI's work, NDLO decision makers now plan pilot training to minimize the effects of the PTC's capacity limitations. In addition, the results that address the total number of F-35 pilot years and the ratio of converted to new F-35 pilots support the choices that were ultimately made about the length of the transition phase.

## Conclusions

As a part of FFI's support to the NDLO in planning for new fighter aircraft for the Royal Norwegian Air Force, we developed an integer linear program to study training plans for pilots transitioning from the F-16 and the F-35 fighter, minimizing the total number of F-35 pilot years. We determined the earliest possible year that the NDLO could achieve a fully operational F-35 fighter capability, taking into account the PTC's educational-capacity limitation. This analysis, in conjunction with other studies that FFI performed for the transition (e.g., cost analysis), enabled FFI to propose a training plan that saved tens of millions of dollars over the solution that NDLO had proposed.

The work we present in this paper shows military acquisition stakeholders that using OR resources can improve both planning and results. The Norwegian Armed Forces saved a substantial amount of money by using OR in the F-35 acquisition process. The final decisions for the transition from the F-16 to the F-35 in Norway were made based on an interplay between our integer linear program, a model showing the cost implications of various training plans,

and military judgement and analysis. The integer linear program quickly generated multiple solutions for the analysts and military staff to study. In our opinion, the key to the success of the analyses conducted by FFI and the NDLO is that they are a combination of the three methods discussed. We also believe that this success was the result of integrating the OR analysts and military personnel into the acquisition team. Therefore, to other organizations attempting to solve similar problems, we recommend integrating analysts into their acquisition teams. In our environment, this allowed the analysts to supplement their OR skills with knowledge of the military domain.

Planning for the F-35 acquisition evolved over more than two decades. Although many analyses were conducted during this period, decisions were often not made until long after the analyses had been completed, when many of the assumptions in the analyses (e.g., costs, budget, operational needs) had changed. We view this as our biggest challenge in showing the relevance and validity of the analyses. We believe, however, that efficient OR tools and methods are critical time-saving measures in a planning process. The method we describe in this paper is relevant for organizations that seek to develop training programs for converting highly specialized personnel, such as military pilots, medical staff, and engineers, to a new generation of technology.

## Appendix. Integer Linear Programming Formulation

In this appendix, we present an integer linear programming formulation, including sections for indices, parameters, and decision variables. All variables are fixed in the first year to capture the initial conditions. We present our model using the notation guidelines in Teter et al. (2015).

### Indices

- $t = 1, \dots, T$ , index on year.
- $j = 1, \dots, J$ , index on level of operational readiness.

### Parameters:

- $T$ —number of years.
- $J$ —number of levels of operational readiness.
- $P^{F16}$ —years a new F-16 pilot flies an F-16.
- $P^{F35}$ —years a new F-35 pilot flies an F-35.
- $P^{\text{conv}}$ —years a converted pilot flies an F-35.
- $T^{\text{conv}}$ —years for conversion and (or) training.
- $N^{F16}$ —number of F-16 pilots in year 1
- $N^{F35}$ —number of F-35 pilots in year 1.
- $V^{\text{min}}$ —minimum number of new F-35 pilots each year.

- $W^{\min}$ —minimum number of converted pilots for the planning horizon.
- $W^{\max}$ —maximum number of converted pilots for the planning horizon.
- $Y^{A_j}$ —the year when the F-35 aircraft must reach full operational capability.
- $A^{\text{MLOR}}$ —number of pilots in the minimum level of operational readiness.
- $A_j$ —number of pilots at operational readiness level  $j$ ;  $j = J$  for full operational capability.
- $C_t$ —flight hours available at the PTC in year  $t$ .
- $H^{\text{new}}$ —number of flight hours needed each year for a new F-35 pilot in training.
- $H^{\text{conv}}$ —number of flight hours needed each year for a converting pilot in training.
- $Q_t$ —number of F-16 pilots that retire in year  $t$ . We estimate this value based on the initial F-16 pilot pool.
- $M$ —large number for modelling purposes.
- $L$ —upper limit on the integer variables  $x_t, y_t, w_t, u_t, v_t, e'_t, e''_t,$  and  $e'''_t$ .
- $D$ —penalty per F-35 pilot above  $A_j$ .
- $E$ —additional penalty per F-35 pilot above  $A_j + 2$ .
- $F$ —additional penalty per F-35 pilot above  $A_j + 6$ .

