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Optimization Model for Robust Acquisition Decisions in the Norwegian Armed Forces

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The Norwegian Defence Research Establishment (FFI) supports the Norwegian Ministry of Defence with quantitative analyses for its long-term defence planning. As part of this activity, FFI has developed a stochastic model for optimizing investments in materiel. The model maximizes the capabilities of the force structure (i.e., military personnel, equipment, and materiel) by allocating materiel acquisition projects over time, given certain constraints and considering the uncertainty of future-year budgets. The objective of the model is to make decisions about current acquisition projects, and also provide flexibility for future decisions. It models future budgets using a set of budget scenarios. The model helps decision makers explore project acquisition choices, and improve their knowledge of the total investment picture and of individual projects. The model has given FFI new insights into the dynamics of acquisition planning and the necessity for considering future uncertainty.

Key words: stochastic optimization; mixed-integer programming; materiel acquisition; budgetary uncertainty; scenario.
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The Norwegian Defence Research Establishment (FFI) has primary responsibility for defence-related research in Norway. It is the chief advisor on defence-related science and technology to the Norwegian Ministry of Defence (MoD) and to the Norwegian Armed Forces (FFI 2012). FFI supports the MoD with quantitative analyses for long-term defence planning. As part of this activity, FFI has developed an optimization model, which it calls optimization of investment decisions (OID), for materiel acquisition (i.e., investment project decisions).

The MoD develops and maintains an investment plan that contains possible future materiel acquisitions for the Norwegian Armed Forces. It carefully considers these investment projects in light of the current and future needs of the Armed Forces, and an assumed future investment budget. The MoD is ultimately responsible for making decisions about which investments to make and when to make them. In this process, FFI has an advisory role to the MoD. Thus, FFI is not a part of the decision-making process; however, our analyses give impartial information to the MoD's decision makers. The MoD also performs its own analyses, and the various services within the Armed Forces make their assessments, which they also provide as input. The MoD evaluates all inputs and requirements to ensure that it makes optimal decisions.

The objective of long-term defence planning at FFI is to identify security challenges that Norway may face in the future—in peace, crisis, or war—and, in light of this, provide guidance for a future force structure (Hennum and Glærum 2008, 2010). This approach is similar to NATO's long-term planning method (NATO 2003), although with a few national adaptations. Based on these and other analyses, and an assessment of the immediate needs of the Norwegian Armed Forces, the MoD maintains the list of desired future acquisition projects. This list typically includes hundreds of projects. Although each step in the long-term planning process has uncertainties (Birkemo 2011), the list of investment projects basically represents the projects in which the MoD wants to invest. The MoD, FFI, and (or) the military services (i.e., Norwegian Army, Navy, and Air Force) have carefully analyzed and evaluated each project;

however, all of the projects may not be affordable. The OID model addresses this budgetary uncertainty.

Investment optimization has been studied in the literature for decades. Traditional capital budgeting methods, which began with the work of Dean (1951), are not applicable to the OID model, because (1) they do not consider budgetary uncertainties, and (2) their objective of maximizing return is not relevant to our OID model. Our goal is to maximize the capabilities of the force structure. The key to meeting this objective is to make investment decisions that allow flexibility if future budget requirements differ from the assumed requirements. Military investment projects typically extend over many years. Therefore, the timing of projects, including their start dates, can have major implications for future investment possibilities.

The current acquisition plan of the MoD is based on assumptions about future investment budgets, a commonly used deterministic approach to planning. However, deterministic planning does not consider future uncertainty, making future flexibility difficult. The acquisition decisions made today are not likely the best decisions at a future time in the planning horizon. Because the MoD assumes that its budget will increase by a specific percentage each year throughout this horizon, its plan becomes infeasible if the budget increases are smaller than assumed. Consequently, the MoD might have to pause, postpone, or stop some projects, including acquisitions that services within the Armed Forces expect and consider important.

The OID model maximizes the capabilities of the force structure by allocating acquisition projects over time, given specific constraints, taking into account the lack of advanced knowledge about future budgets. Given the great number of desired acquisition projects, solving this problem should help us to know which investments to make today to maximize the long-term capabilities of the force structure in the face of future budget uncertainty. The OID model uses a stochastic approach to the problem; its goal is to help us make decisions that are good for today, but allow us enough flexibility to also be able to make good decisions in the future.

