

# Comparison of two methods for automatic range alignment in ISAR imaging

Tor Berger, Steffen Tollisen and Svein-Erik Hamran  
Forsvarets forskningsinstitutt (FFI)  
P.O. Box 25  
NO-2027 Kjeller  
Norway

tor.berger@ffi.no / steffen.tollisen@ffi.no /  
svein-erik.hamran@ffi.no

**Abstract**— A maximum likelihood algorithm for translational motion estimation (TME) in ISAR imaging is described and compared with prominent point processing (PPP). It is suggested that the kurtosis (fourth order central moment) of averaged range profiles can be used to decide on which of the two methods that gives the best results in terms of image sharpness. Experimental data of a small aircraft is collected and processed in short intervals. Results from the PPP and the TME algorithms are compared, automatically in terms of a sharpness measure, and also by user inspection. The results indicate that for low kurtosis regions the TME method should be used, while for high kurtosis regions, the PPP method is preferred.

## I. INTRODUCTION

In inverse synthetic aperture radar (ISAR) imaging the rotation of a target relative to the radar is the basis for producing the reflectivity images of the target. In essence ISAR is conceptually equal to spotlight SAR when imaging algorithms are considered [1], and many of the same algorithms apply in both cases. In an ideal situation the target would rotate around a fixed point in space, and the ISAR image is formed by applying a two dimensional Fourier transform to the observations. If the rotation of the target is small, i.e. within a few degrees, and the extent of the target is limited, i.e. in the order of meters, we may approximate the observed data in the wave number domain to be on a rectangular grid, meaning that the two dimensional Fourier transforms may be replaced by two dimensional Fast Fourier Transforms (FFTs). By applying 2D FFTs the ISAR images can be produced within a reasonable amount of processing time, and we also denote this as Range-Doppler imaging. In a practical situation, however, the turntable assumption is not always valid, as the target in general also experiences translational motion. In ISAR systems, tracking radars aim to compensate for the translational motion by adjusting the data collection window to always include the target. To be able to produce well focused ISAR images, the phase of the received data must be known to within approximately  $\pi/4$  or better [1]. The consequence of this constraint is that the translational motion must be known to within a fraction of a wavelength, typically  $\lambda/16$  or better.

One easy way to focus the data is to look for prominent scatterers, i.e. scatterers that show up clearly in the range profiles, and survive throughout the observation period. If the origin is a single point on the target and we force the corresponding range cell to have zero phase, we place the point at zero Doppler. If we also adjust the phase at all other range bins with the conjugate of the phase found at the prominent range cell, we apply the same phase adjustments to the other range bins as we have done with the prominent range bin. Consequently, if a single prominent scatterer exists, this method would focus the image. One problem occurs if we have a situation where several scatterers interfere within the same range bin. Then such a method would try to force each of them to be at zero Doppler, which they all in general are not, and image smearing would result. Also, if there is no observable prominent point, image smearing would result if we try to force a point with nonzero Doppler to have zero Doppler. In these situations other methods may apply. One such method that has proven effective is the maximum likelihood (ML) method for translational motion estimation [2], [3], [4]. In the ML method two following observations are compared, and the difference in range between the two profiles is estimated by a maximum likelihood approach. The method has been shown to be effective and produce good results when a prominent point type of algorithm has not been successful. The downside to the ML method is that it has increased computational complexity compared to a prominent point method, although it can be shown to be of the same order of complexity;  $\mathcal{O}(N \log_2 N)$  for data profiles of length  $N$  [4].

## II. INVERSE SYNTHETIC APERTURE IMAGING

We assume rotation in a plane given by X and Y axes. The antenna line of sight aligns with the Y-axis, and an object is assumed to rotate at a constant angular velocity  $\Omega$  in the XY-plane. The objective is to reconstruct the object's reflectivity projected onto the XY-plane based on range information at different rotation angles. By assuming that the distance from the antenna to the target centre  $r_a$  is much larger than the

maximum extent of the target, the observed signal can be written as

$$S(f, t) \approx e^{-j\frac{4\pi f r_a(t)}{c}} \iint_{-\infty}^{\infty} \rho(x, y) e^{-j2\pi[k_x x + k_y y]} dx dy \quad (1)$$

In equation (1)  $\rho$  is the ISAR image defined at spatial coordinates  $x$  and  $y$ ,  $f$  is the (range) frequency parameter,  $t$  is the slow time parameter,  $c$  is the wave propagation speed, normally the speed of light, and the distance term  $r_a$  is the time varying translational motion. The wavenumbers are defined as

$$k_x = \frac{2f}{c} \sin(\Omega t) \quad (2)$$

$$k_y = \frac{2f}{c} \cos(\Omega t)$$

The ISAR image is obtained by applying a 2D Fourier transform to the compensated signal, i.e. after multiplying the observed signal with the conjugate of the exponential term in front of the integrals in equation (1). If the movement is small and the extent of the target is limited, the observed points in the wave number domain are approximately on a rectangular grid, meaning that a straightforward 2D FFT is applicable.