#### Decision variables

- $x_t \in \{0, L\}$ —number of F-16 pilots in year  $t$ .
- $y_t \in \{0, L\}$ —number of new F-16 pilots in year  $t$ .
- $w_t \in \{0, L\}$ —number of F-16 pilots that convert in year  $t$  (starts training).
- $u_t \in \{0, L\}$ —number of F-35 pilots in year  $t$ .
- $v_t \in \{0, L\}$ —number of new F-35 pilots in year  $t$ .
- $a'_t \in \{0, J\}$ —level of operational readiness covered by the F-16 in year  $t$ .
- $a''_t \in \{0, J\}$ —level of operational readiness covered by the F-35 in year  $t$ .
- $b'_t \in \{0, 1\}$ —a binary variable with value 1 if the F-35 has reached full operational capability by year  $t$ ; 0 otherwise.
- $b''_t \in \{0, 1\}$ —a binary variable with value 1 if the number of F-35 pilots is higher than the minimum level of operational readiness by year  $t$ ; 0 otherwise.
- $c_{jt} \in \{0, 1\}$ —variable that equals 0 if the number of F-16 pilots is lower than the number of pilots in level of operational readiness  $j$  at time  $t$ ; 1 otherwise.
- $d_{jt} \in \{0, 1\}$ —variable that equals 0 if the number of F-35 pilots is lower than the number of pilots in level of operational readiness  $j$  at time  $t$ ; 1 otherwise.
- $e'_t \in \{0, L\}$ —the number of F-35 pilots greater than  $A_j$  in year  $t$ .
- $e''_t \in \{0, L\}$ —the number of F-35 pilots greater than  $A_j + 2$  in year  $t$ .
- $e'''_t \in \{0, L\}$ —the number of F-35 pilots greater than  $A_j + 6$  in year  $t$ .

The model minimizes the number of F-35 pilots. In addition, if the number of pilots in some years must be more than the number of pilots required for full operational capability, we want to spread this over several years rather than

having more pilots than necessary in some years). Therefore, we impose a penalty for having more pilots than necessary, which increases as the number of pilots increases.

$$\min \left\{ \sum_{t=1}^T u_t + D \sum_{t=Y_{A_j}}^T e'_t + E \sum_{t=Y_{A_j}}^T e''_t + F \sum_{t=Y_{A_j}}^T e'''_t \right\} \quad (1)$$

$$\text{s.t. } x_t = x_{t-1} + y_t - w_t - Q_t |_{t \leq P^{\text{F16}}} - y_{t-P^{\text{F16}}} |_{t > P^{\text{F16}}} \quad t = 2, \dots, T \quad (2)$$

$$u_t = u_{t-1} \quad i = 2, \dots, 3 \quad (3)$$

$$u_t = u_{t-1} + v_t + w_{t-T^{\text{conv}}} - v_{t-P^{\text{F35}}} |_{t > P^{\text{F35}}} - w_{t-P^{\text{conv}}} |_{t > P^{\text{conv}} + T^{\text{conv}}} \quad t = 4, \dots, T \quad (4)$$

$$v_t \geq V^{\min} \quad t = 4, \dots, T \quad (5)$$

$$u_t \geq A_j \quad t = Y_{A_j}, \dots, T \quad (6)$$

$$\sum_{t=1}^T w_t \geq W^{\min} \quad (7)$$

$$\sum_{t=1}^T w_t \leq W^{\max} \quad (8)$$

$$H^{\text{new}}(v_{t+1} + v_{t+2}) + H^{\text{conv}}(w_t + w_{t-1}) \leq C_t \quad t = 2, \dots, T \quad (9)$$