The OID model is related to the traditional knapsack problem (Martello and Toth 1990), in which the goal is to fill a knapsack with objects, constrained by the volume of the knapsack. The complexity of such a model increases when we add a time factor and consider uncertainty, as the OID model does.

Planning Under Uncertainty

Some practitioners do not believe that deterministic planning problems exist; that is, they believe that planning and uncertainty are closely related. The concept of stochastic models is that we must consider that we will attain new knowledge over time that we can use to make better decisions. Therefore, we must be able to amend these decisions as this new information becomes available. In stochastic models, a probability is allocated to different scenarios, and the optimal solution is the one that gives the best expected value, given that any of the scenarios may occur (Wallace 1998).

The objective of the OID model is to optimize the expected value of the acquisition plan, where value represents the capabilities of the force structure, considering that a large set of possible future budget scenarios exists. This can allow us to make investment decisions today, but change course if our budget assumptions prove to be incorrect. This is a new way of thinking for the defence planners in Norway—at FFI, the MoD, and the Armed Forces. To date, we have correctly assumed that our budgets would increase each year, and the OID model has been an important factor in changing that way of thinking (i.e., our assumption that our budget will always increase might be incorrect).

Markowitz (1952) introduced the concept of portfolio optimization (i.e., optimal project selection when costs or revenues are uncertain) and showed how to minimize portfolio risk. Orman and Duggan (1999) show how Markowitz's theories have evolved into modern portfolio optimization. In the OID model, return, per se, is not an issue. Materiel investments represent capabilities that are valuable to us. However, we do not consider any risk associated with this value. Risk considerations are part of the preceding analyses in the long-term planning process, and the list of desired projects or capabilities is a result of methodically considering our requirements and desired projects. We must determine which of these projects we can afford.

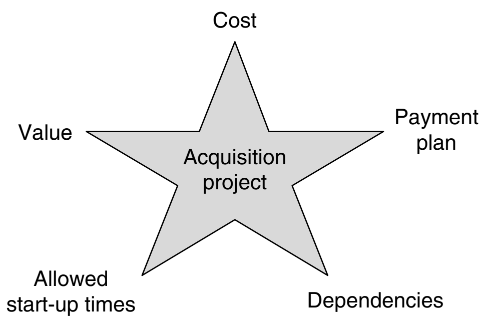

Figure 1: The OID model considers multiple data items for evaluating each investment project.

Model Design and Framework

In this section, we describe the elements that comprise the OID model and outline the analyses for which the model is applicable. The appendix shows the mathematical formulation of the model.

Acquisition Projects

A set of data is associated with each acquisition project (see Figure 1). Each project has a cost, and a payment plan that describes how the cost is distributed over time. Each project has some value to us; this value represents the acquired materiel's capabilities. Given a rational acquisition strategy, a unit's monetary value is a proxy for its capabilities. Thus, we say that a project's value is equal to the sum of its costs. The project has value to us each year from the time the materiel is acquired.

The projects may also be weighted relative to each other; therefore, we adjust the value based on the project's utility (i.e., importance). For each project, we must define a first and last allowable start-up time. Within this time interval, constraints in the model allow it to determine when the project should start. A project may have dependencies; it may depend on other projects and other projects may depend on it. All such relationships must be given as input to the model.

Investment Budget

In theory, an infinite number of budget scenarios exists; therefore, we must select a set of those scenarios. In stochastic programming, we commonly generate scenarios based on historical data, mathematical and (or) statistical models, and expert opinions, and we frequently use a combination of these methods. In the OID model, scenarios could be generated based on expert opinion. However, experts have difficulty in considering budgets beyond a few years into the future. Therefore, using a combination of mathematical or algorithmic methods for the later years may be necessary. Thus, we also developed a fully algorithmic method for scenario generation. For simplicity, we say that in each year, the budget may either (1) increase by a specific percentage, or (2) not increase at all, and we give the probability of each of these two possible options as input to the model. This allows us to view the scenario structure as a binary tree (see Figure 2); this example shows year 0 as the base year and three additional years—year 1 to year 3. Thus, it generates the following eight (i.e., 2^3) scenarios: (1) flat, flat, flat; (2) flat, flat, increase; (3) flat, increase, flat; (4) flat, increase, increase; (5) increase, flat, flat; (6) increase, flat, increase; (7) increase, increase, flat; and (8) increase, increase, increase.