### III. PROMINENT POINT PROCESSING - PPP

The success of a prominent point type of method for focusing ISAR data depends mainly on the existence of a prominent scatterer visible throughout the observation period, and the assumption that the phase errors in the observed data is independent of range. The last assumption means that a single phase adjustment can be applied to all range bins for each slow time step. Consider now the case where a target consists of two point scatterers,  $p_0=(0,0)$  and  $p_1=(x_1, y_1)$  rotating without translational motion about  $p_0$ , and with amplitudes  $\rho_0$  and  $\rho_1$ , respectively. The observed signal including phase error  $\varphi_{err}$  is

$$S(f, t) \approx \rho_0 e^{-j\frac{4\pi f}{c} \varphi_{err}(t)} + \rho_1 e^{-j\frac{4\pi f}{c} [x_1 \sin(\Omega t) + y_1 \cos(\Omega t) + \varphi_{err}(t)]} \quad (3)$$

Having assumed independent contributions from the point scatterers, the first term of equation (3) consist only of the phase error as  $p_0$  is at the origin of the rotation, while the other term of equation (3) also includes the phase change caused by the rotation of  $p_1$  about  $p_0$ . As we know that  $p_0$  is at the origin,

the phase should be zero during the observation period, and by multiplying equation (3) by the conjugate of the first term of this equation, focused data is obtained and 2D FFT can be applied to produce the ISAR image. Since the multiplying term is constant along range, one could equally well multiply the range profiles, i.e. the 1D inverse FFT of  $S$  in equation (3) along the frequency parameter  $f$ . If the prominent point is not at the centre of the rotation, the result is shifted in the projected XY-plane according to the choice of prominent point. Introducing translational motion means that the observed signal must be multiplied by the conjugate of the first exponential term in equation (1), and further processing would be as described above. In an automated ISAR image generating algorithm there is still a question of choosing the proper prominent point, or to be more precise; choosing the range bin containing what we believe is the best prominent point. We may assume that by envelope correlation or other methods we have aligned the range profiles so that scatterers of the target do not move across resolution cells during the observation period. Then if a single prominent point exists throughout the observation period, we would expect the amplitude variations along slow time to be small compared to the case where several scatterers interfere within the same range resolution bin. By choosing the maximum of the mean to standard deviation ratio we ensure that the range bin selected contains some energy (mean value) with as little variation as possible (standard deviation). In the following, this measure is used in the automated ISAR imaging algorithm for the PPP method.

### IV. TRANSLATIONAL MOTION ESTIMATION - TME

We apply a maximum likelihood approach to estimate the translational motion from one observed radar return to the next. The method is described in [2]-[4], and we show the main parts in the following. The TME method estimates the range value  $r_a$  from equation (1) in a maximum likelihood sense. The motion of a point target at position  $(x, y)$  at slow time  $t$  is composed of a translational and a rotational motion as defined in equation (4).

$$r(t) = r_a(t) + r_\Omega(t) = r_a(t) + x \sin(\Omega t) + y \cos(\Omega t) \quad (4)$$

We look at two following radar returns, at time  $t-T$  and  $t$ , where  $T$  is the pulse repetition interval (PRI). Also, we define delta time parameters as in equation (5).

$$\Delta r_a(t) = r_a(t) - r_a(t-T)$$

$$\Delta r_\Omega(t) = r_\Omega(t) - r_\Omega(t-T) \quad (5)$$

We may write the observed signal at time  $t$  as

$$S(f, t) \approx e^{-j\frac{4\pi f}{c}\Delta r_a(t)} e^{-j\frac{4\pi f}{c}r_a(t-T)} \cdot \iint_{-\infty}^{\infty} \rho(x, y) e^{-j\frac{4\pi f}{c}r_\Omega(t-T)} e^{-j\frac{4\pi f}{c}\Delta r_\Omega(t)} dx dy \quad (6)$$