$$b'_t A^\theta \leq u_t \quad t = 2, \dots, T \quad (10)$$

$$u_t + 1 \leq A^\theta + M b'_t \quad t = 2, \dots, T \quad (11)$$

$$w_t + M b'_t \leq M \quad t = 2, \dots, T \quad (12)$$

$$b''_t A_j \leq u_t \quad t = 2, \dots, T \quad (13)$$

$$u_t + 1 \leq A_j + M b''_t \quad t = 2, \dots, T \quad (14)$$

$$y_t + M b''_t \leq M \quad t = 2, \dots, T \quad (15)$$

$$a'_t + a''_t \geq A^\theta \quad t = 2, \dots, Y_{A_j} \quad (16)$$

$$j c_{jt} \leq a'_t \quad t = 2, \dots, T, j = 2, \dots, J \quad (17)$$

$$A_j c_{jt} \leq x_t \quad t = 2, \dots, T, j = 2, \dots, J \quad (18)$$

$$j d_{jt} \leq a''_t \quad t = 2, \dots, T, j = 2, \dots, J \quad (19)$$

$$A_j d_{jt} \leq u_t \quad t = 2, \dots, T, j = 2, \dots, J \quad (20)$$

$$u_t - A_j \leq e'_t \quad t = Y_{A_j}, \dots, T \quad (21)$$

$$u_t - A_j - 2 \leq e''_t \quad t = Y_{A_j}, \dots, T \quad (22)$$

$$u_t - A_j - 6 \leq e'''_t \quad t = Y_{A_j}, \dots, T \quad (23)$$

$$x_1 = N^{\text{F16}} \quad (24)$$

$$y_1 = 0 \quad (25)$$

$$u_1 = N^{\text{F35}} \quad (26)$$

$$v_t = 0 \quad t = 1, \dots, 3 \quad (27)$$

$$w_1 = 0 \quad (28)$$

- Constraint set (2) tracks the number of F-16 pilots each year.
- Constraint set (3) initializes the number of F-35 pilots in years 2 and 3 to make it equal to the number of F-35 pilots in year 1. For most scenarios considered (and those reported in this paper), the earliest time that new F-35 pilots can be fully operational is year 4.
- Constraint set (4) tracks the number of F-35 pilots each year starting in year 4.
- Constraint set (5) requires the number of new F-35 pilots each year to be at least  $V^{\min}$ .
- Constraint set (6) ensures the number of F-35 pilots each year remains at full operational capability (Level 5) starting in year  $Y_{A_i}$ .
- Constraint set (7) is the lower limit on the number of converted pilots.
- Constraint set (8) is the upper limit on the number of converted pilots.
- Constraint set (9) restricts the capacity at the PTC. In a given year  $t$ , the pilots who are training at the PTC are pilots starting conversion in year  $t$ , pilots starting conversion in year  $t-1$ , new pilots who will be combat ready in year  $t+1$ , and new pilots that will be combat ready in year  $t+2$ .
- Constraint sets (10)–(12) ensure that no pilots are converted after the F-35 has reached the minimum level of operational readiness.
- Constraint sets (13)–(15) ensure that no new F-16 pilots are converted after the number of F-35 pilots has reached that required for full operational capability.
- Constraint set (16) ensures the readiness level of the F-16 and the F-35 together satisfy the minimum level of operational readiness.
- Constraint sets (17)–(20) link the  $x$  and  $u$  variables with  $a'_i$  and  $a''_i$ .
- Constraint sets (21)–(23) determine different levels of F-35 pilots that exceed the number needed for full operational capability (Level 5).
- Constraint sets (24)–(28) initialize some of the variables.