In long-term planning at FFI, we look about 20 years forward in time. With only two budget possibilities each year and a period of 20 years (i.e., year 0 to year 19), we generate 2^{19} scenarios. However, solving a model that has so many scenarios is impossible because the solution would consume a huge amount of computer memory. Thus, we must reduce the number of scenarios to a manageable size, as we describe next.

We always select the best and worst scenarios in the scenario tree as the first two scenarios. As the third scenario, we choose the one whose budget value in the final year is closest to the average value of all the budget scenarios. As the fourth scenario, we pick the opposite scenario, and call the third and fourth scenarios symmetric. For example, "flat, flat, increase" and "increase, increase, flat" scenarios are opposite (i.e., symmetric). Having selected these four scenarios, we randomly choose, in symmetric pairs, the remainder of the scenarios to be used in the model run. Thus, these scenarios will vary from one run to another.

We always choose symmetrical pairs because spanning the scenario space is important; we want our solutions to involve both the best and the worst budget scenarios to allow us to argue for the robustness of the solution.

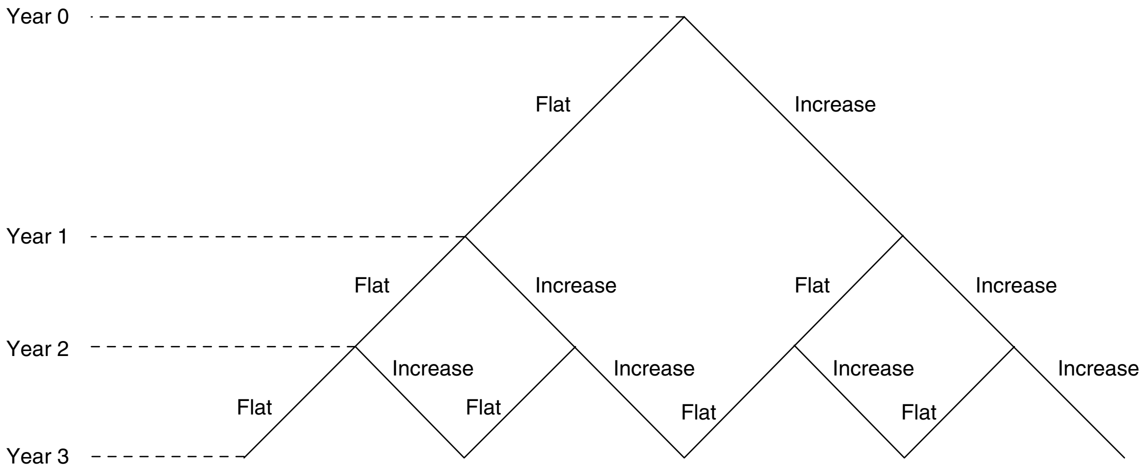

Figure 2: The binary tree depicts scenarios for yearly budget changes from year 0 to year 3.

Objective Function

The model should maximize the expected capabilities of the acquired materiel. In addition, we must ensure that the model does not place investments randomly along the timeline, but places them as close as possible to their earliest allowable start-up times; this is more valuable to us because it optimizes the force structure's capabilities. The objective function must represent these two goals.

The expected value of the acquired materiel equals the sum of the expected value of each acquisition project. A project's value the year that it begins is equal to the sum of its costs. For each subsequent year, the value decreases based on the rate of depreciation. We calculate the project's expected value by multiplying its total value by the probability that the project will start. If a project is selected in each scenario, it has a probability of 100 percent. To ensure that projects begin as early as possible, we adjust each project's value by a number between zero and one raised to the power of the year the materiel is acquired. This ensures that the model will choose to acquire the materiel as early as it can afford it. Otherwise, the model would place acquisitions randomly along the timeline.

The projects in a project portfolio may have different levels. They are assigned weights in accordance with their importance; the lowest level represents investments in the projects that we deem to be the most important. We invest in projects based on (1) the project's weighted value, and (2) the allocated budget. We assign the level when we enter the input data. Projects with the lowest level are assigned the highest weight; thus the model tends to choose these projects before it selects projects with higher levels.