If now  $e^{-j\frac{4\pi f}{c}\Delta r_\Omega(t)} \approx 1$ , i.e. the translational motion due to rotation only is small from one radar return to the next, then equation (6) is written as

$$S(f, t) \approx e^{-j\frac{4\pi f}{c}\Delta r_a(t)} S(f, t-T) \quad (7)$$

The maximum likelihood estimate of the range difference between two radar returns at time  $t-T$  and  $t$  is the value that minimizes the measure  $J$  defined in equation (8) below.

$$J(r(t)) = \left\| S(f, t) - e^{-j\frac{4\pi f}{c}r(t)} S(f, t-T) \right\|^2 \quad (8)$$

When dealing with real data, a discrete version of the above has to be applied. We define the discrete observed signal as  $S_{m,n}$ . The bandwidth is  $B$ , divided into  $M$  frequency bins. The total observation time is  $NT$ , with  $N$  distinct observations (slow time samples). Writing out equation (8) it is easily shown that minimizing  $J$  in equation (8) is the same as maximizing the measure  $I_n$  in equation (9) [4].

$$I_n(r) = 2\text{RE} \left\{ e^{-j\frac{4\pi f_0}{c}r} \frac{1}{M} \sum_{m=0}^{M-1} S_{m,n} \bar{S}_{m,n-1} e^{-j2\pi\left(\frac{2Br}{c(M-1)}\right)m} \right\} \quad (9)$$

The sum in equation (9) is the discrete Fourier transform of the product of  $S_{m,n}$  and the complex conjugate of  $S_{m,n-1}$ . Equation (9) is in most cases more efficiently calculated by applying the Chirp Z-transform [5] utilizing its zooming abilities to perform a discrete Fourier transform on parts of the unit circle with increased data resolution. This is described in more detail in [4].

## V. CHOICE OF PROCESSING METHOD

One main question is how to choose between the PPP and TME processing methods. The PPP method is likely to perform well in the case of a few sharp peaks in the mean to standard deviation ratio. One could then think of using the standard deviation of collapsed (averaged along slow time) range profiles as a measure for choosing methods, i.e. apply the PPP method for the case of high standard deviation. This could work well in some cases, but in general the standard deviation measure would not be sensitive enough to cover the

case of one strong scatterer and other parts of the target producing return signals at a much lower level. Therefore we believe that a higher order statistical moment would be better. The fourth order central moment, also denoted *kurtosis*, of a distribution  $x$  is defined as

$$k = \frac{E[(x-\mu)^4]}{\sigma^4} \quad (10)$$

In equation (10)  $\mu$  is the mean value and  $\sigma$  is the standard deviation of the distribution  $x$ . The kurtosis is a measure of how outlier-prone a distribution is, and the kurtosis of a Gaussian distribution is 3. The kurtosis of a distribution more outlier-prone than a Gaussian is higher than 3, and vice versa for a distribution less outlier-prone than a Gaussian. Often the kurtosis is defined by subtracting 3 from equation (10), to bring the kurtosis of a Gaussian to zero. We would expect that data where the PPP method would perform well has collapsed range profiles of high kurtosis, while data where the TME method would perform better has collapsed range profiles of lower kurtosis.

In order to compare the two processing methods described in sections III and IV, we use a sharpness measure to assess the quality of the ISAR images produced by the PPP and TME methods. Several sharpness measures have been described and proposed [6], and care must be taken when using these types of metrics for image quality comparison. In general they are not scale independent and require equal conditions. The sharpness measure used here is the mean value to standard deviation ratio of the squared absolute value, or intensity, of the resulting image. For robustness it is also ensured that processing parameters and geometries are equal for the PPP and TME processing methods. What we observe when applying such a sharpness measure is to some extent a lack of consistency. In most cases the image quality as perceived by a person correlates with the sharpness measure, but in some cases an image of higher sharpness measure than another is perceived as being of lower quality when assessed by a person. For this reason a user evaluation of image quality is performed for the outputs of methods. To have a statistical sound result, one should present the images to a number of people for quality assessment. In this paper though, one person's assessment is used. The overall conclusions are therefore based on the combined results of sharpness measure evaluation and user evaluation.