## References

- AIMMS (2014) AIMMS modeling guide—Employee training problem. Accessed August 25, 2014, [http://main.aimms.com/aimms/download/manuals/aimms3om\\_employeetraining.pdf](http://main.aimms.com/aimms/download/manuals/aimms3om_employeetraining.pdf).
- Baumgarten PB (2000) Optimization of United States Marine Corps officer career path selection. Accessed May 9, 2016, <http://calhoun.nps.edu/handle/10945/32952>.
- Brucker P, Qu R, Burke E (2011) Personnel scheduling: Models and complexity. *Eur. J. Oper. Res.* 210(3):467–473.
- Castillo-Salazar JA, Landa-Silva D, Qu R (2016) Workforce scheduling and routing problems: Literature survey and computational study. *Ann. Oper. Res.* 239(1):39–67.
- Daniel JM (1967) A planning model for the optimal utilization of manpower resources. Accessed May 9, 2016, <http://calhoun.nps.edu/handle/10945/12841>.
- Davidson P (2011) Robust analysis of the joint strike fighter integrated training center pilot scheduling. Master's thesis, Naval Postgraduate School, Monterey, CA.
- Holloway SV (2010) Optimizing Marine Corps pilot conversion to the Joint Strike Fighter. Accessed May 9, 2016, <http://calhoun.nps.edu/handle/10945/5285>.
- Holz BW, Wroth JM (1980) Improving strength forecasts: Support for Army manpower management. *Interfaces* 10(6):37–52.
- IBM (2014) CPLEX optimizer. Accessed May 9, 2016, <http://www-01.ibm.com/software/integration/optimization/cplex-optimizer/>.
- Jasperson KS (1999) Scheduling aircrew training at United States Navy fleet readiness squadron HC-3 during replacements of H-46D helicopters by CH-60S. Master's thesis, Naval Postgraduate School, Monterey, CA.
- Matar YM (2012) Optimizing Marine Corps maintenance personnel conversion to the Joint Strike Fighter. Accessed May 9, 2016, <http://calhoun.nps.edu/handle/10945/6828>.
- Teter MD, Newman AMA, Weiss M (2015) Consistent notation for presenting complex optimization models in technical writing. *Surveys Oper. Res. Management Sci.* Forthcoming.
- Wang J (2005) A review of operations research applications in workforce planning and potential modelling of military training. DSTO Systems Science Laboratory, Edinburgh, Australia. Accessed January 20, 2016, <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.528.621&rep=rep1&type=pdf>.

## Verification Letter

Arne Petter Bartholsen, Project Manager, Norwegian Defence Research Establishment, 2027 Kjeller, Norway, writes:

“The Norwegian Defence Research Establishment (FFI) has developed an optimization model for studying training schemes for pilots in the Royal Norwegian Air Force in the transition phase between F-16 and F-35 combat aircraft. This work led to important input to the analysis that FFI did to support the Norwegian Ministry of Defence and the Norwegian Defence Logistics Organisation in the acquisition process of the new combat aircraft. The model was time-saving, and led to useful insights about the problem. Also, it made the results of our analysis more reliable.”

**Maria Fleischer Fauske** is a senior scientist at the Norwegian Defence Research Establishment (FFI). She holds a Master of Science in Information and Communications Technology from the Norwegian University of Science and Technology, where she specialized in operations research, including optimization. She has several years of experience in long-term defence planning from FFI.

**Erlend Øby Hoff** is a principal scientist at the Norwegian Defence Research Establishment (FFI). He holds a Master of Science in Physical Chemistry from the University of Oslo. At FFI he has specialized both in cost analysis and military operations research, where he has worked on projects related to defence planning, mainly in the land and air domain. He has had a leading role in Norwegian F-35 cost analysis and worked on various F-35 projects since 2004. He has also co-chaired a NATO study on cost-efficiency implications of international cooperation.