Constraints

The OID model includes several constraints that limit the solution space:

- The budget cannot be exceeded by more than a set percentage in any year or in any scenario.
- Over a preset period of years, each budget must average out such that a budget overrun in any given year is offset by budget shortfalls in other years. This constraint ensures that the model represents any dynamics in the budget and payment plans.
- Two budget scenarios that have equal costs up to a given year must also have equal investment decisions up to that year.
- A project cannot begin before the earliest allowed start-up time, or after the latest allowed start-up time.
- If a project has a prerequisite project, the time between the start dates of these two projects must be a minimum number of years. Thus, a particular project cannot be selected if its prerequisite project has not been selected.
- A project may require another project to begin a maximum number of years after it has begun.

- Two projects may be mutually exclusive. That is, only one of the projects can be selected; neither project may be selected if the other project has been selected.
- A project may be defined as mandatory; this project must be chosen.
- A project can be started only once in each scenario.

Possible Analyses

The OID model can be used to look at the acquisition decision problem in various ways, and it may answer multiple questions. The most important question relates to the investments that should be made now. The model provides a list of such acquisitions. It may select some acquisition projects in most or all scenarios. These are probably robust acquisitions; by investing in these projects, we will probably not confine ourselves to an inflexible path.

Finding Robust Acquisition Projects

The MoD's acquisition plan (PPM) is based on an assumption of a specific budget increase each year. The PPM is infeasible if the budget does not increase or increases by a smaller amount than the MoD assumed. In this section, we show how the OID model's decisions differ from those of the PPM. The OID model finds good decisions, even when the budget increase is flat. However, these decisions could have consequences. Next, we show an example based on data from the PPM and from FFI's databases; note that because the details about materiel, acquisition projects, and their costs are confidential, we cannot include them in this paper. Instead, we provide general information with as much detail as our confidentiality requirements allow.

Because the model uses random scenarios, each model run is unique. To show the model's stability, we run it several times, and report the differences in the decisions and the objective value. We solved the model at FFI using a software tool based on IBM ILOG CPLEX, and used ILOG CPLEX Concert technology for Java to implement the model. Using this technology, the model is formulated within a Java application and Java calls CPLEX directly.

Acquisition Projects: Example

This example includes 189 acquisition projects; based on their sizes, we categorize them as large, medium, and small. Each of the 33 large projects has total investment costs that are higher than NOK 2.0 billion ($333 million). Each of the 31 medium-sized projects has investment costs between NOK 0.5 billion and NOK 2.0 billion ($83 million and $332 million). Each of the 125 small projects has investment costs of less than NOK 0.5 billion. In year 0, the investment budget is about NOK 8.5 billion ($1.5 billion). We must spend a large part of this budget on projects that are already in operation.

We ran the OID model five times for each of 16, 32, and 64 scenarios that the budget selection algorithm, which we describe previously in the *Investment Budget* subsection, selected. The period in this example is 20 years; therefore, the scenarios were chosen from a set of 2^{19} scenarios in which the budget either did not increase or increased by 1.5 percent in each year. We set the probability of each possibility to 50 percent for each year. We stopped the runs after 900 seconds or when we obtained an EpGap of 1.00 percent. The differences between the best and worst objective values obtained were:

- 0.42 percentage points in the series of runs with 16 scenarios;
- 0.34 percentage points in the series of runs with 32 scenarios;
- 0.64 percentage points in the series of runs with 64 scenarios;
- 2.05 percentage points in all the runs.

Because the model is an integer optimization problem, it will probably not find the exact optimal solution within a reasonable time. Therefore, we instruct it to stop when it finds a solution that is within a preset percentage (i.e., gap limit) of the optimal solution. For example, if we set the gap limit to 0.5 percent, then the model stops when it finds a solution that is proven to be within 0.5 percent of the optimal solution. When we ran the model using 16 scenarios, the model reached the gap limit within our time limit (i.e., 900 seconds) in each of the five runs. Using 32 scenarios, the model reached the gap limit in two of the five runs; in the other three runs, the gap was just above 1 percent. Using 64 scenarios, the model reached the gap limit once; in the other four runs, the gap was between 1.5 and 2.7 percent. In all runs, except two, over 99 percent of the budget was used in year 0. In the two runs, about 98.5 percent was used.

Comparing the OID's decisions in year 0 to those in the PPM, we observed the following in all of the model runs.