## VI. COMPARISON OF PROCESSING METHODS

Data from the TIRA system at Fraunhofer FHR<sup>1</sup> has been processed. TIRA is a Ku-band radar system and the selected waveform has 0.8 GHz bandwidth. Two different segments of four minutes each were selected, and five seconds sub intervals were used for imaging. Images were produced by the two processing methods for each interval, and the overlap between images was four seconds, i.e. 80 %. For each sub interval the range profiles were aligned and the kurtosis of the collapsed range profile was calculated. The PPP and TME processing methods were both used to produce ISAR images, and the sharpness measure described earlier was calculated for both images. In Figure 1 and Figure 2 histograms are shown indicating whether PPP (red bars) or TME (blue bars) gives higher sharpness measure as a function of kurtosis value. The results shows a tendency with blue bars shifted to the left and red bars shifted to the right. This means that it is more likely that for high kurtosis values the PPP processing method produces sharper images than the TME processing method, as we expected. A few snapshots of the image sequences can be shown to illustrate the quality of both processing methods. First, a sub interval of low kurtosis is chosen. The collapsed range profiles are shown in Figure 3. The corresponding ISAR images produced by the PPP and TME processing methods are shown in Figure 4. For convenience the two images are put together in the same frame.

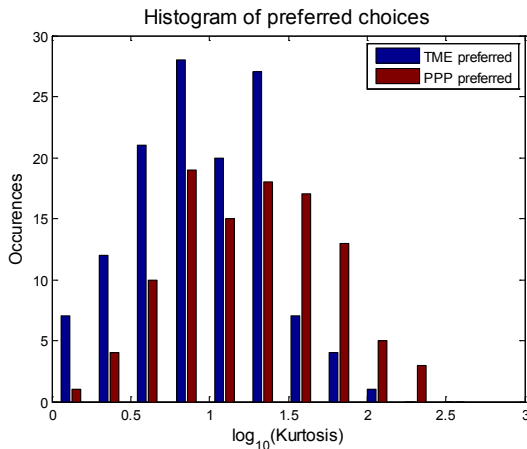


Figure 1: Histogram of preferred choices for the first interval considered. Automatic evaluation of image quality by sharpness measure.

From Figure 4 we see that the TME image is of slightly higher quality than the PPP image, and the sharpness measure also supports this.

<sup>1</sup> Fraunhofer Institut für Hochfrequenzphysik und Radartechnik located in Wachtberg, close to Bonn, Germany

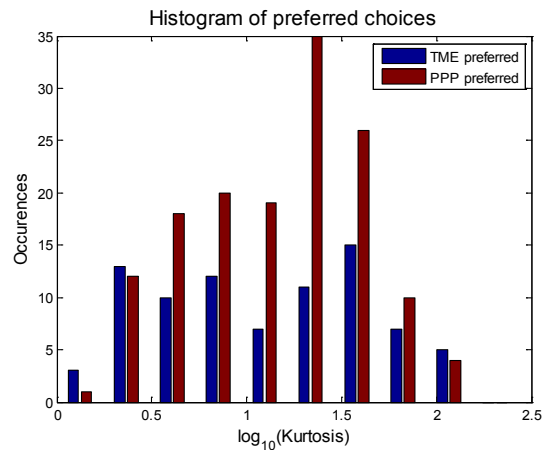


Figure 2: Histogram of preferred choices for the second interval considered. Automatic evaluation of image quality by sharpness measure.

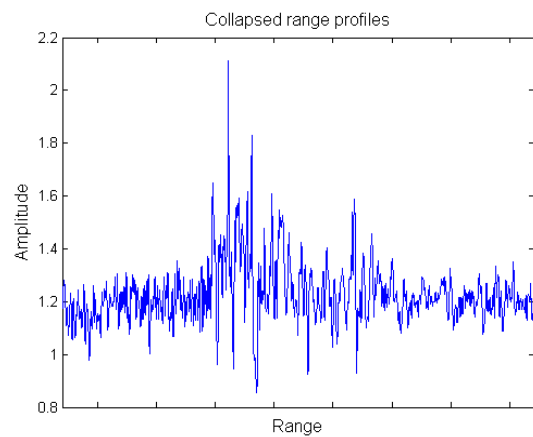


Figure 3: Collapsed range profile of a sub interval with low kurtosis value equal to 6.0.

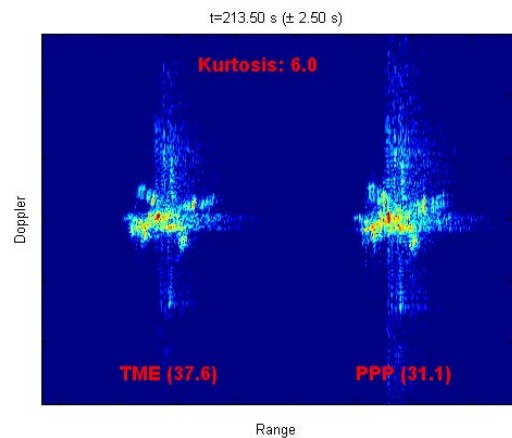


Figure 4: ISAR image results for the TME (left) and PPP (right) processing methods. Kurtosis and sharpness values are shown on the figure.