- In year 0, the OID did not select two medium and two small projects that the PPM had selected.
- In year 0, the OID selected two medium and two small projects that the PPM had not selected.

Additionally, in each run, two to four small projects differed in the OID model and the PPM; however, in the 15 runs, the specific projects differed.

We could not find any specific pattern in the project decisions across the runs using different numbers of scenarios. We saw that the objective value increased when the number of scenarios decreased; our explanation is that because the model is easier to solve with fewer scenarios, we obtain a better value within the given time limit.

Conclusions

The OID model provides information about the selection of good decisions today—given that future budgets are uncertain—and shows that these decisions influence future investment possibilities. The OID model has given FFI management new insights into the dynamics of acquisition planning, and demonstrated that it should not overlook future uncertainty in making investment decisions. The model has shown how the timing of projects can have a large impact on investment possibilities, and that small variations in acquisition choices may provide flexibility for the choices that must be made in the future.

The OID model represents a new way of thinking for FFI, the MoD, and the Armed Forces. It has been a part of one long-term defence planning cycle. Although the MoD is solely responsible for making decisions about acquisition strategies, it highly regards FFI's analyses and considers the results of these analyses an important part of its decision criteria.

Working with the investment data, adapting these data, and organizing them to be applicable as input to the OID model, are time-consuming tasks. However, they force the decision makers to carefully evaluate each potential investment project. This exercise is valuable because it raises the decision maker's knowledge about each investment, its importance, its dependencies on other investments and the dependencies of other investments on it, and the stringency associated with the timing of an investment. By introducing stochastic thinking to decision makers, their awareness of future uncertainty will improve, helping them to understand how future budgets might affect possible choices and decisions. This is a powerful argument for using the OID model.

Appendix. Model Formulation

Input Parameters

T: number of years.
S: number of budget scenarios.
I: number of investment projects.
C_{it}: investment costs for project i, the tth year after start up.
V_{it}: operational value of project i, the tth year after start up.
FT_i: the earliest allowed start-up time for project i.
LT_i: the latest allowed start-up time for project i.
E_{ij}: minimum number of years project j must start before project i starts.
L_{ij}: maximum number of years project j can start after project i starts.
K_{st}: budget size in year t under budget scenario s.
P_{st}: probability of being in scenario s after t years.
D: number where $0 \leq D \leq 1$.
R: percentage of the budget size that gives the allowed budget overrun each year.
A: the number of years that the budget must be equalized if the budget is exceeded in any year.
M: a large number.
E_i: the set of all projects that must be started before project i can be started.
L_i: the set of all projects that must be started after project i, if project i has started.
G_i: the set of all projects that are mutually exclusive to project i.
I_A: the set of all mandatory projects.
S_t: the set of scenarios where each is the lowest scenario in the group of scenarios that have the same budget to some point in time t.
S_t: the set of scenarios that have the same budget as scenario s up to time t.

The model's decision variables are:

$$x_{ist} = \begin{cases} 1, & \text{if investment project } i \text{ is started under budget scenario } s \text{ in year } t \\ 0, & \text{else,} \end{cases}$$

y_{st} = budget overrun at time t in scenario s.

The model's formulation, with its objective function and constraints, is:

$$\max \sum_{i=1}^{I}\sum_{s=1}^{S} V_i \sum_{t=1}^{T} x_{isn} D^t P_{st} \tag{1}$$

$$\text{s.t.}\quad x_{ist} = x_{is't} \qquad i=1,\ldots,I,\ s\in S_t,\ s'\in S_{ts},\ t=0,\ldots,T, \tag{2}$$

$$\sum_{i=1}^{I}\sum_{\tau=1}^{t} C_{i(t-\tau)} x_{is\tau} - y_{st} \le K_{st} \qquad s=1,\ldots,S,\ t=0,\ldots,T, \tag{3}$$

$$\sum_{\tau=t}^{t+A-1} y_{s\tau} \le 0 \quad s=1,\ldots,S,\ t=0,\ldots,T-A+1, \tag{4}$$

$$\sum_{t=1}^{T}(t-M)x_{ist} \ge \sum_{t=1}^{T} t x_{jst} + E_{ij} - M \qquad i=1,\ldots,T,\ s=1,\ldots,S,\ j\in E_i, \tag{5}$$

$$\sum_{t=1}^{T} x_{ist} \le \sum_{t=1}^{T} x_{jst} \quad i=1,\ldots,I,\ s=1,\ldots,S,\ j\in E_i, \tag{6}$$

$$x_{ist} \le \sum_{\tau=t}^{\min\{t+L_{ij},\,T\}} x_{js\tau} \qquad i=1,\ldots,I,\ s=1,\ldots,S,\ t=0,\ldots T,\ j\in L_j, \tag{7}$$

$$\sum_{t=FT_i}^{LT_i} x_{ist} = 1 \quad s=1,\ldots,S,\ i\in I_A, \tag{8}$$

$$\sum_{t=1}^{T} x_{ist} + \sum_{t=1}^{T} x_{jst} \le 1 \qquad i=1,\ldots,I,\ s=1,\ldots,S,\ j\in G_i, \tag{9}$$

$$x_{ist} = 0 \qquad i=1,\ldots,I,\ s=1,\ldots,S,\ t=0,\ldots,FT_i-1, \tag{10}$$

$$x_{ist} = 0 \qquad i=1,\ldots,I,\ s=1,\ldots,S,\ t=LT_i+1,\ldots,T, \tag{11}$$

$$\sum_{t=1}^{T} x_{ist} \le 1 \quad i=1,\ldots,I,\ s=1,\ldots,S, \tag{12}$$

$$x_{ist} = \{0,1\} \qquad i=1,\ldots,I,\ s=1,\ldots,S,\ t=0,\ldots,T, \tag{13}$$

$$y_{st} \le K_{st} R. \tag{14}$$

The objective function in (1) maximizes the expected value of the project portfolio. Constraint (2) says that for two budget scenarios that have equal development up to some point in time, investment decisions up to this point in time must also be equal. Constraint (3) ensures the budget is not overrun by more than y_{st} each year. Constraint (4) ensures that the budget overrun is equalized over a period of A years. Constraint (5) says that a project i that depends on a project j to start first, must start at least E_{ij} years after project j. Constraint (6) specifies that project i cannot start if project j has not started first. Constraint (7) says that if project i requires project j to start at a later time, then project j can, at the latest, start L_{ij} years after project i. Constraint (8) ensures that all mandatory projects are started in each scenario. Constraint (9) says that two mutually exclusive projects cannot both be started. Constraints (10) and (11) specify that projects cannot start before the first allowed start-up time or after the last allowed start-up time. Constraint (12) ensures that a project can only be started once in each scenario. Constraint (13) defines the variables x_{ist} as binary. Constraint (14) specifies that the variables y_{st} cannot be larger than R percent of the budget in any given year.

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Verification Letter

Espen Berg-Knutsen, Director of Research, Norwegian Defence Research Establishment, N-2027 Kjeller, Norway, writes:

"The Norwegian Defence Research Establishment (FFI) developed the OID model as a part of its long-term defence planning activities. The purpose of the model is to take future budgetary uncertainty into account in the investment planning process. The model gives us insight into the robustness of the proposed investment plan, and can be used to investigate different solutions. FFI has an advisory role toward the Norwegian Ministry of Defence, who ultimately makes all investment decisions. The OID model is one of several tools we use in long-term defence planning at FFI."

Maria Fleischer Fauske is a senior scientist at the Norwegian Defence Research Establishment (FFI). She holds a Master of Science degree in information and communications technology from the Norwegian University of Science and Technology, where she specialized in operations research, including optimization. She has several years of experience in long-term defence planning from FFI.

Magnar Vestli holds a Master of Science degree in industrial economics and technology management from the Norwegian University of Science and Technology, where he specialized in managerial economics and operations research. From 2009 to 2012 Vestli worked as a scientist at the Norwegian Defence Research Establishment, supporting the Ministry of Defence's long term planning. Since 2012 Vestli has worked at Pöyry Management Consulting (Norway), developing power market models.

Sigurd Glærum graduated from the University in Oslo with a PhD in applied mathematics in 1994. He has subsequently worked for the Norwegian Defence Research Establishment and—between 2000 and 2005—for the NATO C3 Agency (presently NCI Agency) before returning to FFI. He is project manager for a number of projects supporting the Norwegian MoD in its long-term defence planning. Fields of research for these projects include: capability based planning, scenario development and analysis, cost analysis, and Russian studies.