Next we look at a snapshot where the kurtosis is high (238.2). In this case we clearly see the prominent point on the collapsed range profile in Figure 5. The PPP processing method produces an image of slightly higher visual quality than the TME processing method as seen in Figure 6. Although not shown here, what we in general observe when going through hundreds of images produced is some lack of symmetry, meaning that the difference in quality of images is higher for low kurtosis regions than for high kurtosis regions. This suggests that the TME processing method is more robust than the PPP method. This would be reasonable as the PPP processing method requires a prominent point surviving throughout the observation period, while the TME processing methods has no such restrictions.

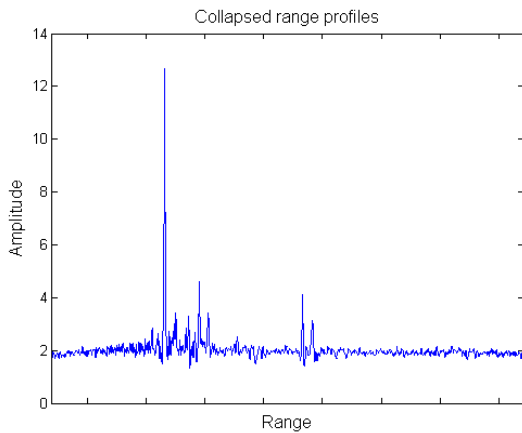


Figure 5: Range profiles (top) and collapsed range profile (bottom) of a sub interval with high kurtosis value equal to 238.2.

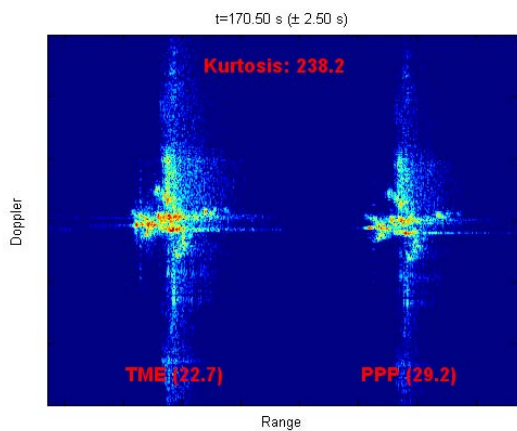


Figure 6: ISAR image results for the TME (left) and PPP (right) processing methods. Kurtosis and sharpness values are shown on the figure.

When going through the images of the two processing intervals we also note a slight inconsistency with the sharpness measure we have used. Although the visual image quality in general correlates with image sharpness measure, this is not always the case. An example of this is shown in

Figure 7. The visual quality of the image produced by the TME processing method is higher than the visual quality of the image produced by the PPP processing method. In this case the sharpness of the TME image is lower than the sharpness of the PPP image, and in an automatic evaluation the PPP is voted the best method to use. We do see in this example that the kurtosis is quite low, and the visual inspection supports our hypothesis of preferring TME to PPP.

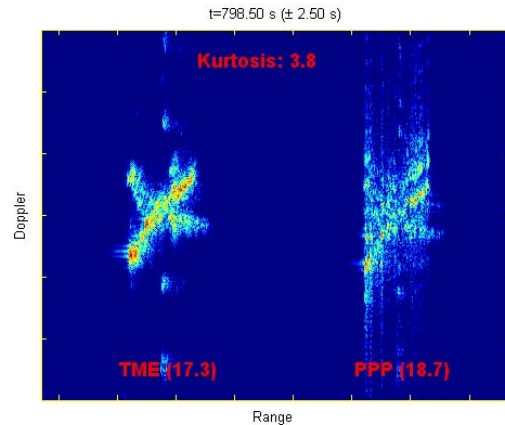


Figure 7: Example of conflict between image sharpness measure and visual quality.

The question then is how the results are changed when using a purely visual quality measure, i.e. a user inspection of image quality. We have applied user inspection to assess the quality of all the images in both intervals. Similar to the automatic evaluation the histogram results for the two intervals are shown in Figure 8 and Figure 9. We see that the red bars are shifted to the right, and the blue bars are shifted to the left. This puts further emphasis on suggesting TME for low kurtosis regions and PPP for high kurtosis regions.

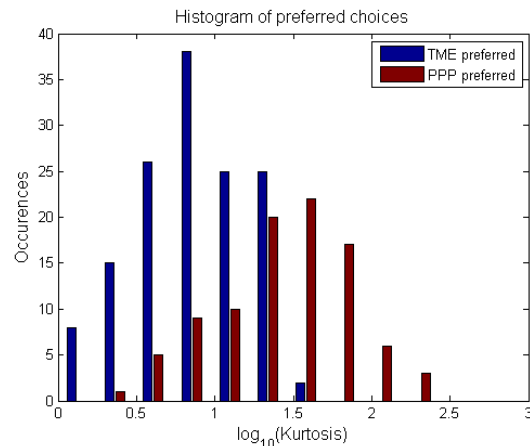


Figure 8: Histogram of preferred choices for the first interval. User evaluation of image quality has been applied.

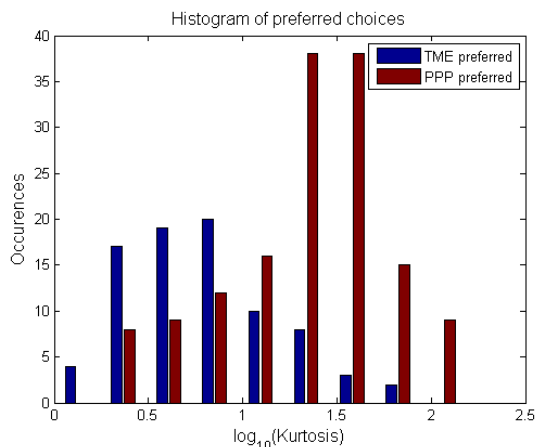


Figure 9: Histogram of preferred choices for the second interval. User evaluation of image quality has been applied.

The threshold for choosing between the two methods seems to be in the range 10-30, i.e. a value of 1-1.5 on the horizontal axis of Figure 8 and Figure 9. The results shown here suggest that for kurtosis values below about 20, the TME processing method should be applied, while for kurtosis values above about 20, the PPP processing method should be applied.

## VII. CONCLUSIONS

A kurtosis measure has been proposed as a criterion for choosing between two processing methods for ISAR range alignment. The prominent point processing (PPP) method seems to be best adapted to data where collapsed range profiles have high kurtosis, while a maximum likelihood procedure, here denoted translational motion estimation (TME), seems to work best in low kurtosis regions. The PPP is faster than the TME, and is preferred when processing speed is an issue, but the TME is more robust since it does not require strong scatterers being traceable throughout the observation period. Observations with these and other experimental data suggests that the TME produces better results on average, but when PPP works well it outperforms TME. Two simple image quality assessment methods were used to evaluate the methods; image sharpness and user inspection. Both support the hypothesis of preferring TME for low kurtosis regions and PPP for high kurtosis regions. In this context low kurtosis means kurtosis values less than about 20. By applying this criterion the total processing time can be reduced for multi image sequences, as we use the PPP method whenever we can and the TME method whenever we have to.

## VIII. ACKNOWLEDGEMENTS

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## REFERENCES

- [1] Jakowatz, Jr, C.V., Wahl, D.E., Eichel, P.H., Ghiglia, D.C., and Thompson, P.A.: 'Spotlight-Mode Synthetic Aperture Radar: A Signal Processing Approach' (Springer, New York, 1996)
- [2] Simmons, S., Evans, R.: 'A new approach to motion estimation for ISAR imaging', 1994 IEEE International Conference on Acoustics, Speech and Signal Processing, 19-22 April 1994, vol. v, pp. V/201 – V/204
- [3] Simmons, S., Evans, R.: 'Maximum likelihood autofocus of radar images', Record of the IEEE 1995 International Radar Conference, 8-11 May 1995, pp. 410-415
- [4] Berger, T., Hamran, S-E.: 'An efficient scaled maximum likelihood algorithm for translational motion estimation in ISAR imaging', 2010 IEEE Radar Conference, 10-14 May 2010, pp. 75-80
- [5] Rabiner, L. R., Schafer, R. W.: 'The Chirp z-transform algorithm', IEEE Trans. on Audio and Electro acoustics, vol. AU-17, no. 2, June 1969, pp. 86-92
- [6] Berizzi, F., Martorella, M., Haywood, E., Dalle Mese, E., and Bruscoli, S.: 'A survey on ISAR autofocus techniques' 2004 International Conference on Image Processing, 24-27 Oct 2004, vol. 1, pp. 9-12